

# Shape-Based Analysis of Image Ensembles

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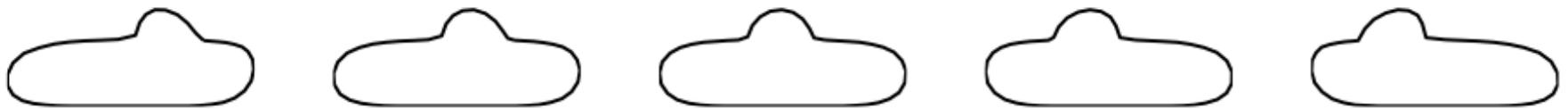
University of Utah

# Shape and Pathology



# Quantitative Shape Analysis

- What is the mean of these shapes?



- Quantify variability



# Historical Perspective

- D'Arcy Thompson, On Shape and Form, 1917

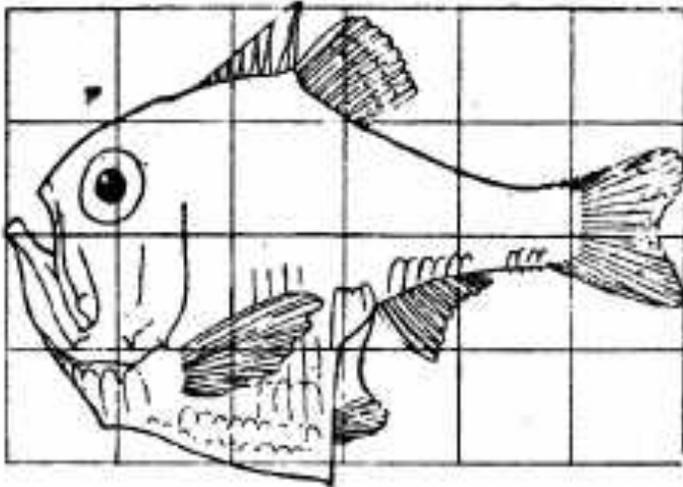


Fig. 517. *Argyropelecus Olfersi*.

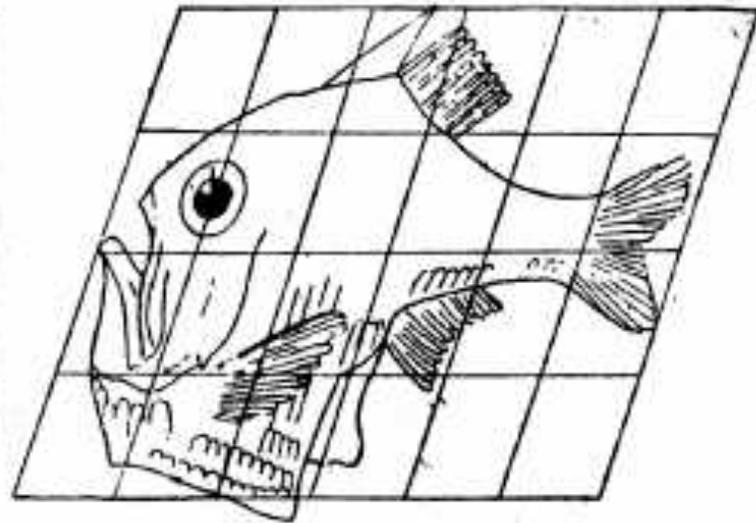
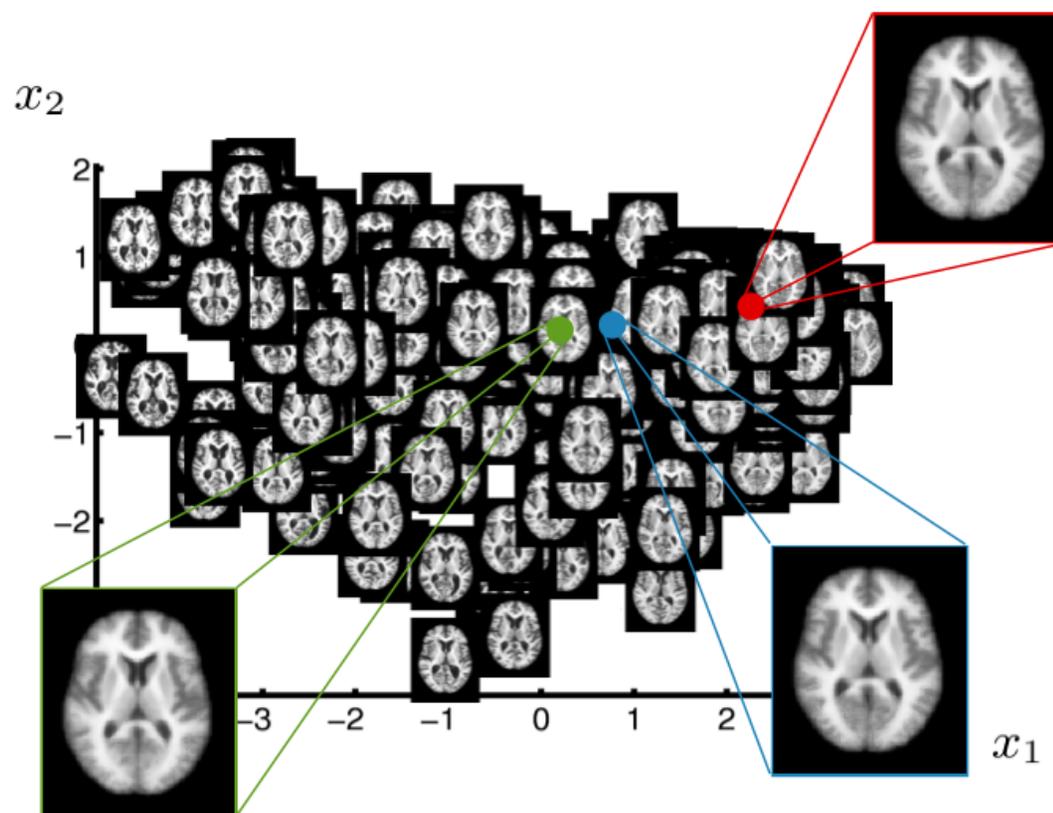


Fig. 518. *Sternoptyx diaphana*.

What do we do with shape  
statistics?

# Big Data, Images, Shapes

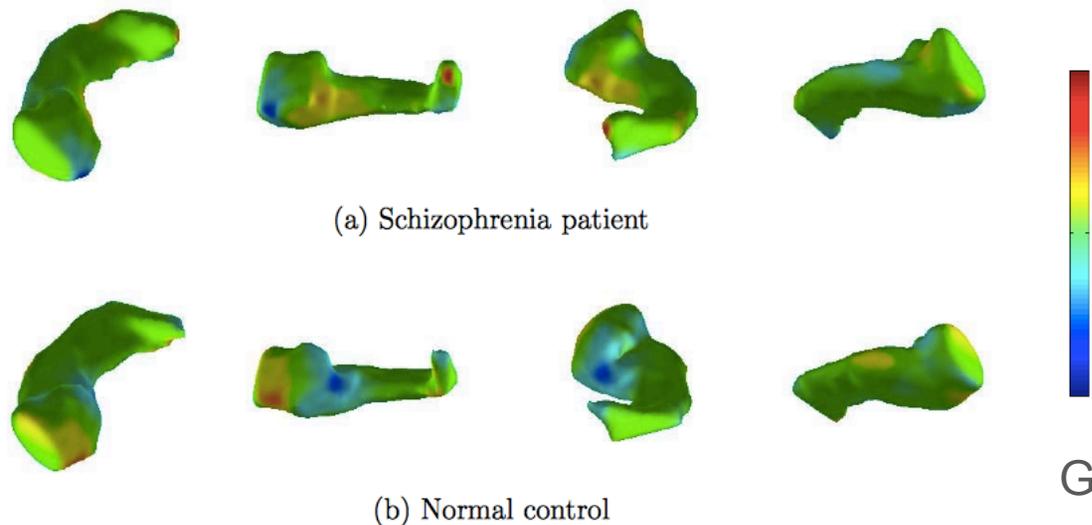
- Build tools for statistical analysis of shape and operate on large databases of images
- E.g. ADNI – Alzheimer's Disease Neuroimaging Initiative



Greber et al., 2010

# Shape Analysis – Statistical Comparisons

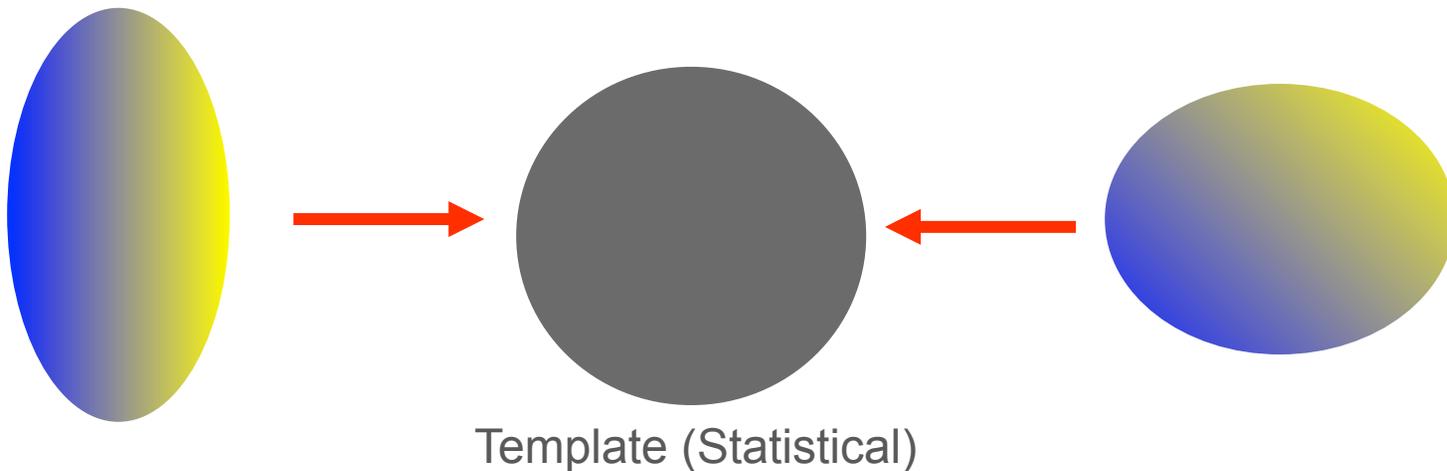
- Hypothesis testing on ensembles of shapes
  - Does one group demonstrate a different morphology?
  - What does that mean?



Golland, 2001

# Shape Analysis – Common Coordinate System

- Alignment of different organisms to template
- Compare other values
  - E.g. mechanical stresses or field strengths



# Shape Analysis – Segmentation/ Detection

- Shape leads to “appearance”
- Bayesian formulation

$$P(S|I) \propto P(I|S)P(S)$$

Posterior

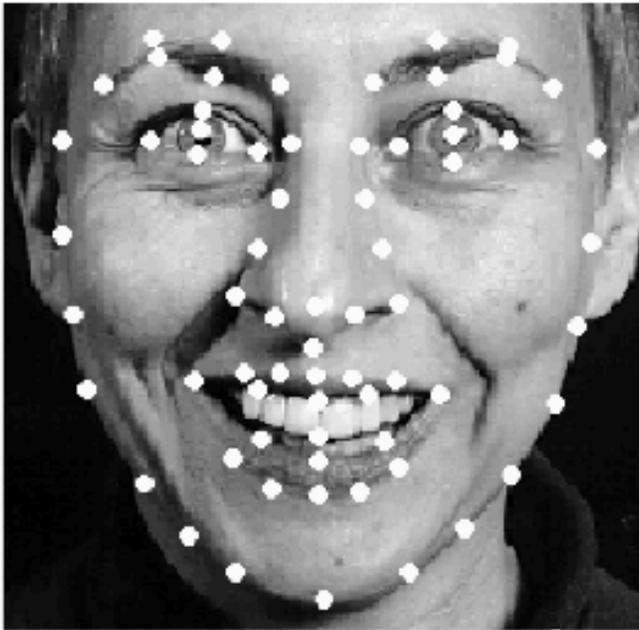
Likelihood

Prior

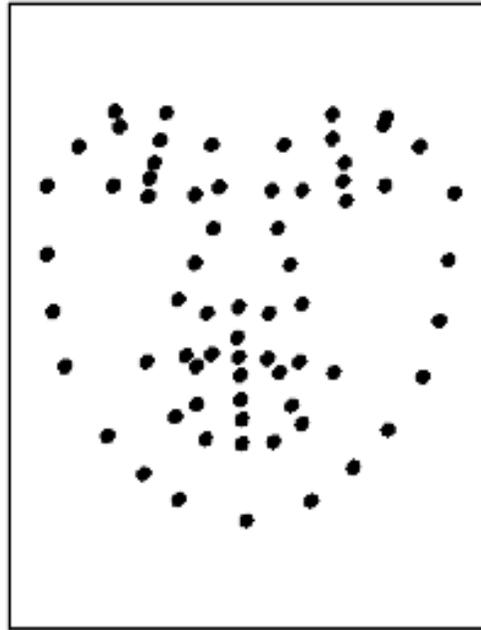
$$\log P(S|I) = \log P(I|S) + \log P(S)$$

For Gaussian distributions, these logs lead to quadratics

# Active Appearance Models



Landmarks on  
training data



Shape Model  
(Incl. Variability)

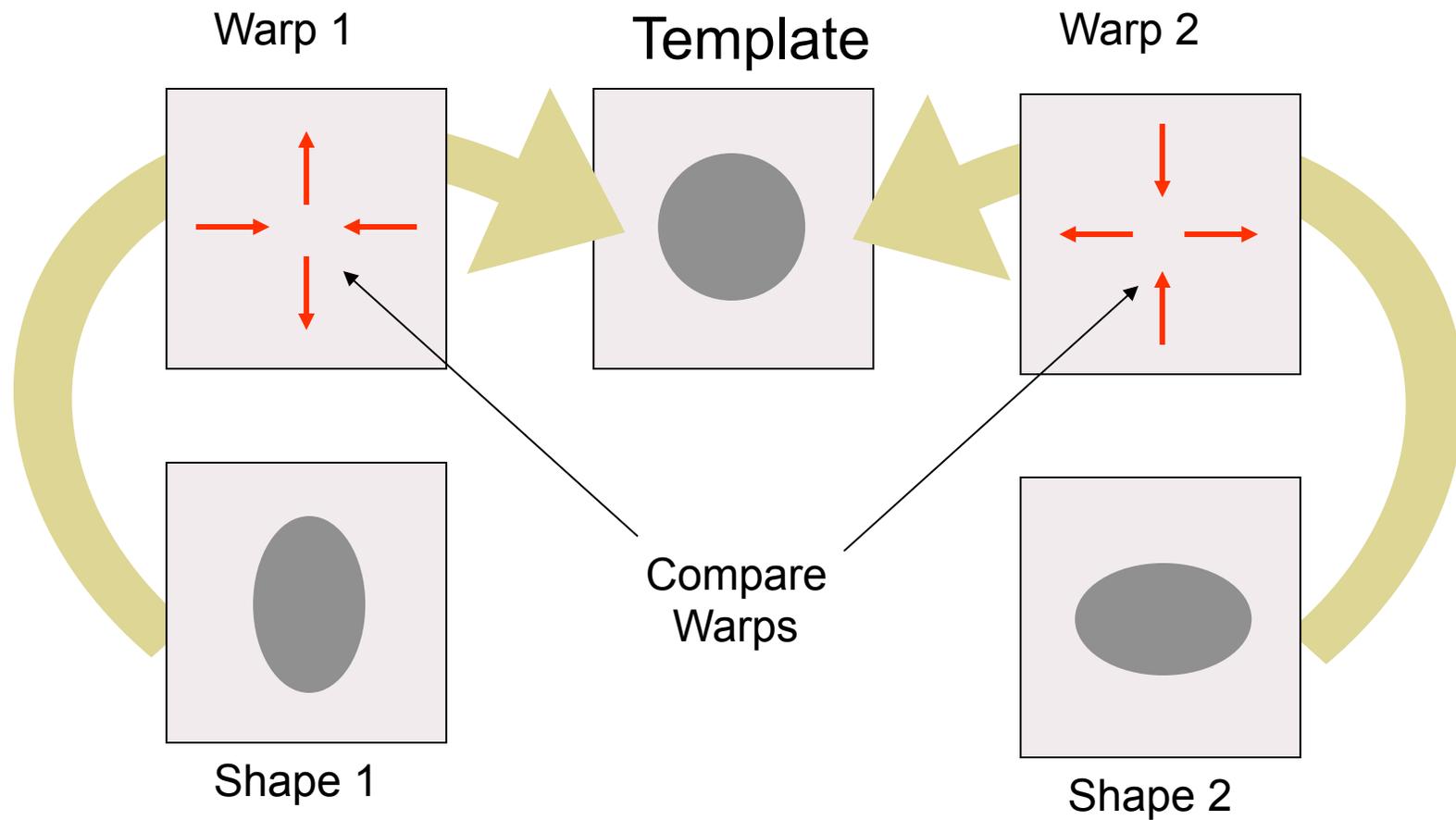


Appearance Model  
(Incl. Variability)

# Other Applications of Shape Analysis

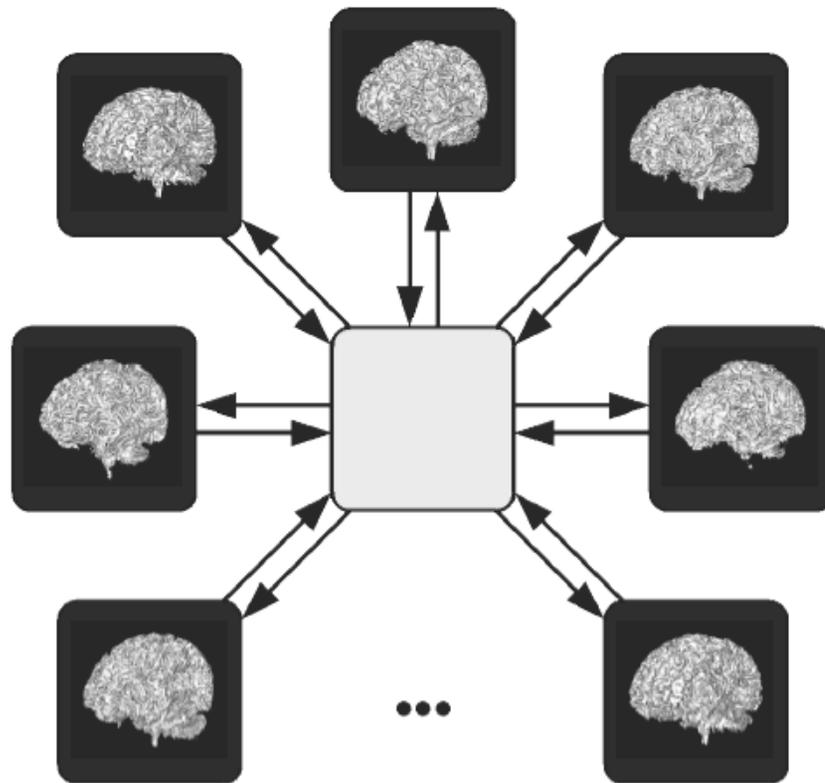
- E.g. sizes/shapes of prosthetics or implants
- Evaluating how a patient differs from population (z-score)
- Morphology and genetics
  - E.g. developmental, phenotypes
- Shape and function
  - How does shape impact physiology?

# Image Deformation Approach to Shape



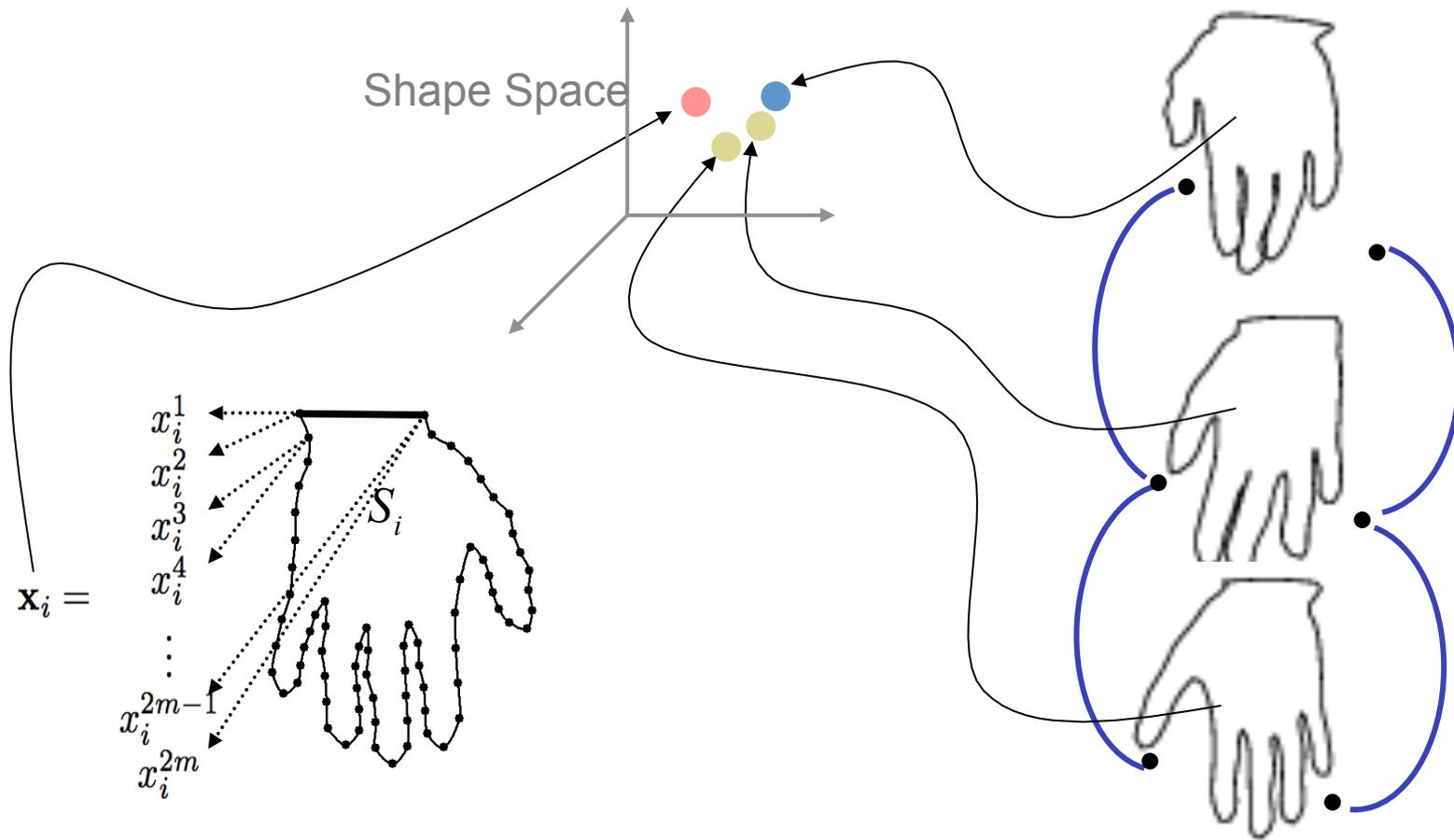
# Unbiased Diffeomorphic Atlases

- Find a template that minimizes sum of distances to ensemble



Joshi et. al, 2004

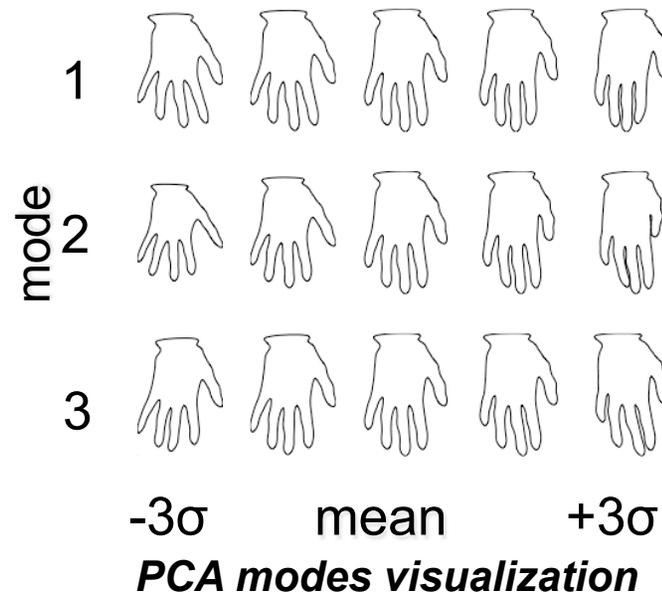
# Shape from Correspondences/ Landmarks



Point Distribution Models (PDMs)

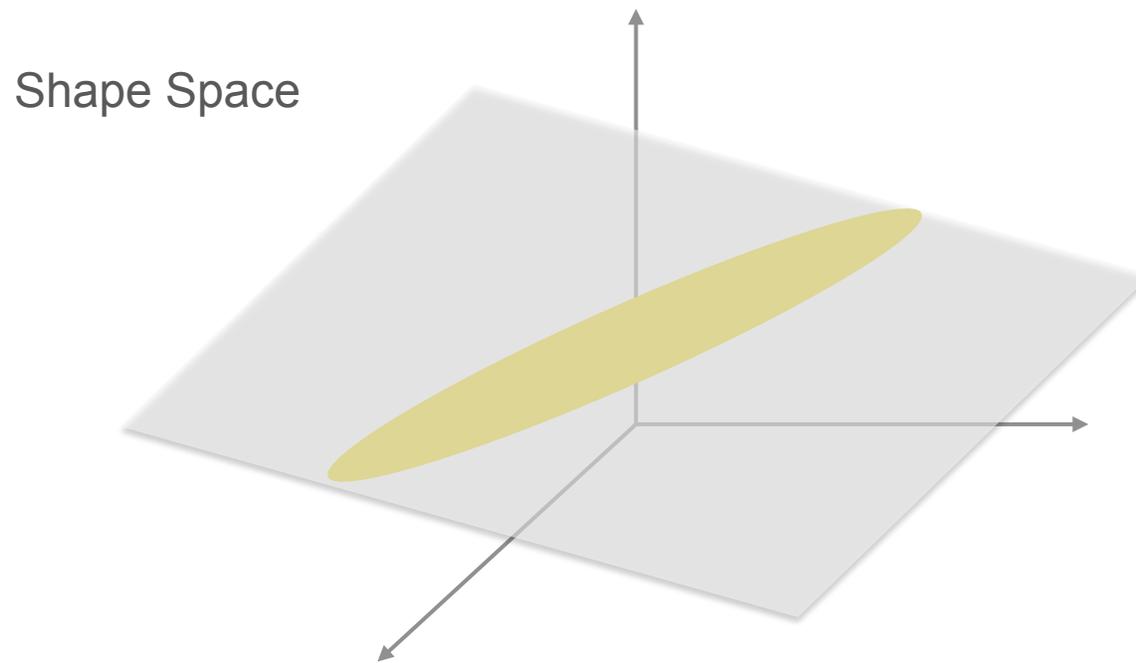
# Hands: A Pattern Theoretic Study of Biological Shapes

- Ulf Grenander, 1990
- Pick landmarks on sets of hands
- A few PCA modes capture variability



# Statistics in Shape Space

- PCA for dimensionality in shape space
- Gaussian models on linear subspace



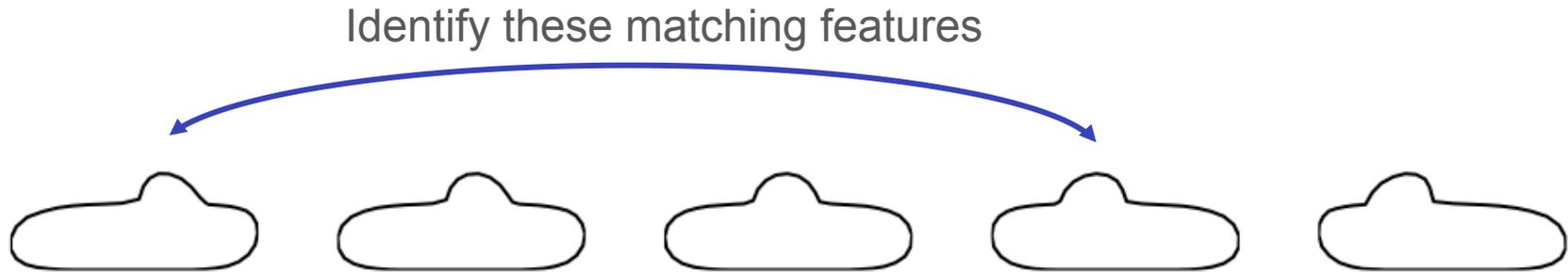
# Correspondences and Shape

- The choice matters
  - Defines the shape space
- Manual landmarks
  - Not practical
  - 3D, not clear
  - User error
- Need: automatic 3D correspondence placement



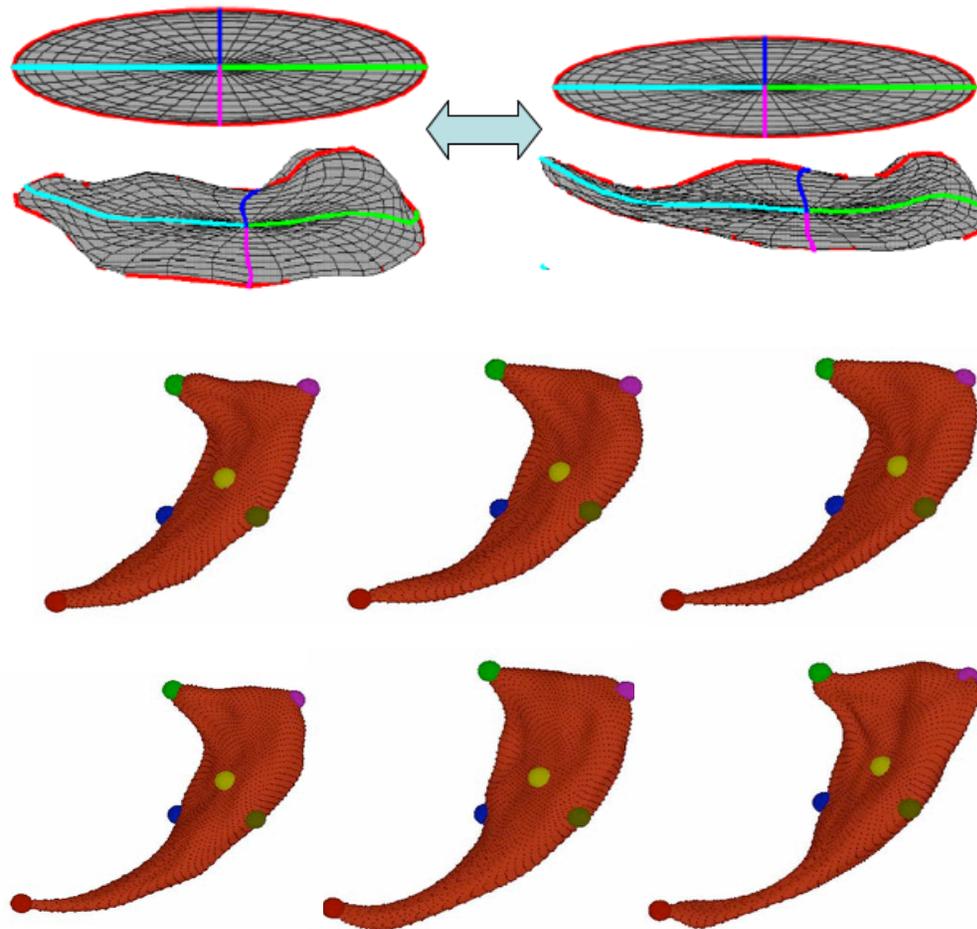
# Shape Correspondence

- Feature matching
  - E.g. Guibas et al.



# Smooth Mappings to Primitives

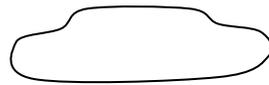
- E.g. SPHARM-PDM, Steiner et al., 2006



# Correspondence Depends on the Population



- Optimize local/nearest differences



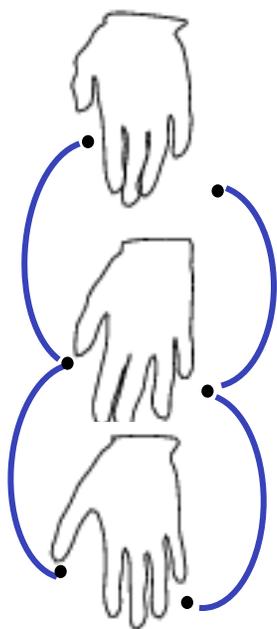
- Account for the trends in the ensemble



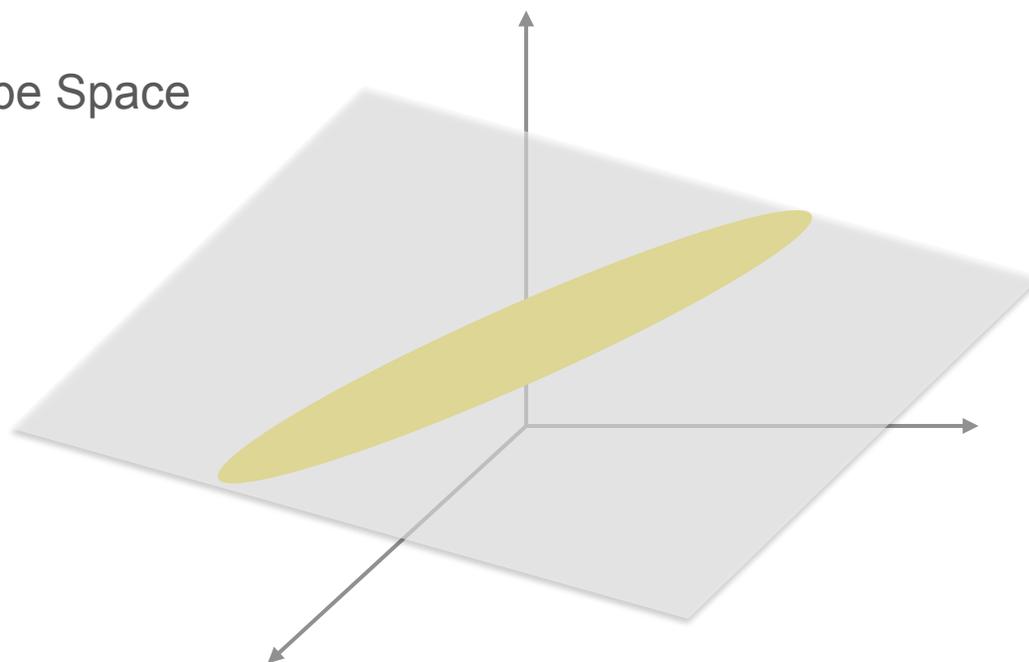
- Davies et al. 2000 (MDL)
  - Parameterization should be such to minimize *description length* of ensemble

# Basic Idea

- Landmark (LM) positions are not unique
- LMs control behavior of shape space
- Strategy: position LMs to optimize the statistics in shape space

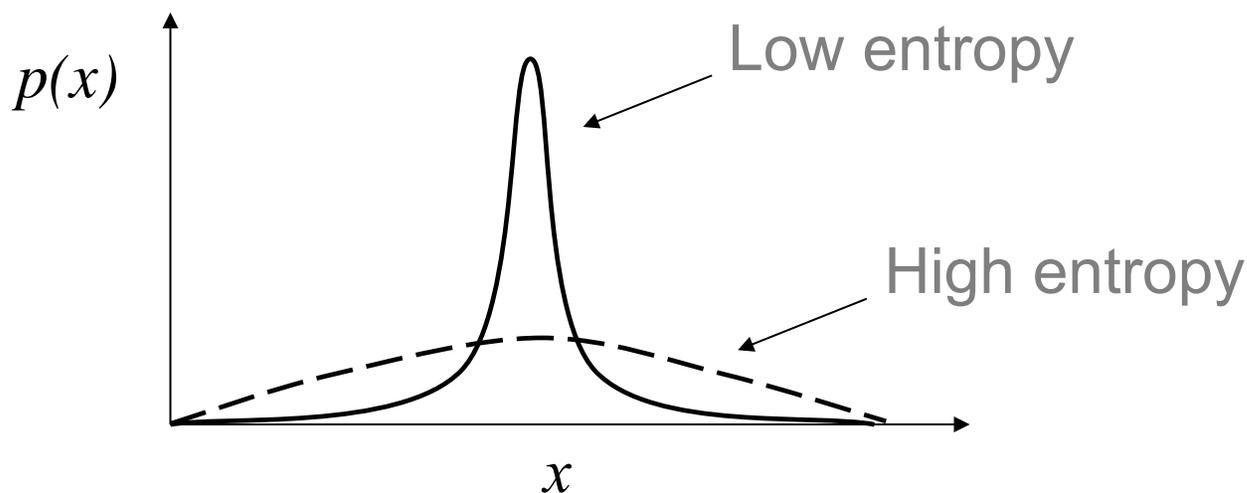


Shape Space



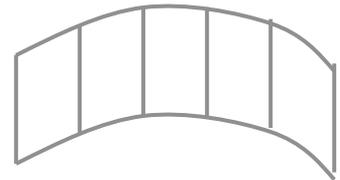
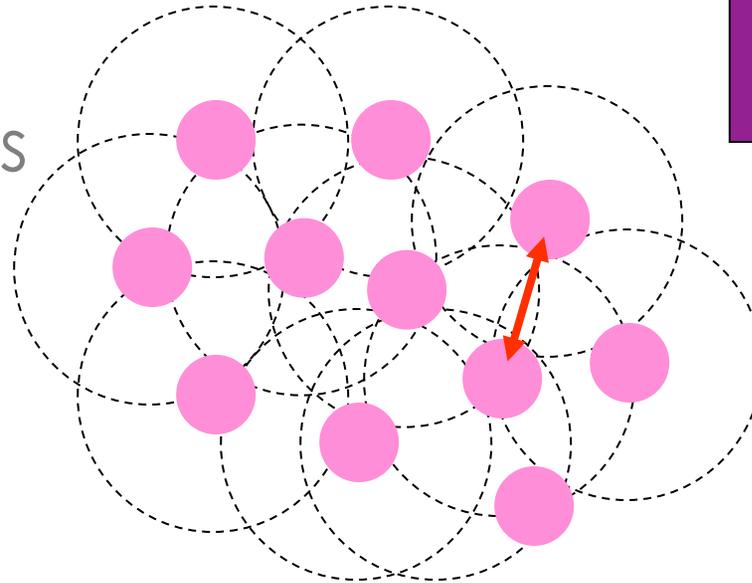
# Occam's Razor

- Minimize entropy of distribution
  - Measure of uncertainty – information content of a sample



# Particle Systems

- Particles distributed to minimize potential
  - Particle-particle interactions
  - Minimize through incremental updates (dynamic)
- Satisfy constraints
  - E.g. isosurfaces



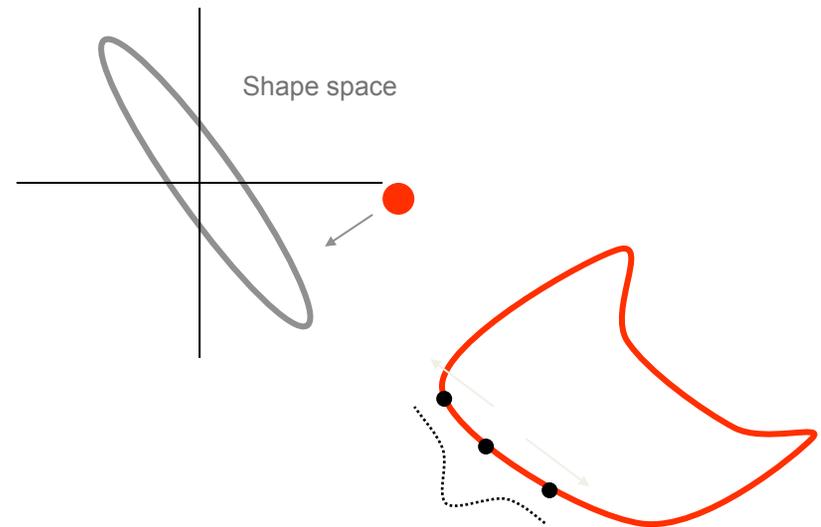
# Particle-Based Shape Correspondences

- Shapes as a set of interacting particle systems
- Compact models, but balanced against geometric accuracy (good, adaptive samplings)
- Optimize particle positions by minimizing an entropy cost function

$$Q = H(Z) - \sum_k H(P^k)$$

↑  
Entropy of the  
shape ensemble

↑  
Potential  
energies for  
particle-particle  
interactions

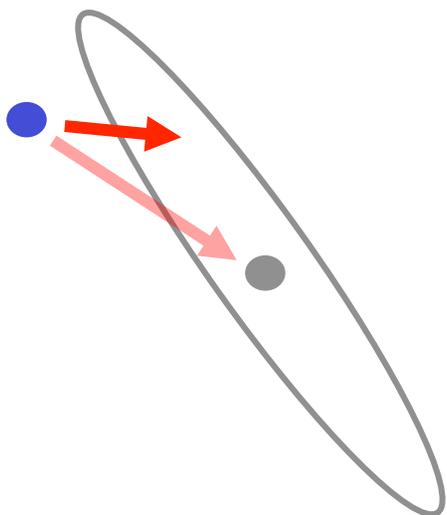


# Minimizing Ensemble Entropy

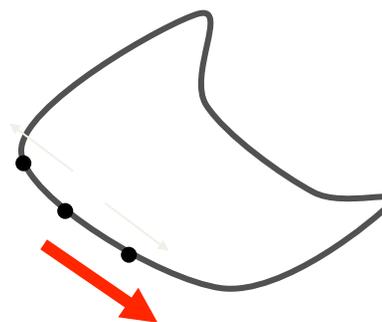
- Weighted movement toward mean

$$\frac{dX_i(t)}{dt} = \Delta t \Sigma^{-1} (\mu - X_i(t))$$

(+ forces between particles)



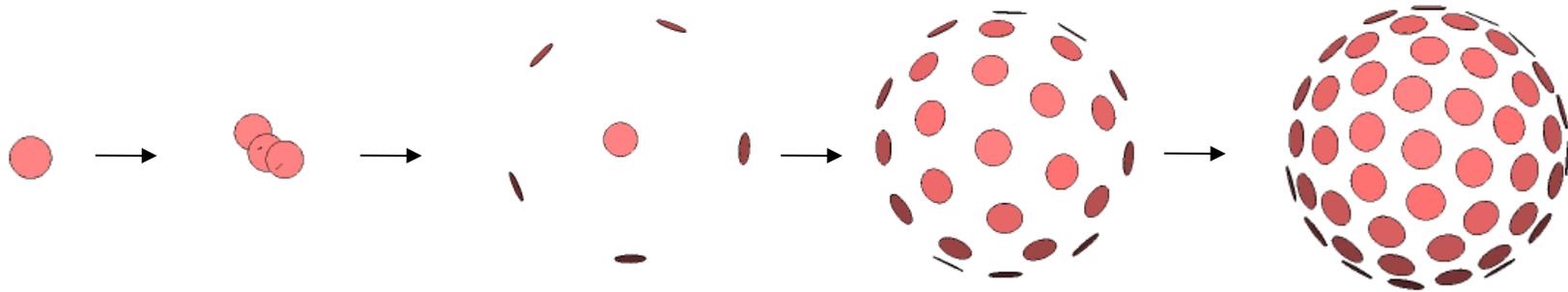
Shape Space



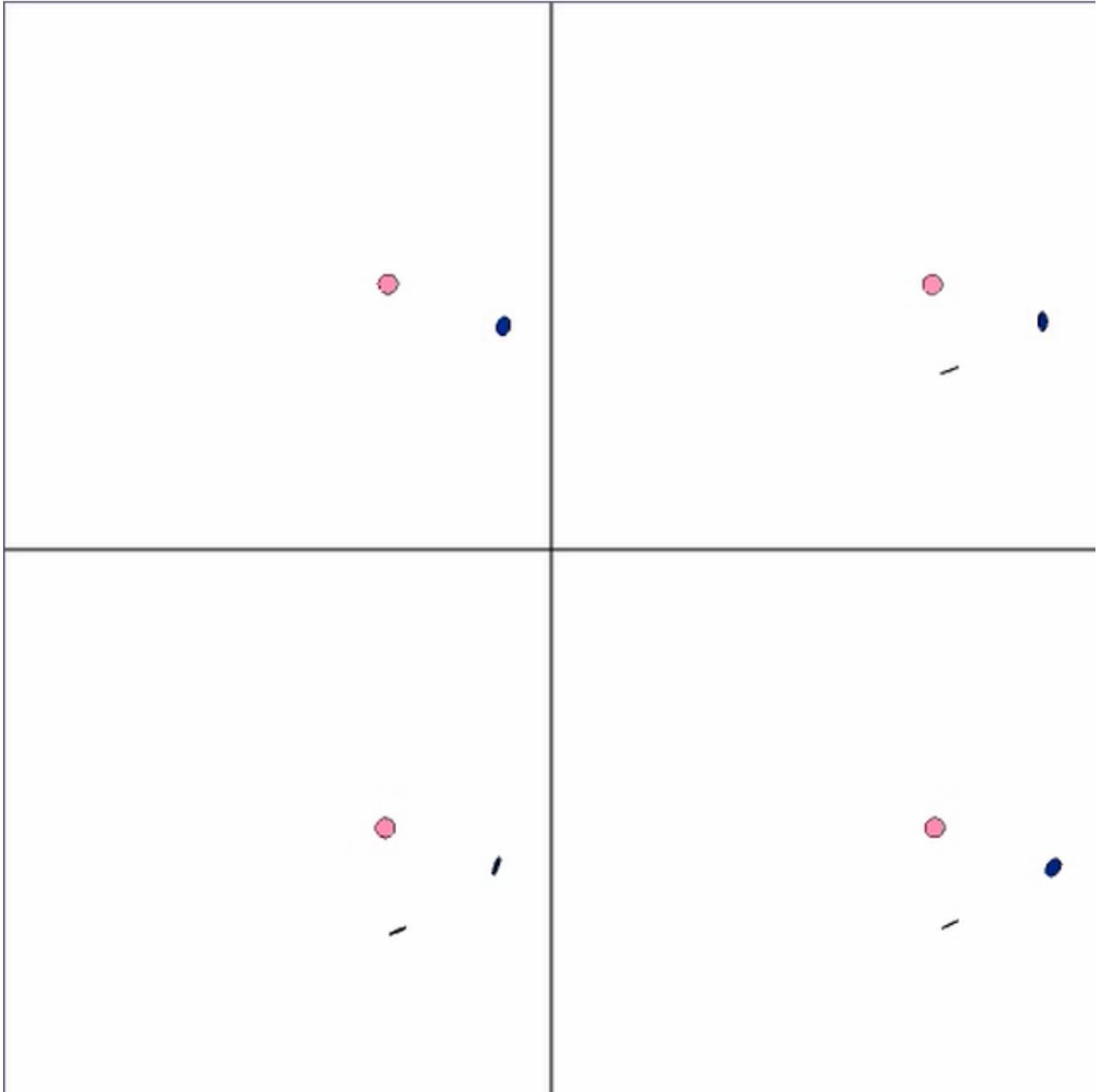
Object Space

# Entropy-Based Surface Sampling

- Big issue: local minima and how to initialize
  - Lots of *tricks*



*Surface sampling with a max entropy particle system.  
Particles split under optimization until entire surface is sampled.*

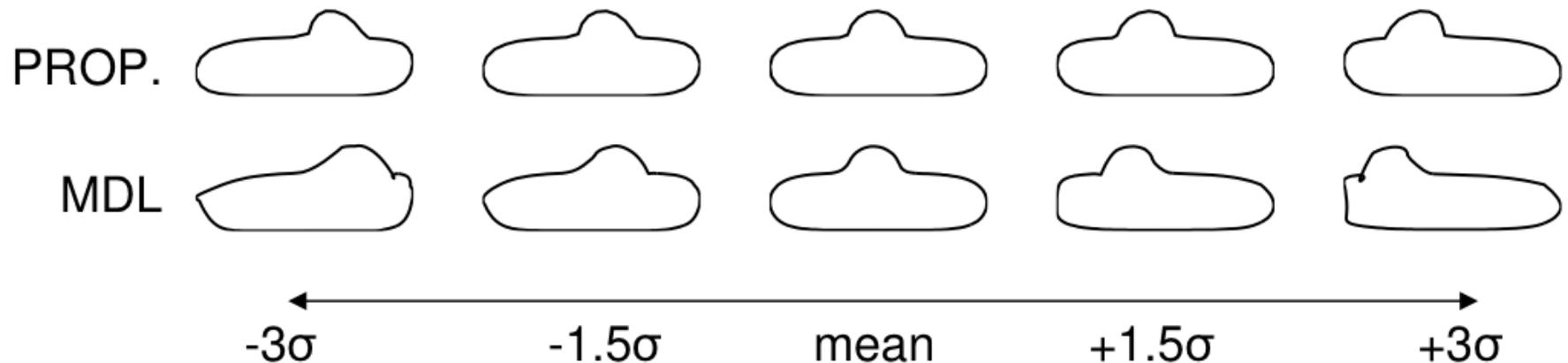


# Shape Analysis Pipeline

1. Segmentation process -> binary volumes
2. Align segmentations
3. Distance transform from binary interface w/ smoothing
4. Initialize models (splitting under optimization)
5. Optimize Correspondences (w/ Procrustes)
6. Statistical Analysis

# Box-Bump

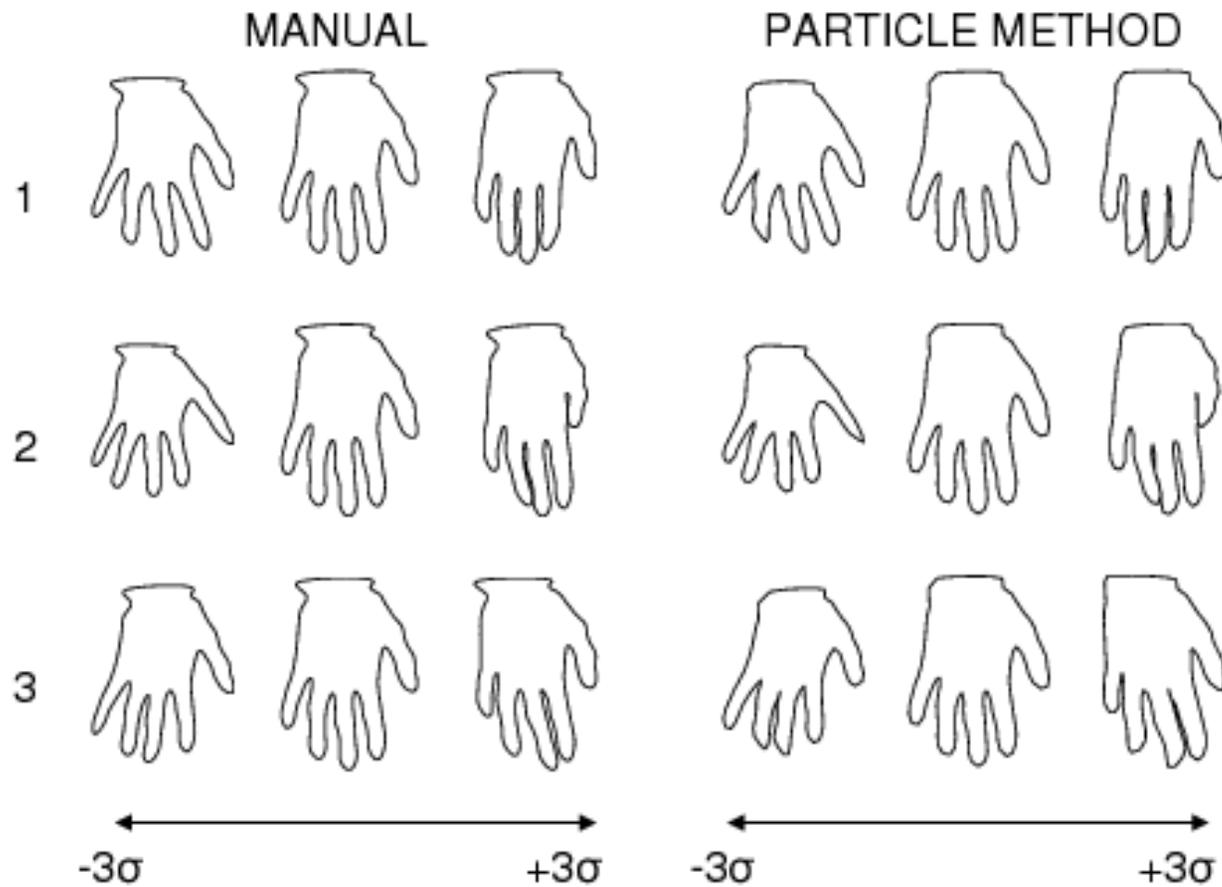
- Results
  - Single major mode of variation
  - Particle system: 0.0015% leakage



\* See Thodberg, IPMI 2003 for details

# Hand Contours

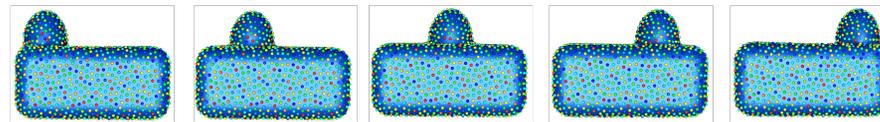
- Particle method similar to manual method



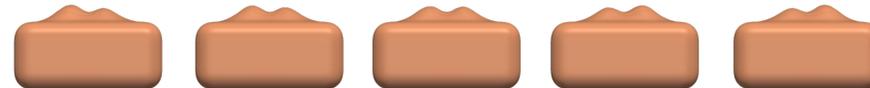
# The Importance of Ensembles in Determining the *Metric*



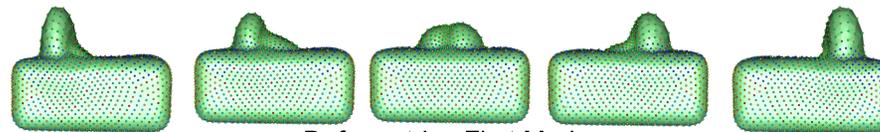
Box-bump Samples



ShapeWorks First Mode



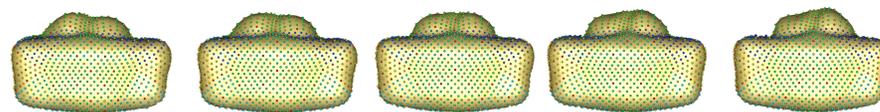
AtlasWerks First Mode



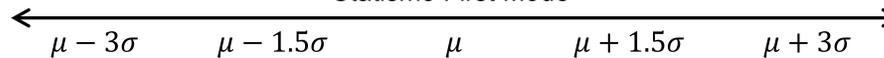
Deformetrica First Mode



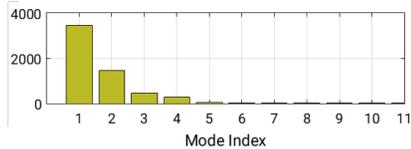
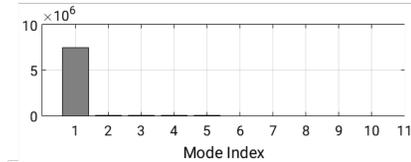
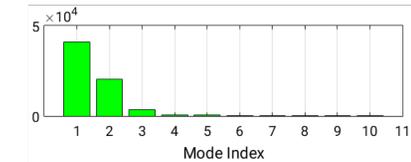
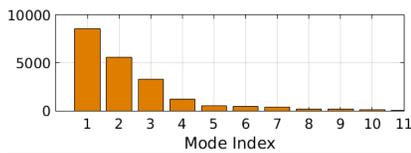
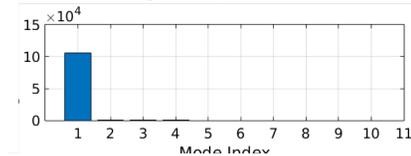
SPHARM-PDM First Mode



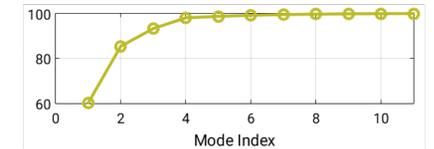
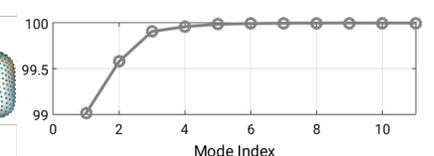
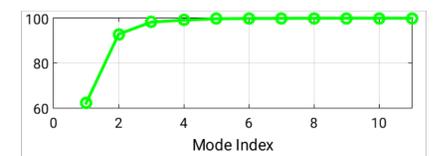
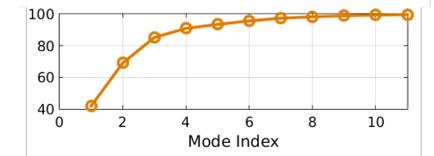
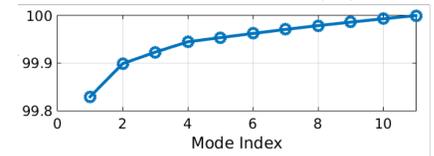
Statismo First Mode



Eigen Spectrums



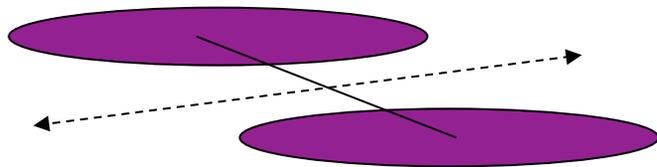
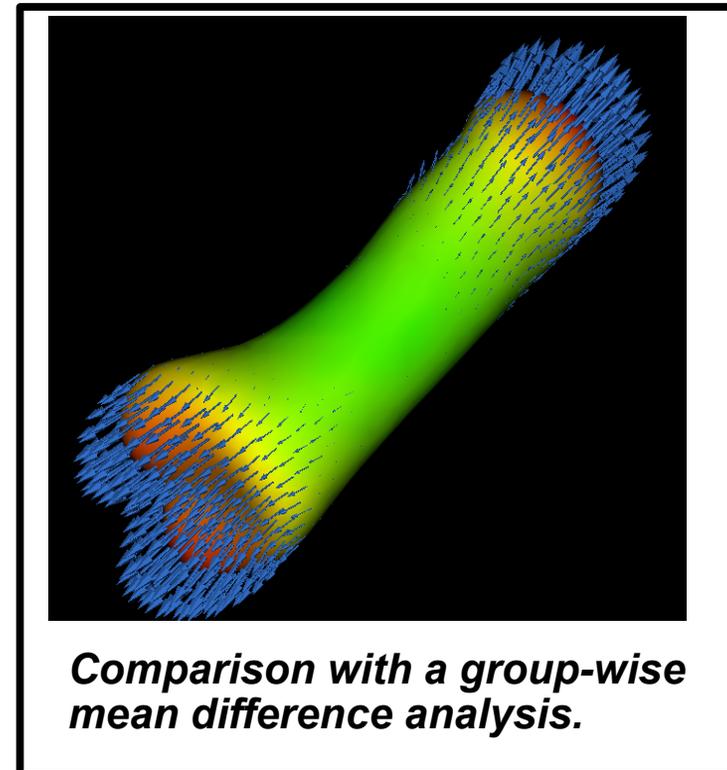
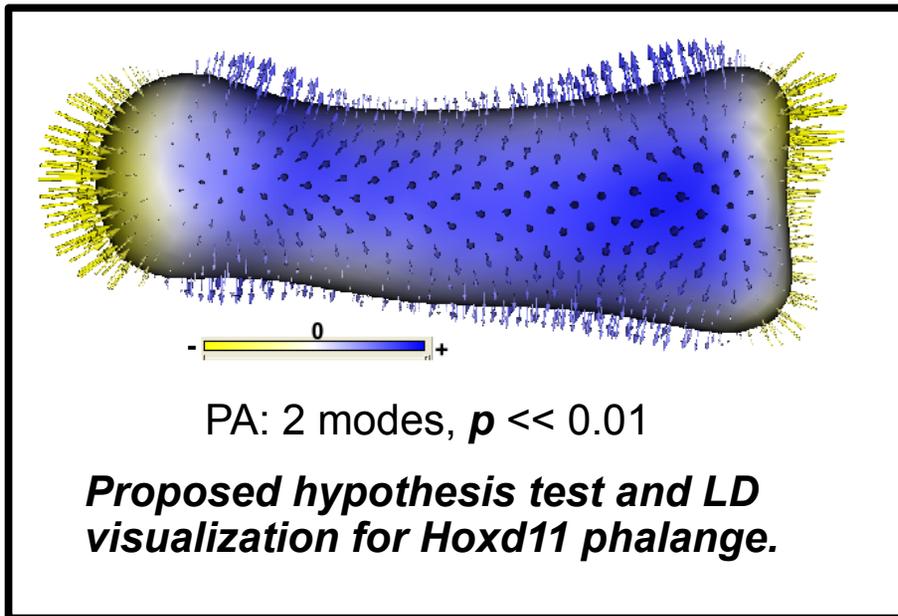
Variance Captured (%)



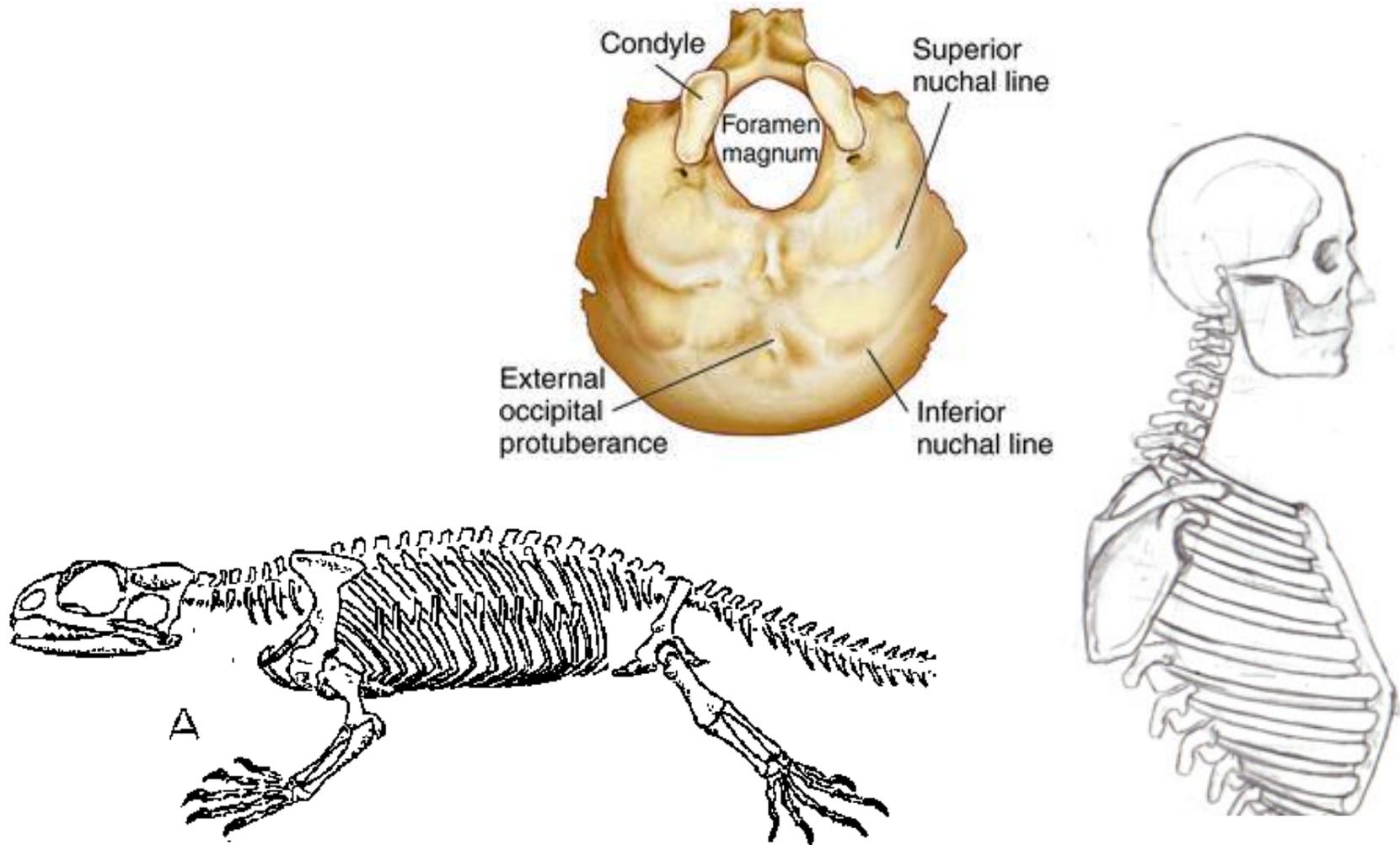
What can we do with this  
technology?

# Genetic Phenotyping Results

## Hoxd11 Phenotyping

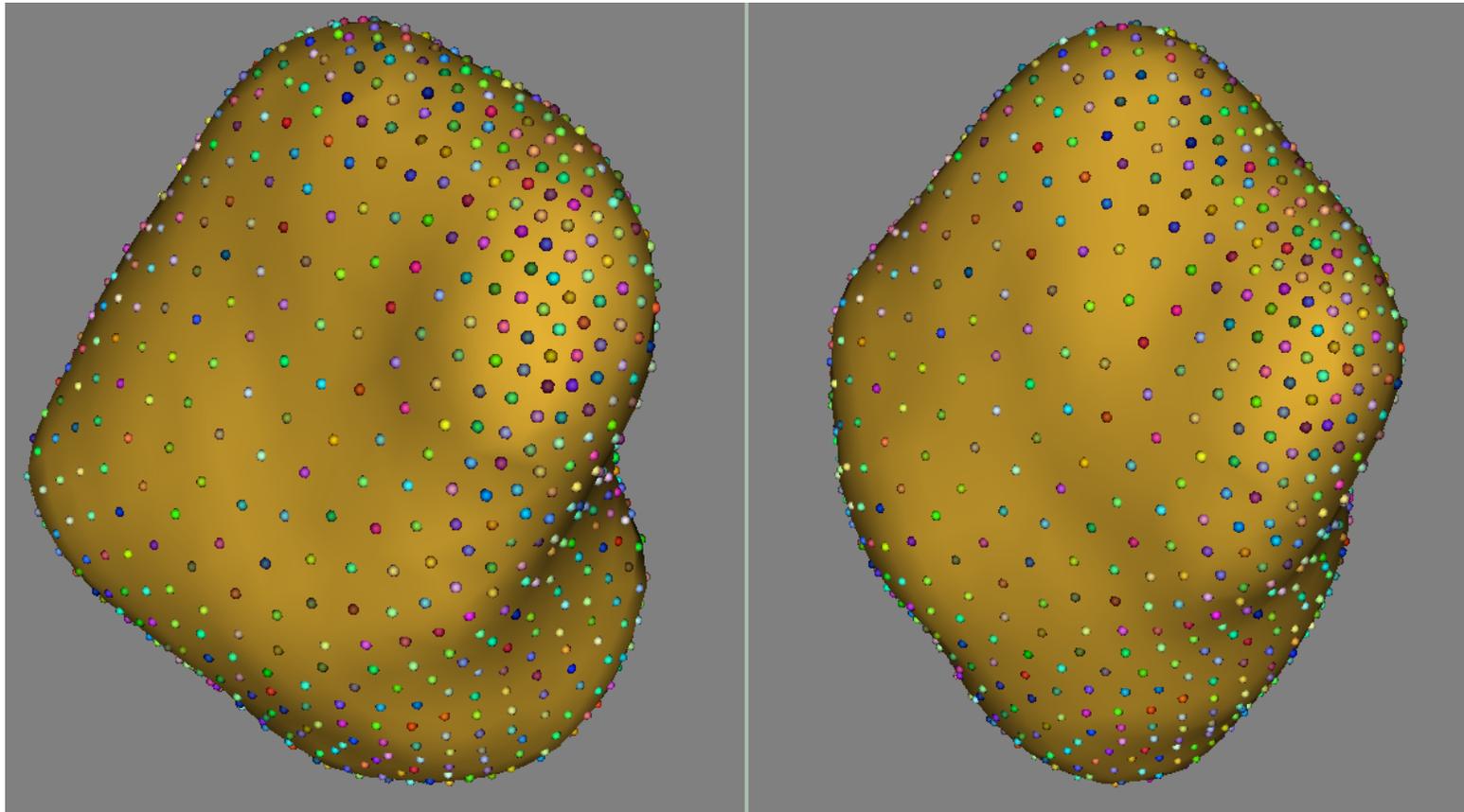


# Baso-Occipital Bone and the Foramen Magnum

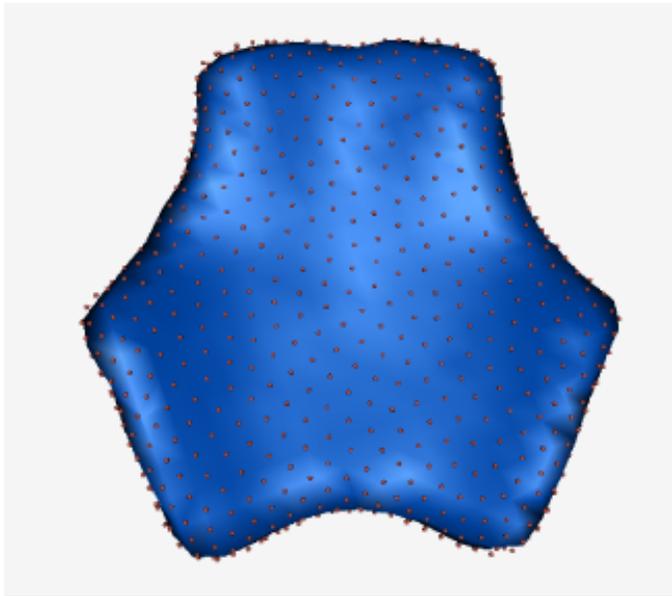


Can we understand the phenotypes associated with known genotypes for BO?

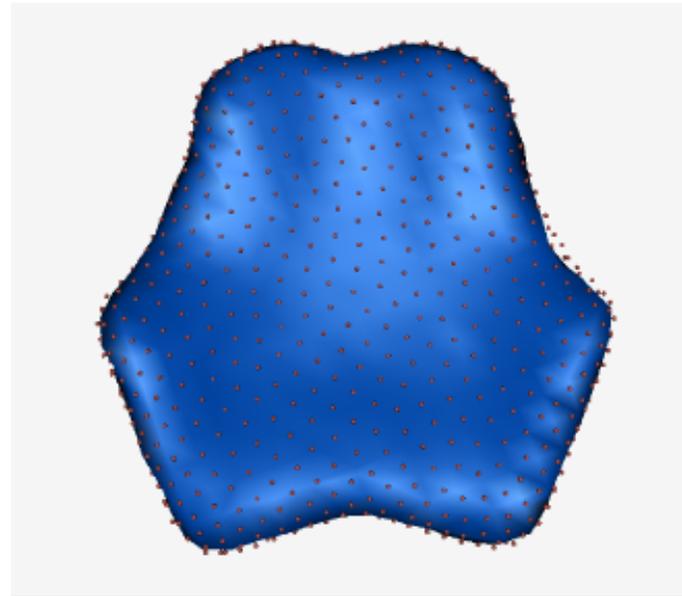
# Baso-Occipital Shape Correspondence



# Pax7 KO Mouse



*Mean shape: LacZ -/-*

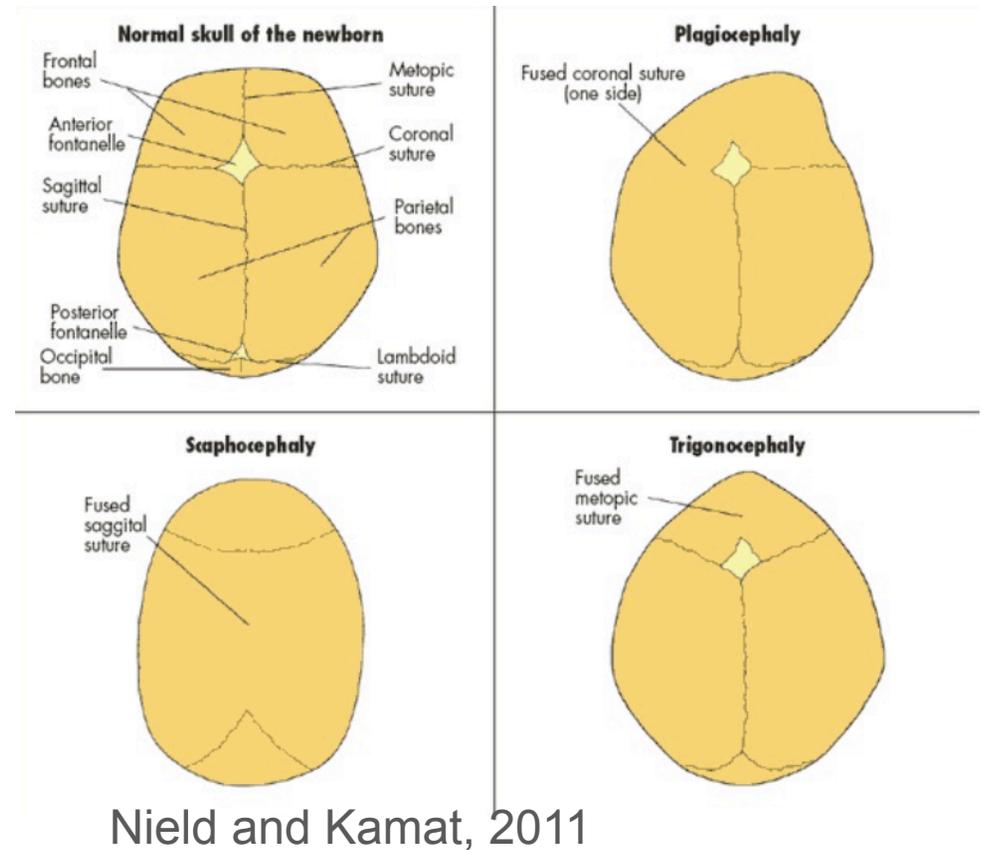
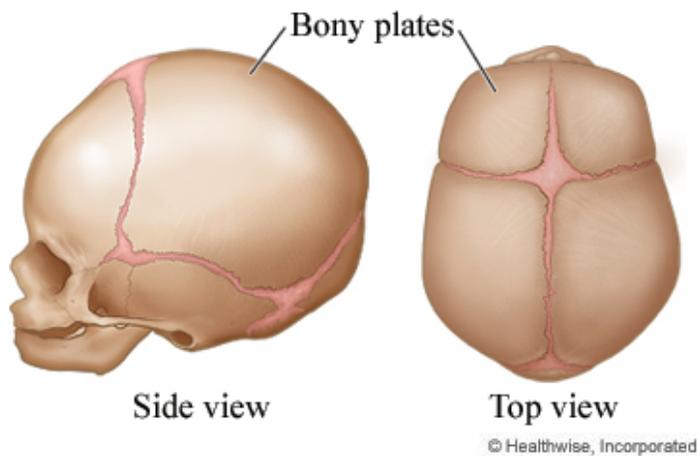


*Wild Type*

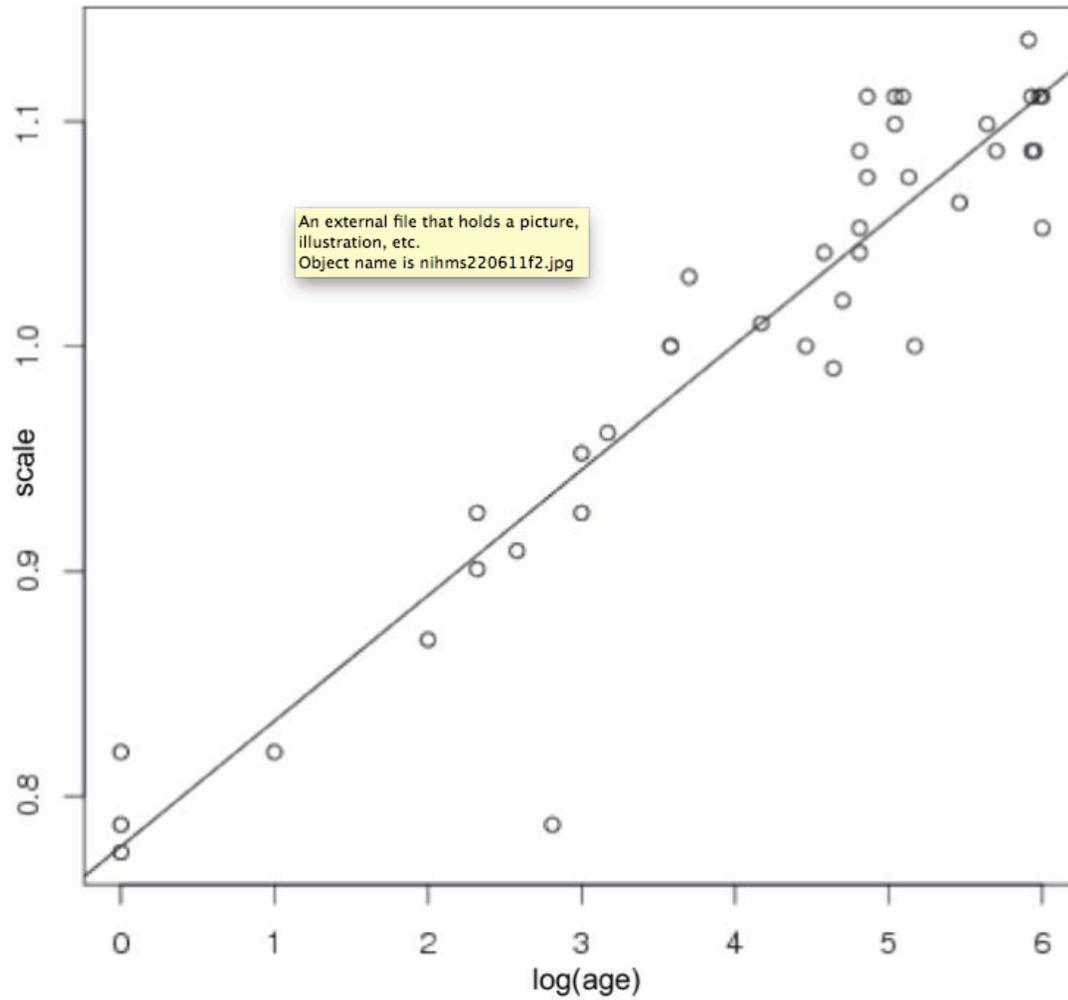
Work with C. Keller, J. Cates, L. Nevel

# Pediatric Head Shape

- Insight into normal and abnormal development

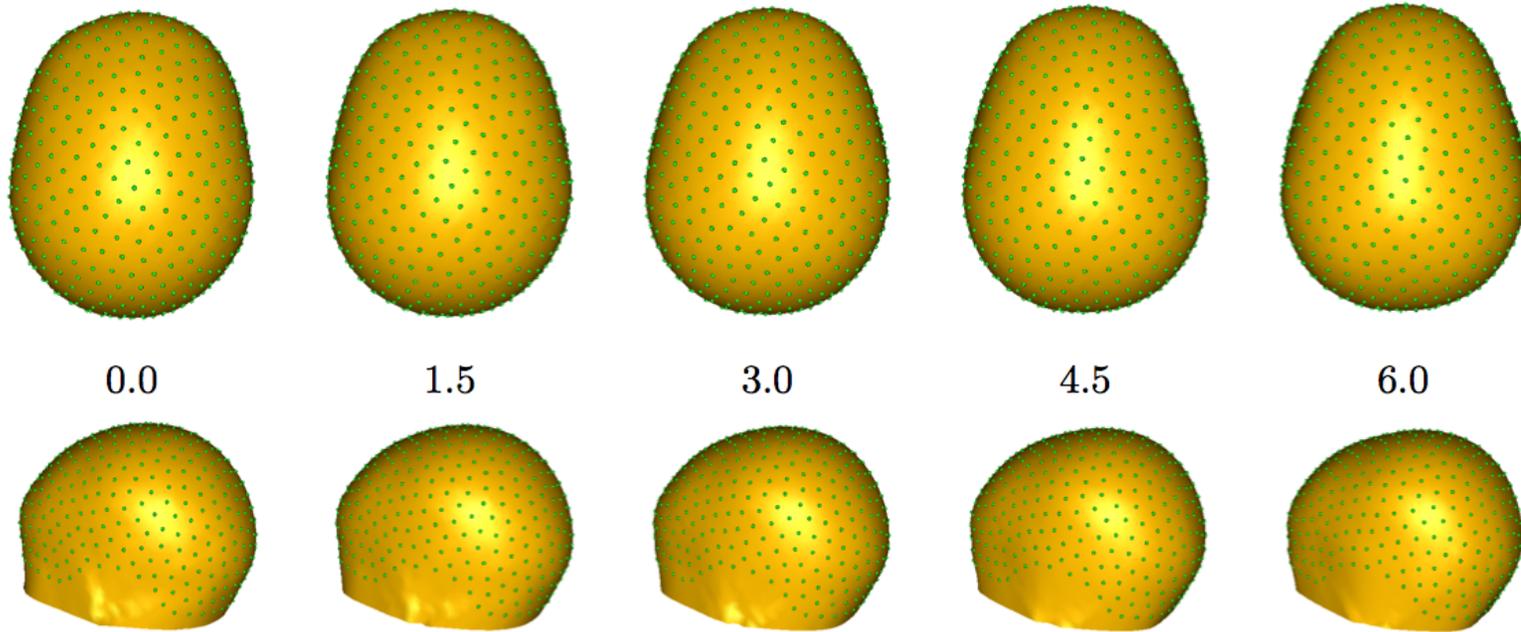


# Kids Heads – Size



# Pediatric Head Shape Size Normalized

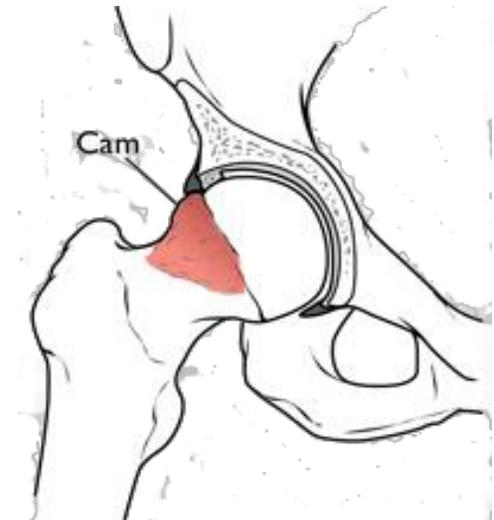
Datar et al., 2009



$$\arg \min_{\mathbf{a}, \mathbf{b}} E(\mathbf{a}, \mathbf{b}) = \frac{1}{2} \sum_k [(\mathbf{a} + \mathbf{b}t_k) - \mathbf{z}_k]^T \Sigma^{-1} [(\mathbf{a} + \mathbf{b}t_k) - \mathbf{z}_k]$$

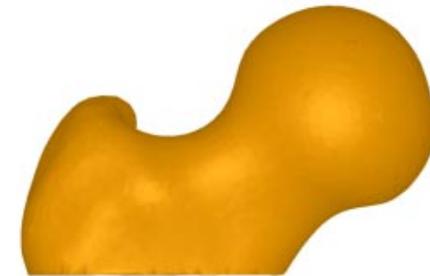
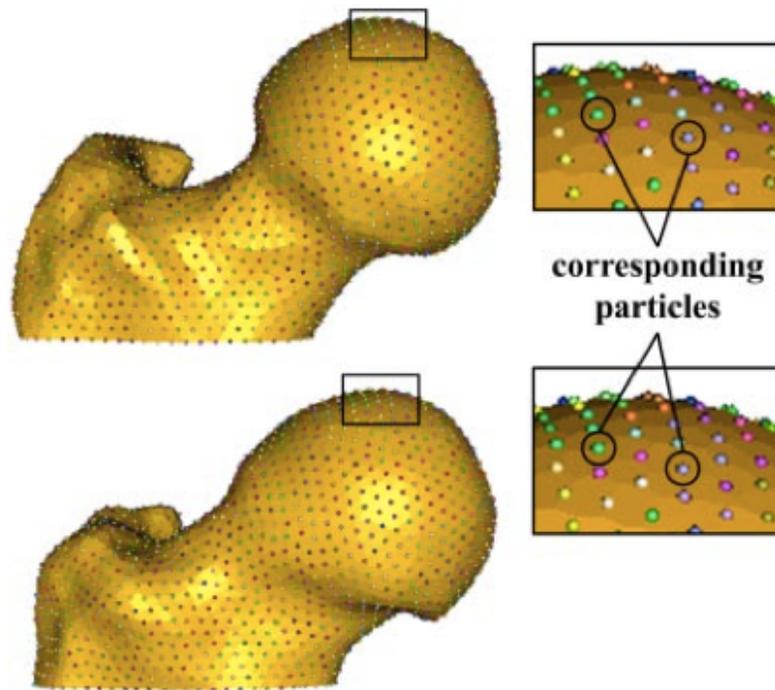
# Orthopedics

- Can we characterize normal and pathological shape for joints?
- E.g. femoral-acetabular impingement
  - Segmented CT volumes
    - Normals, controls – approx. 30 of each
  - Build models -> hypothesis testing on point sets

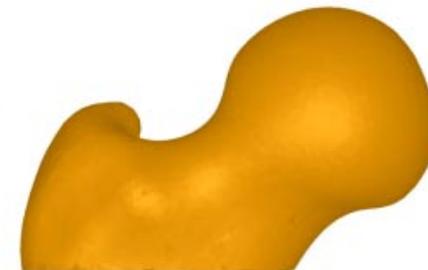
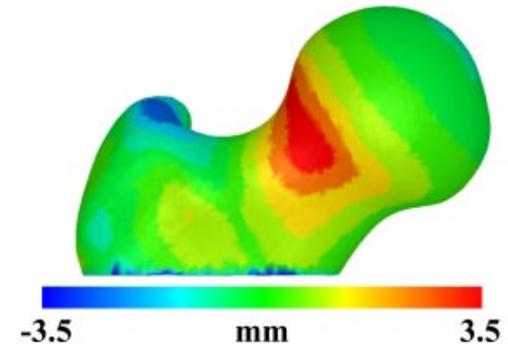


# Hip Pathoanatomy

Harris et. al, 2013



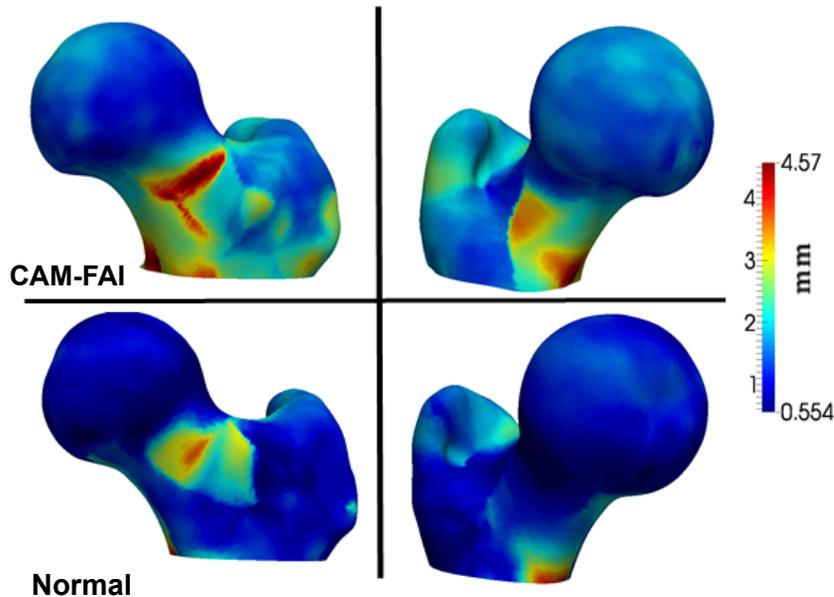
**control**



**cam**

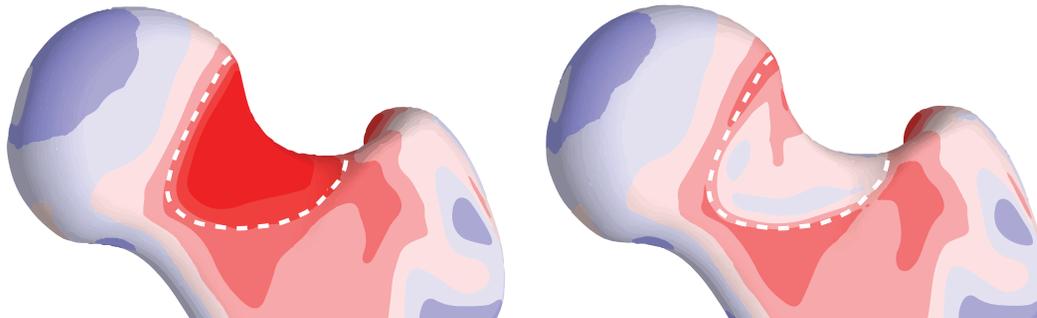
# Cortical Thickness, Resection, Biomechanics

## Cortical Thickness

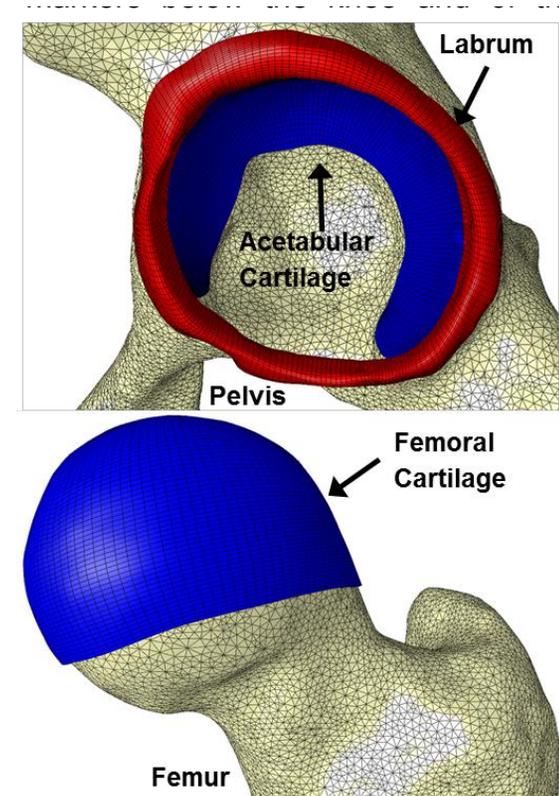


## Resection/ treatment

-2 mm 2 mm

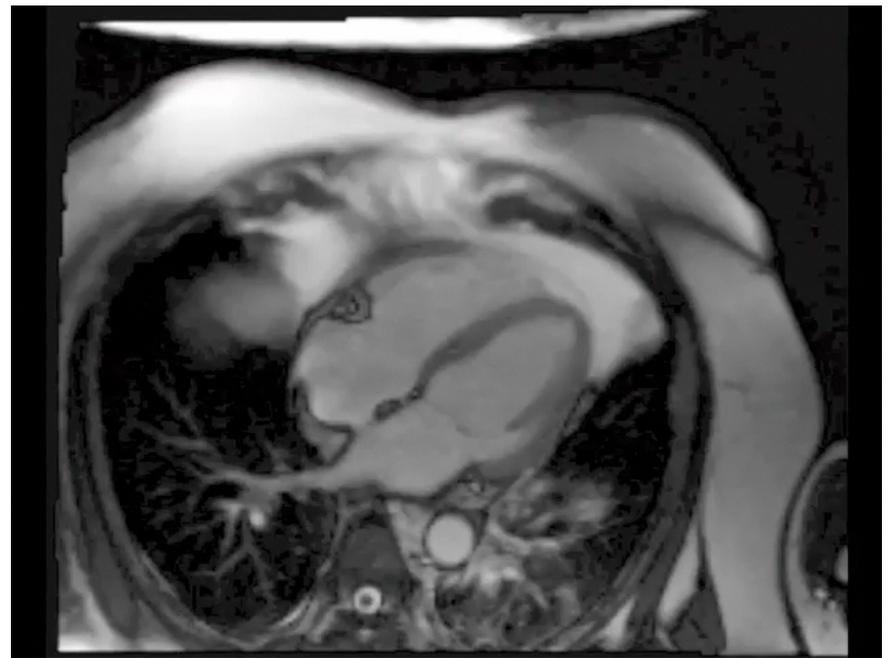
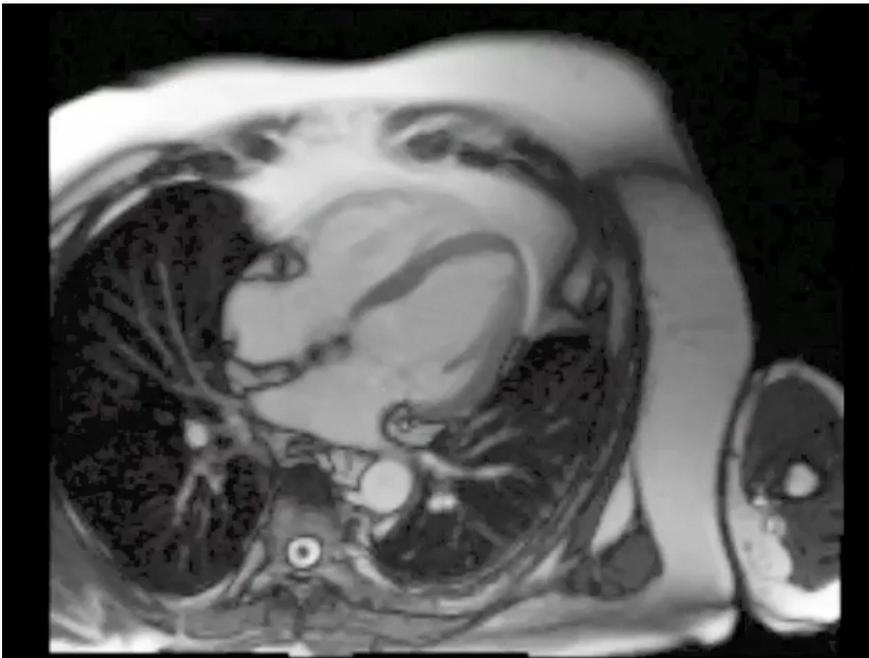


## Biomechanics



Work with P. Atkins, A. Anderson, J. Weiss, P. Agarwal, S. Elhabian

# Cardiac Imaging and Atrial Fibrillation

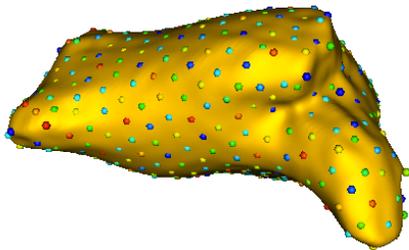


N. Marrouche, R. MacLeod

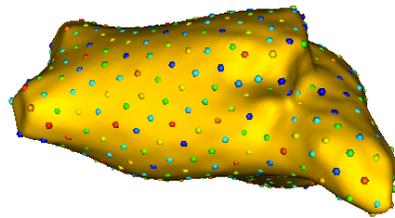
# Shape and Clinical Outcomes

## Statistical Shape Analysis

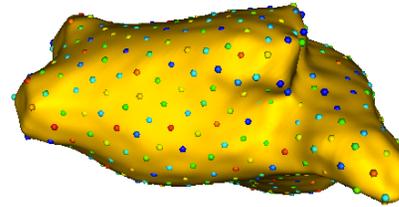
- Left atrium shape in Afib
- Does shape help predict treatment outcomes?



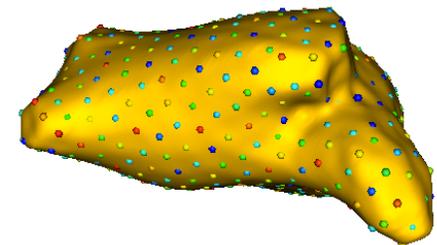
Class 1



Class 2



Class 3



Class 4

# Shape Analysis Software

- Shapeworks (Cates, Whitaker – Utah)
- SHARM PDMs (Styner – UNC)
- Deformetrica (Durleman – ICM, Paris)
  - <http://www.deformetrica.org>
- Given sets of landmarks
  - The shapes package : statistical shape analysis in R

# Thanks

- Collaborators and staff:
  - Josh Cates, Andy Anderson, Jeff Weiss, Nassir Marrouche, Shireen Elhabian, Penny Atkins, Chris Peters, Steve Aoki, Manasi Datar, Rob MacLeod, ...
- Sponsors:
  - NIH P41-GM103545-17, R01-GM083925