Quantum Like Structuring in Gravitational Systems.

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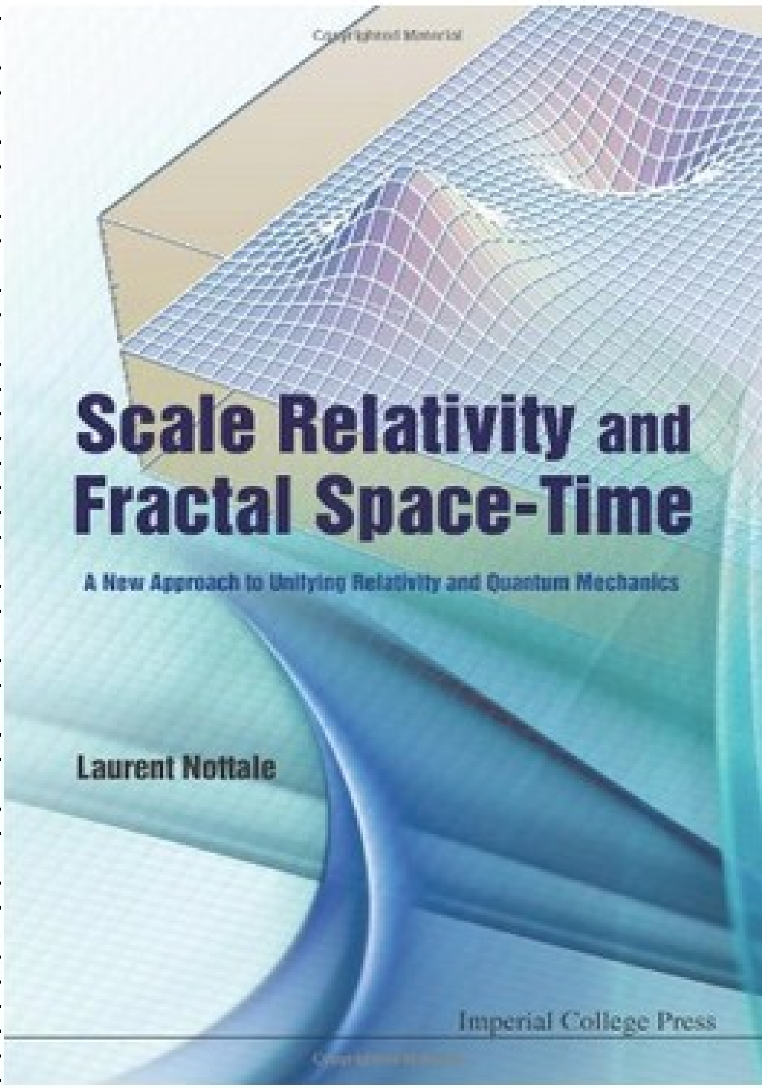

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Derivation of the Schrödinger Equation from Newtonian Mechanics*

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We examine the hypothesis that every particle of mass m is subject to a Brownian motion with diffusion coefficient $\hbar/2m$ and no friction. The influence of an external field is expressed by means of Newton's law $\mathbf{F}=m\mathbf{a}$, as in the Ornstein-Uhlenbeck theory of macroscopic Brownian motion with friction. The hypothesis leads in a natural way to the Schrödinger equation, but the physical interpretation is entirely classical. Particles have continuous trajectories and the wave function is not a complete description of the state. Despite this opposition to quantum mechanics, an examination of the measurement process suggests that, within a limited framework, the two theories are equivalent.

I. INTRODUCTION

WE shall attempt to show in this paper that the radical departure from classical physics produced by the introduction of quantum mechanics forty years ago was unnecessary. An entirely classical derivation and interpretation of the Schrödinger equation will be given, following a line of thought which is a natural development of reasoning used in statistical mechanics and in the theory of Brownian motion.

Consider an electron in an external field. The electron is regarded as a point particle of mass m in the sense of Newtonian mechanics. Our basic assumption is that any particle of mass m constantly undergoes a Brownian motion with diffusion coefficient inversely proportional to m. We write the diffusion coefficient as $\hbar/2m$ and later identify $\hbar$ with Planck's constant divided by 2π. As in the theory of macroscopic Brownian motion, the influence of the external force is expressed by means of Newton's law $\mathbf{F}=m\mathbf{a}$, where $\mathbf{a}$ is the mean acceleration of the particle. The chief difference is that in the study of macroscopic Brownian motion in a fluid, friction plays an important role. For the electron we must assume that there is no friction in order to preserve Galilean covariance. The kinematical description of Brownian motion with zero friction is the same as the description used in the Einstein-Smoluchowski theory[1] (the approximate theory of macroscopic Brownian motion in the limiting case of infinite friction).

The picture which emerges is the following. If we have, for example, a hydrogen atom in the ground state, the electron is in dynamical equilibrium between the random force causing the Brownian motion and the attractive Coulomb force of the nucleus. Its trajectory is very irregular. Most of the time the electron is near the nucleus, sometimes it goes farther away, but it always shows a general tendency to move toward the nucleus, and this is true no matter which direction we take for time. This behavior is quite analogous to that of a particle in a colloidal suspension, in dynamical equilibrium between osmotic forces and gravity. However, the electron in the hydrogen atom has other states of dynamical equilibrium, at the usual discrete energy levels of the atom.

The equations of motion which we derive are nonlinear, but if the wave function ψ is introduced, in a way simply related to the kinematical description of the motion, we find that ψ satisfies the Schrödinger equation. Every solution of the Schrödinger equation arises in this way.

Our theory is by no means a causal theory, but probabilistic concepts enter in a classical way. The description of atomic processes is by means of classical ideas of motion in space-time, and so is contrary to quantum mechanics. However, we show that for observations which may be reduced to position measurements, the two theories give the same predictions. This, and a discussion of von Neumann's theorem[2] on the impossibility of hidden variables, is contained in Sec. IV. The same argument shows that some features of our description are incapable of observation. It is well known[3] that macroscopic Brownian motion imposes limits on the precision of measurements if the measuring instruments are subject to it. If, as we are assuming, every system is subject to a Brownian motion, this implies an absolute limit restricting some measurements.

The discussion in this paper is restricted to the nonrelativistic mechanics of particles without spin, in the presence of external fields.

Our work has close connections with some previous work on classical interpretations of the Schrödinger equation. A comparison with other hidden-variable theories is contained in Sec. V.

II. STOCHASTIC MECHANICS

Stochastic processes occur in a number of classical physical theories.[4] Statistical mechanics is based on a

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[1] A. Einstein, *Investigations on the Theory of the Brownian Movement*, translated by A. D. Cowper (Methuen and Company, Ltd., London, 1926); M. v. Smoluchowski, *Abhandlungen über die Brownsche Bewegung und verwandte Erscheinungen* (Akademische Verlagsgesellschaft, Leipzig, 1923).

[2] J. von Neumann, *Mathematical Foundations of Quantum Mechanics*, translated by R. T. Beyer (Princeton University Press, Princeton, New Jersey, 1955).

[3] R. B. Barnes and S. Silverman, Rev. Mod. Phys. 6, 162 (1934).

[4] A more detailed account of this subject is contained in lecture notes by the author, *Dynamical Theories of Brownian Motion* (Princeton University Press, Princeton, New Jersey, to be published).

Similarity between quantum path and random motion

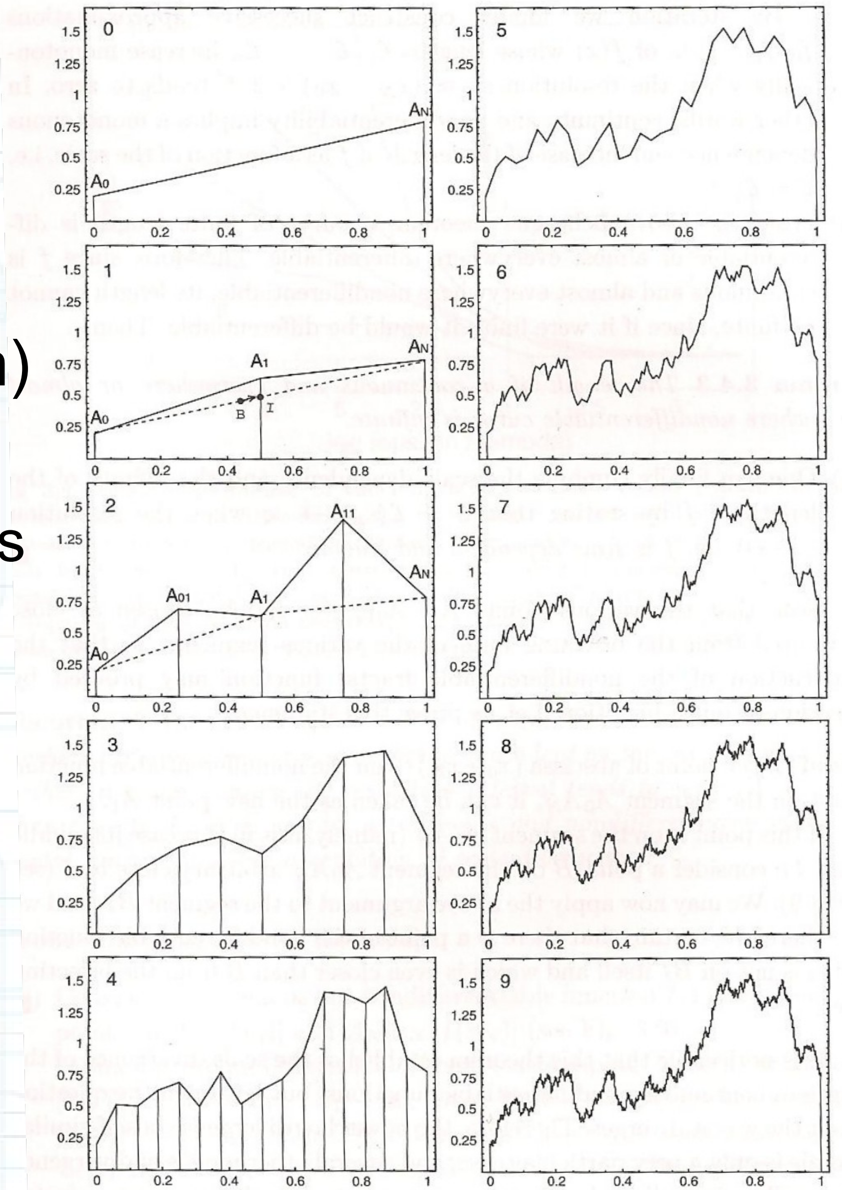
- Quantum Uncertainty

$$\Delta\chi\Delta\rho \geq \frac{\hbar}{2}$$

- Similar to Random (Brownian) Motion

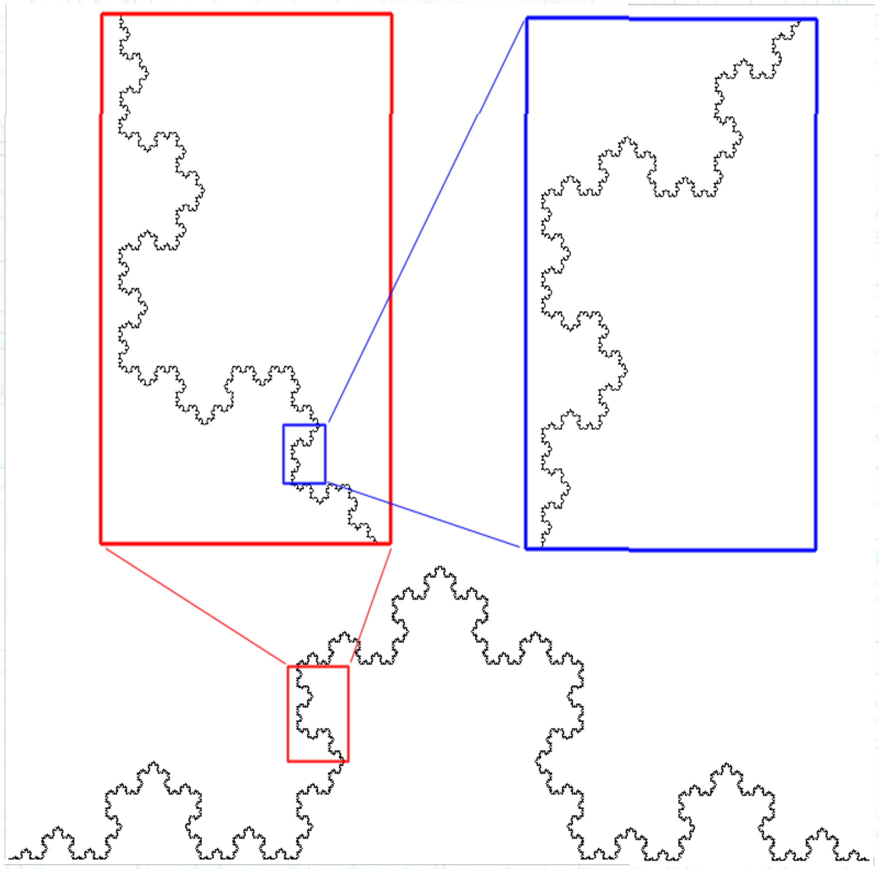
  - Going from point A to point B is done in steps glued together.

Ex: Brownian Random Walk



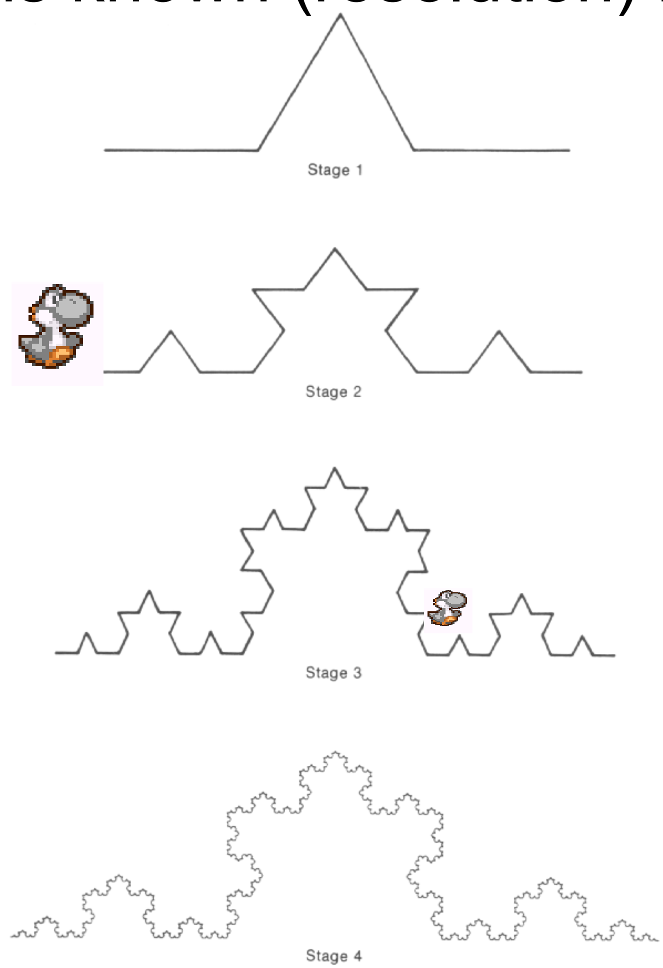
Random/Fractal path

- Attributes vary depending on resolution and scale.
- Non-smooth structure repeats.



Fractal Path

- Distance traveled (length) known depends on how well position is known (resolution) is known



Adapted from Benoit Mandelbrot, *Fractals.*

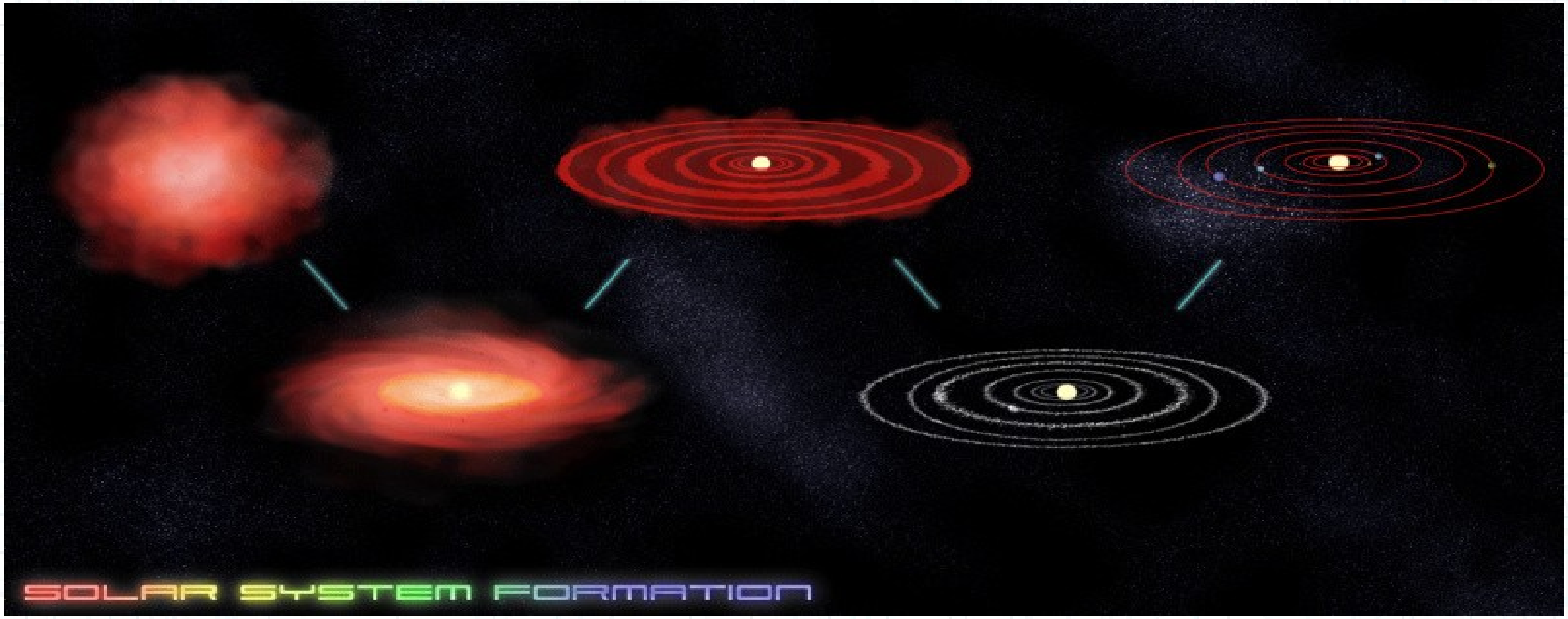
- The length of a Brownian path increases by 4 for every factor of two we zoom in.

Which large scale systems act like quantum particles?

- System must also depend on poperties related to quantum behavior:
  - Chaotic (Brownian) structuring.
  - A force applicable to all scales.

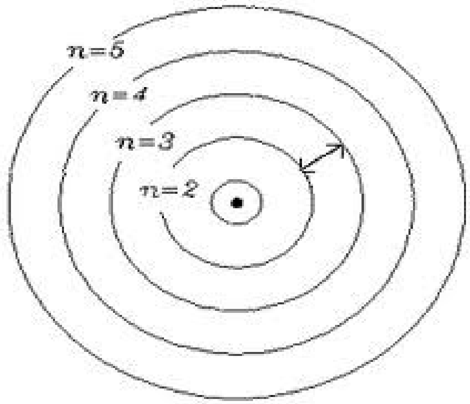
Celestial Systems

- Gravity affects all micro/macro scales
- Condensation

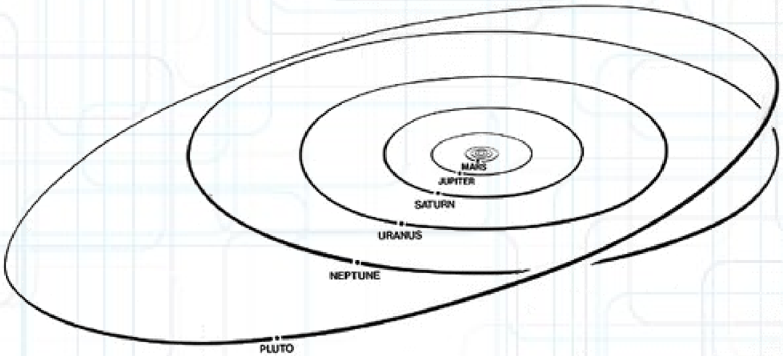


We Ask:

- Do quantum like properties apply to gravitational (Keplarian) systems?
- If so planets would structure in discrete states as seen in quantum mechanics.

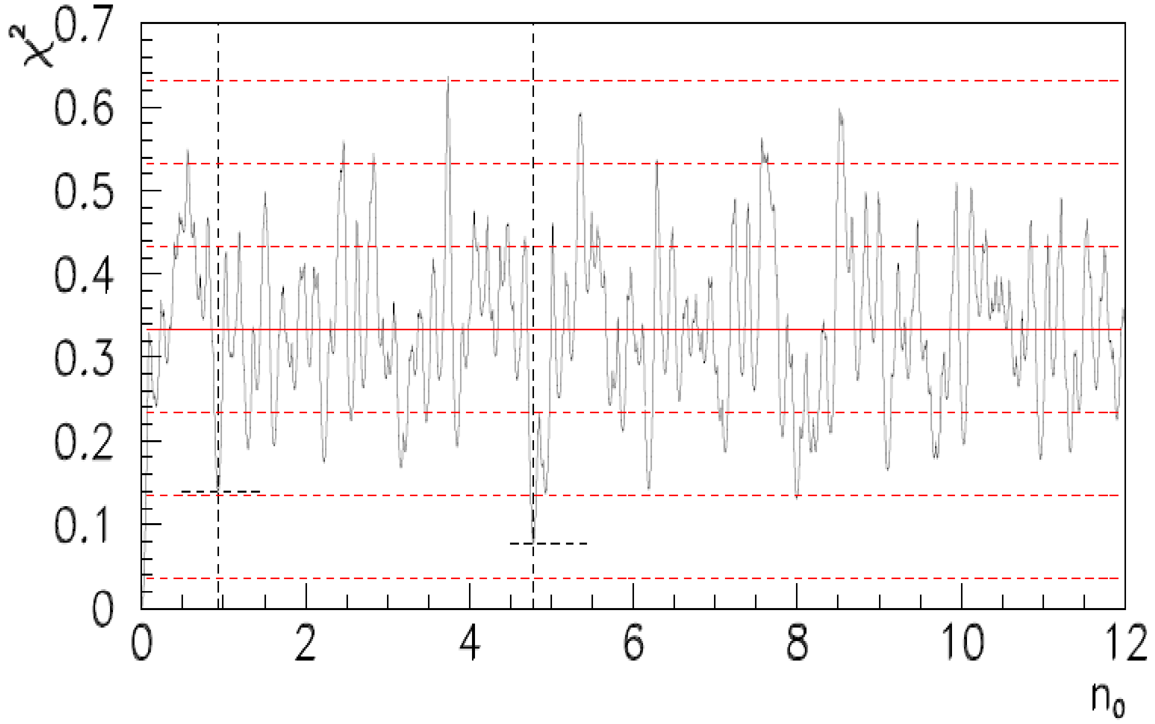


$$n = n_o\left(\frac{T/T_o}{M/M_o}\right)^{\frac{1}{3}}$$



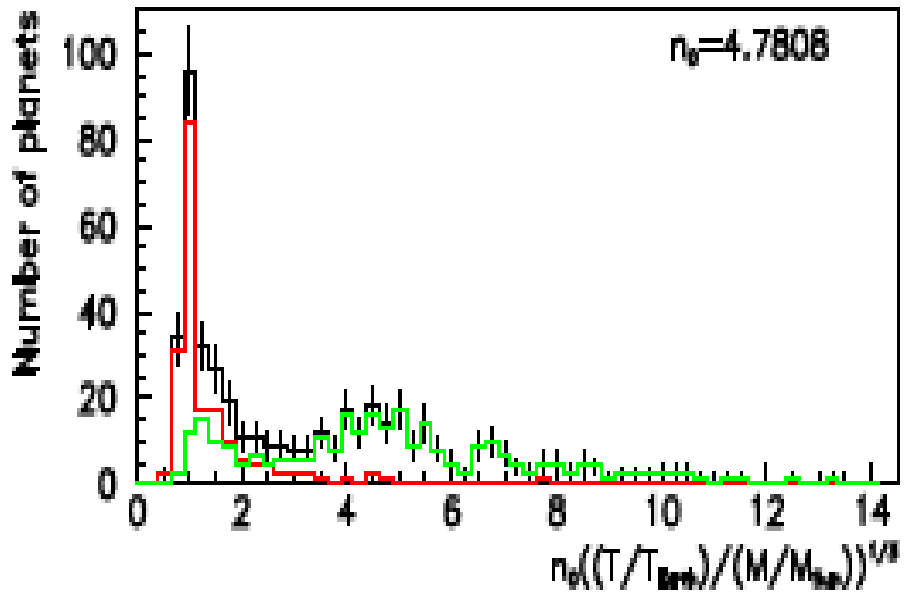
Observations

- For our solar system we find n = 4.8.



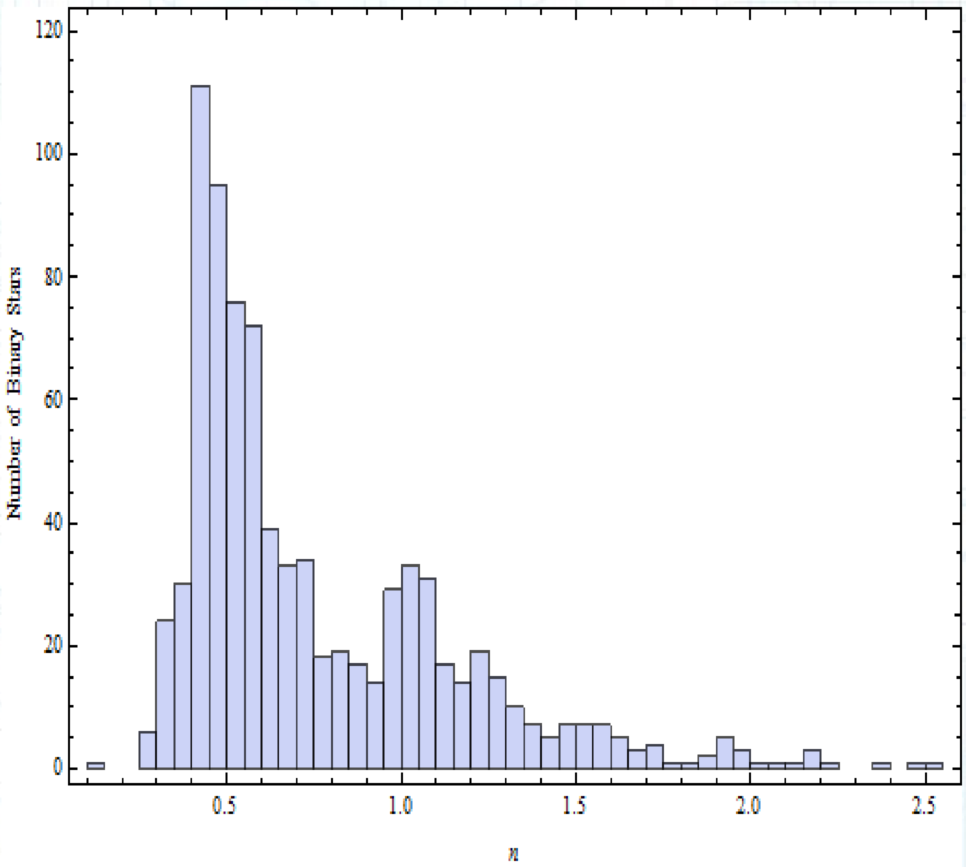
Observations

- For our solar system we find n = 4.8, applying this to a database of 477 exoplanets:



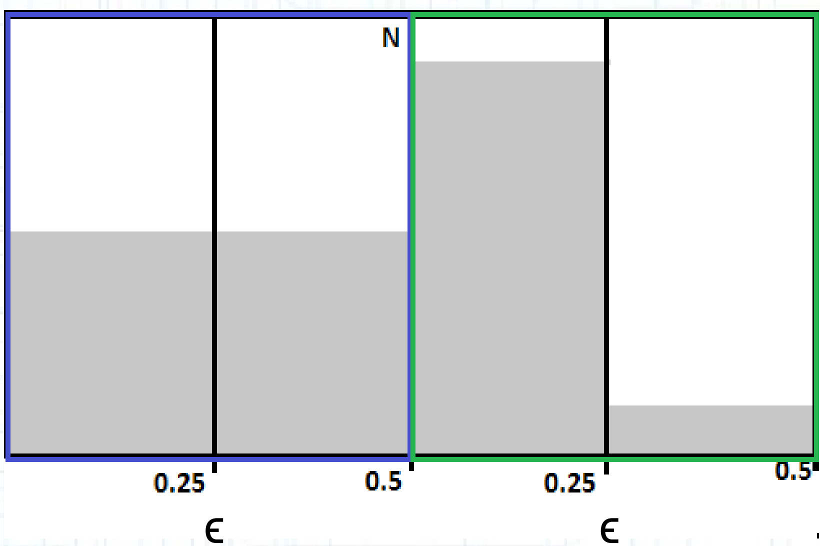
Binary Stars

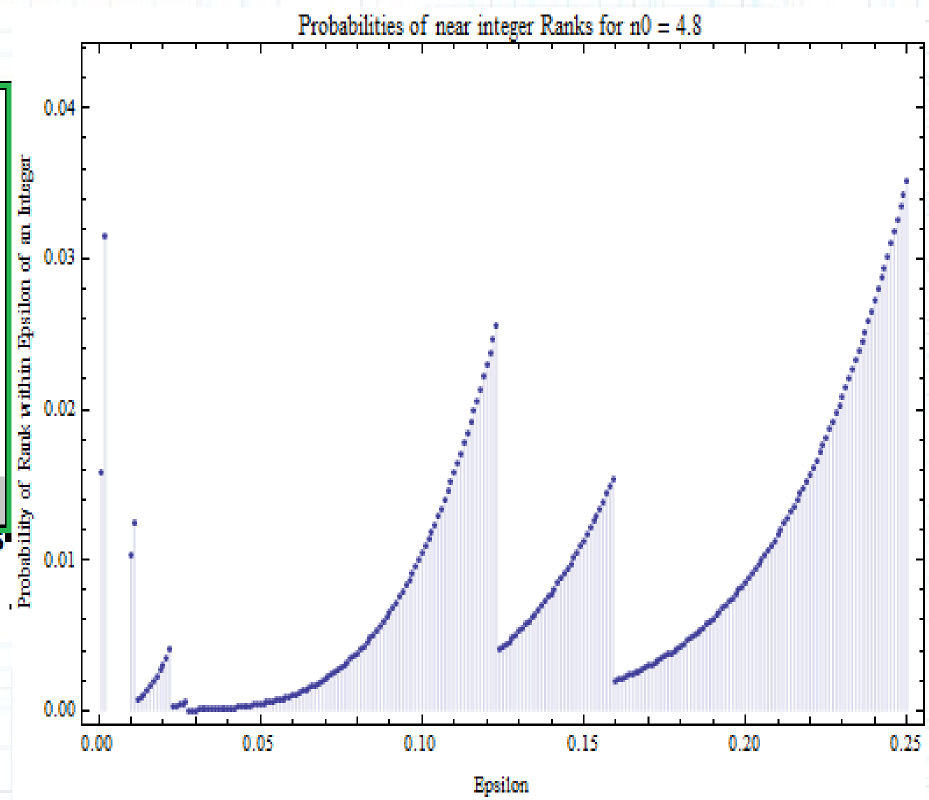
- Showing the number of star systems which share rank values.



Are our results chance?

- What is the probability that our results are not coincidence or caused by measurement error?





Potentials

- Binary stars require a different potential since non-Keplerian (no central mass).

$$n = \frac{n_o}{T^{2/3}} \frac{(m_1 m_2)^2}{(m_1 + m_2)^{4/3}} - \frac{1}{2}$$

- Physics could potentially be scale dependent.

Questions?

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Images:

Koch Curve:
http://blackhistoryfactorfiction.com/wp-content/uploads/2012/01/RecursiveFractals11.gif

Orbital Diagram:
http://thisquantumworld.com/wp/the-technique-of-quantum-mechanics/the-hydrogen-atom/

Planetary Formation:
http://astro.unl.edu/naap/esp/introduction.html

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