Homework 1: Probability Basics

Due: Thursday, January 23 (Your answers are due at the beginning of the class on the due date. The preferred mode of submission is as a *PDF file* via Gradescope. Please see the instructions on the course webpage. These instructions will tell you how to create a Markdown document in R, and how to "knit" it into a PDF file.)

Note. As per course policy, discussing the concepts behind the problems is OK, but you must write up the answers entirely by yourself. Please see the course policy document (on Canvas or the course webpage) for details.

Q1. Suppose A and B are two **disjoint** events in the sample space is Ω. Suppose $P(A) = 0.4$. **Justify your answers using a step-by-step reasoning or by drawing a Venn diagram.**

a. Is it possible to have $P(B) = 0.7$?
b. Find $P(A \cap B^c)$ (recall that B^c is the event that is the complement of B)

Q2. Let the outcomes of a coin toss be denoted H, T (for heads, tails). Write down answers for the following (you do **not** need to provide any justification).

a. What is the sample space for the experiment of tossing the coin *three* times?
b. Write down the set of outcomes corresponding to the events:
    1. we throw tails exactly two times
    2. we throw tails at least two times
    3. tails did not appear before a head appeared

Q3. Consider the experiment of sampling from a population, and let Ω be the set of people (i.e., the sample space or the set of outcomes). Define events $R =$ "person is rich" and $F =$ "person is famous". Write an expression for each of the following events using set operations involving the events R and F . **You can just give the answer, and do not need to show any work.**

a. Person is not rich.
b. Person is either rich or famous (or both).
c. Person is either not rich, or not famous.

Q4. Suppose that $P(R) = 0.1$, $P(F) = 0.07$, and $P(R \cap F) = 0.03$ in the above problem. Find the numerical values for the rest of the probabilities. (**You need to show all the steps involved, or reason using a Venn diagram.**)

Q5. Suppose we have 16 bags, each of which contains two balls. Of these, 7 bags have two blue balls, 5 bags have one red and one blue ball, and 4 bags have two red balls. Consider the following experiment: we first pick one of the 16 bags at random (every bag is equally likely to be chosen), and then we choose one ball from the bag at random (again, each of the two balls in the bag is equally likely to be chosen).

Calculate the following quantities:

a. The probability that the ball chosen in the end is blue.
b. Given that the ball chosen in the end is blue, the probability that the *other* ball in the chosen bag is also blue.

You need to show all the steps involved in obtaining the answer.