Homework 3: Random variables

Due: Thursday, February 27 (Your answers are due at the beginning of the class on the due date. The preferred mode of submission is as a *PDF file* via Gradescope. Please see the instructions on the course webpage. These instructions will tell you how to create a Markdown document in R, and how to "knit" it into a PDF file.)

Note. As per course policy, discussing the concepts behind the problems is OK, but you must write up the answers entirely by yourself. Please see the course policy document (on Canvas or the course webpage) for details. **You need to show all the steps involved in obtaining the answer.**

Q1. Suppose we toss 4 coins (each having heads probability $= 1/2$). Let X denote the random variable: (number of heads) - (number of tails).

(a) What is the range of X? (give exact upper and lower bounds along with a line of explanation)

(b) What is the probability mass function of X?

(c) What is the cumulative density function of X?

Q2. Using R, simulate tossing 4 coins as above, and compute the random variable X defined in problem 1. Estimate the probability mass function you computed in part (b) of problem 1 by simulating 1000 times and averaging.

Q3. Suppose you toss n "fair" coins (i.e., heads probability $= 1/2$). For every coin that came up tails, suppose you toss it one more time. Let X be the random variable denoting the number of heads in the end.

(a) What is the range of the variable X (give exact upper and lower bounds)

(b) What is the distribution of X? (Write down the name and give a convincing explanation.)

[*Hint:* focus on what happens to a single coin]

Q4. Suppose we have a face recognition system with an accuracy of 90%. Suppose it is deployed in a security system that observes 40 people. Compute the following probabilities (*You should write down the expression mathematically, and an R expression that returns the answer*):

(a) It **fails** for exactly 6 people.

(b) It fails for *at most* 2 people.

(c) It fails for *at least* 8 people.

[*Hint:* Recall our derivations involving binomial coefficients in class!]