## Test 1: Similarity, Clustering, Frequent Items - PRACTICE

NAME:
FINAL SCORE:
This test allows one $8 \times 11.5$ inch notes sheet, front and back. Nothing electronic that can transmit/receive information is not (e.g., computers, phones, calculators, ipads). Unlimited blank scratch paper is allowed.

Absolutely no talking allowed, unless a TA is present and you are asking a question. Talking students will have their tests confiscated.

## $1 k$-Grams and Jaccard (35 points)

Consider the following two phrases:

$$
\begin{aligned}
& P_{1}: \text { mississippiriver } \\
& P_{2}: \text { mississippistate }
\end{aligned}
$$

A: (15 points) List the sets of distinct 3-character-grams for phrases $P_{1}$ and $P_{2}$. This should produce two sets $S_{1}$ and $S_{2}$.

B: (10 points) What is the Jaccard Similarity $\mathrm{JS}\left(S_{1}, S_{2}\right)=\frac{\left|S_{1} \cap S_{2}\right|}{\left|S_{1} \cup S_{2}\right|}$ between $S_{1}$ and $S_{2}$ ?

C: (10 points) What is the Andberg Similarity $\operatorname{Andb}\left(S_{1}, S_{2}\right)=\frac{\left|S_{1} \cap S_{2}\right|}{\left|S_{1} \cap S_{2}\right|+2\left|S_{1} \triangle S_{2}\right|}$ between $S_{1}$ and $S_{2}$ ?

## 2 MinHashing (20 points)

Consider three sets $T_{1}, T_{2}$, and $T_{3}$, and the 10 -dimensional min-hash vectors for each set.

$$
\begin{aligned}
& T_{1}: v_{1}=(17,3,4,11,07,03,02,10,72,18) \\
& T_{2}: v_{2}=(11,3,4,01,17,30,12,11,41,24)
\end{aligned}
$$

(Note: each entry in a vector can be thought of as the result of creating a table, permuting the rows, and then returning the index of the highest element in the set. )

A: (10 points) Use $v_{1}$ and $v_{2}$ to estimate the Jaccard Similarity between $T_{1}$ and $T_{2}$ ?

B: (5 points) Will changing the 8 th entry in $v_{2}$ (shown in bold) from an 11 to a 10 decrease the estimated Jaccard Similarity between $T_{1}$ and $T_{2}$ ? Explain your answer.

C: (5 points) Say we get a new set $T_{3}$, and we still remember the hash functions used to create $v_{1}$, and $v_{2}$. So we create a min-hash vector $v_{3}$ to represent $T_{3}$. What part of $v_{2}$ if any, do we need to recalculate to estimate the Jaccard similarity between $T_{3}$ and $T_{2}$ ? Explain your answer.

## 3 Clustering (45 points)

Consider 5 points, and mainly running a form of hierarchical agglomerative clustering (HAC) using the Euclidean distance:
$q_{1}=(2,5)$
$q_{2}=(1,4)$
$q_{3}=(2,2)$
$q_{4}=(1.5,-1)$
$q_{5}=(2,1)$
We will consider two ways to measure the distance between clusters $S_{1}$ and $S_{2}$ :

- Single-Link uses $d\left(S_{1}, S_{2}\right)=\min _{\left(s_{1}, s_{2}\right) \in S_{1} \times S_{2}}\left\|s_{1}-s_{2}\right\|_{2}$ it measures the closest pair of points.
- Complete-Link uses $d\left(S_{1}, S_{2}\right)=\max _{\left(s_{1}, s_{2}\right) \in S_{1} \times S_{2}}\left\|s_{1}-s_{2}\right\|_{2}$
 it measures the furthest pair of points.

A: (10 points) Run HAC using Single-Link until there are 3 clusters. Report the resulting 3 clusters as sets.

B: (5 points) Run HAC using Single-Link until there are 2 clusters. Report the resulting 2 clusters as sets.

C: (10 points) Run HAC using Complete-Link until there are 3 clusters. Report the resulting 3 clusters as sets.

D: (5 points) Run HAC using Complete-Link until there are 2 clusters. Report the resulting 2 clusters as sets.

E: (10 points) Run Gonzalez Algorithm (the greedy algorithm for $k$-center clustering, assignment-based clustering) starting with the first center $c_{1}=q_{1}$ until there are 2 clusters. Report the resulting 2 clusters as sets.

F: (5 points) Run Gonzalez Algorithm (the greedy algorithm for $k$-center clustering, assignment-based clustering) starting with the first center $c_{1}=q_{1}$ until there are 3 clusters. Report the resulting 3 clusters as sets.

