

Secure Nearest Neighbor Revisited

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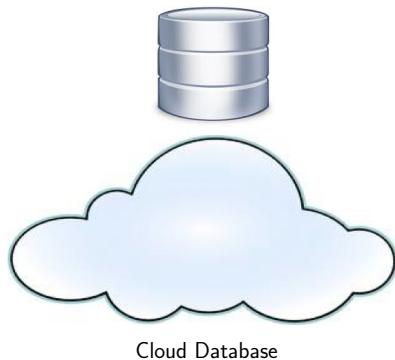


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July 11, 2013

The Motivation

- Cloud databases: Google Cloud SQL, Microsoft SQL Azure, Amazon SimpleDB.



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- Service providers (SP) answer queries from different clients.



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- Service providers (SP) answer queries from different clients.
- Data owner might not want to reveal data values to SP; clients might not want SP to learn their queries and/or the query results.



Database Encryption



Cloud Database

Hakan Hacigumus, Balakrishna R. Iyer, Chen Li, Sharad Mehrotra: Executing SQL over encrypted data in the database-service-provider model. SIGMOD 2002



cloud server

Introduction and Motivation

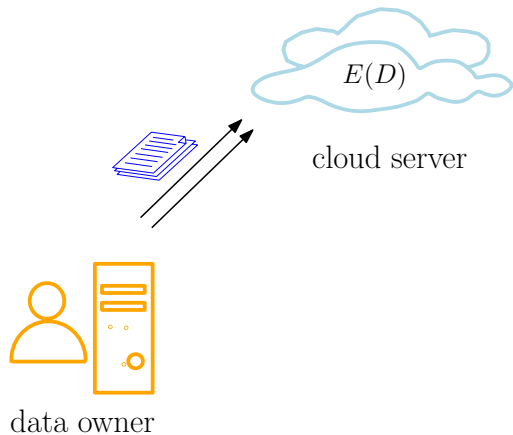


cloud server

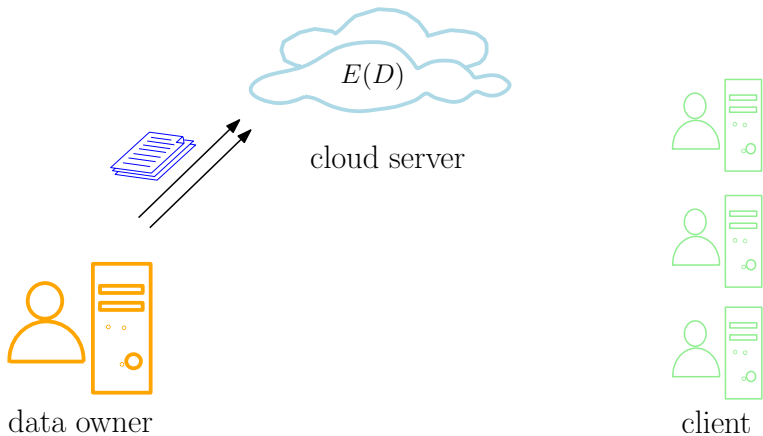


data owner

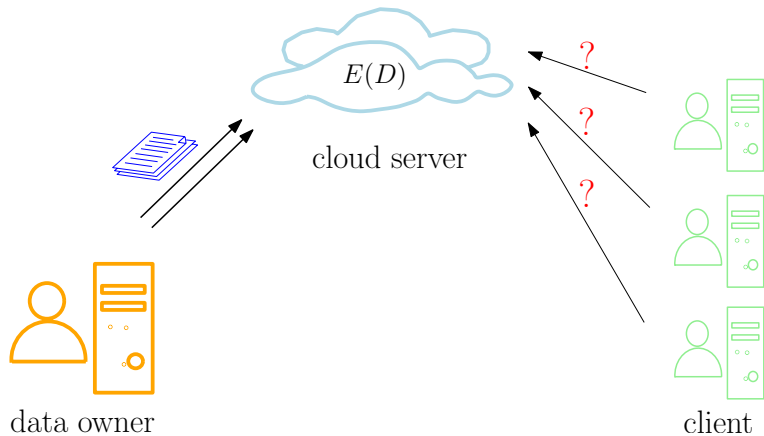
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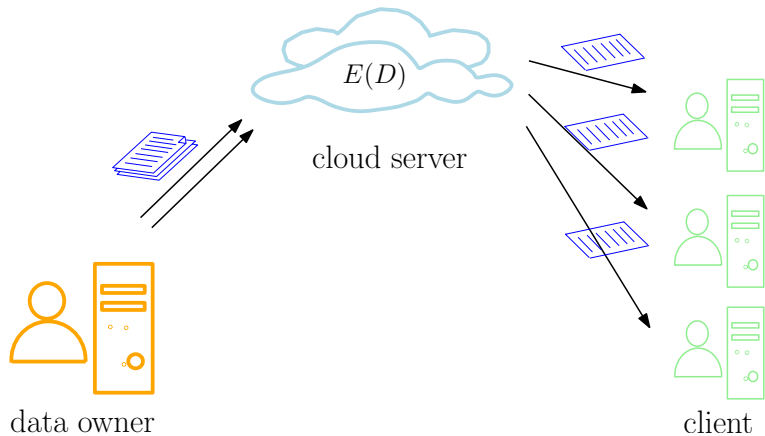
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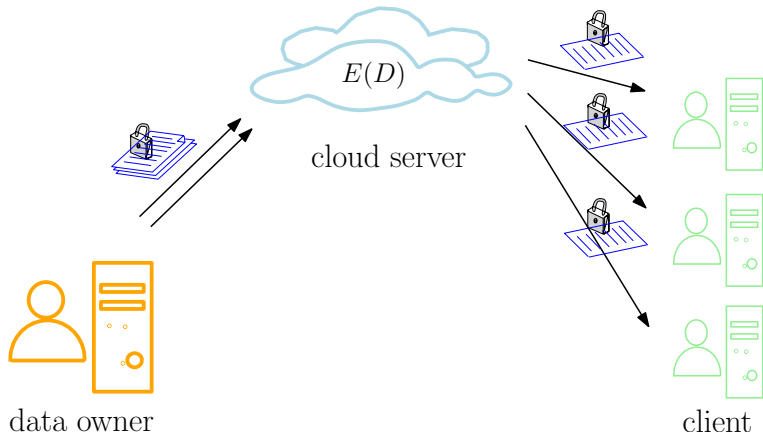
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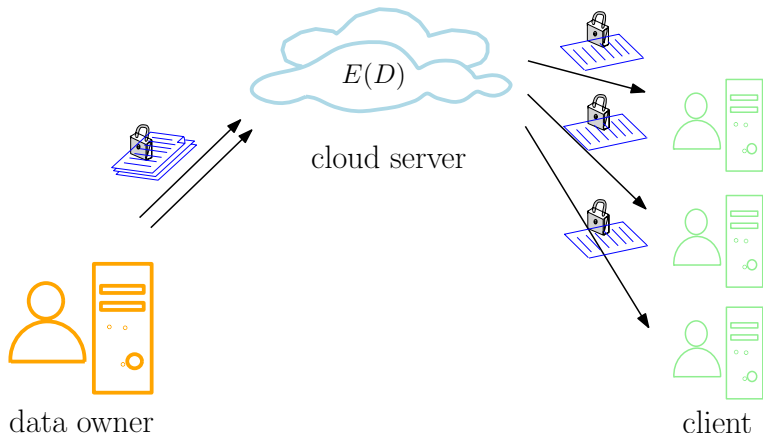


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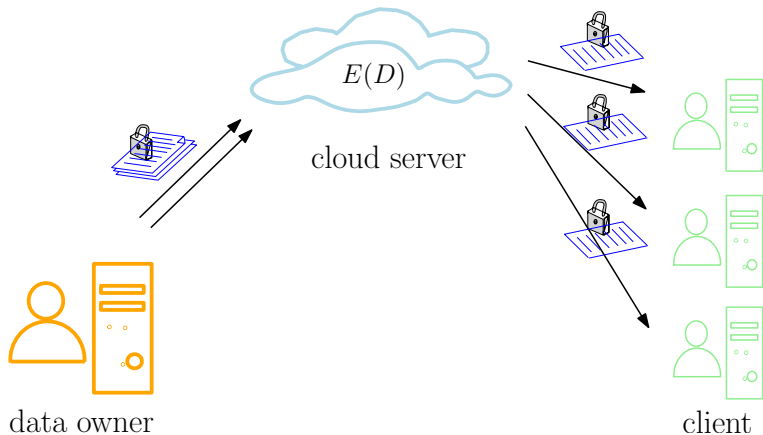
Introduction and Motivation

- Secure Query Processing



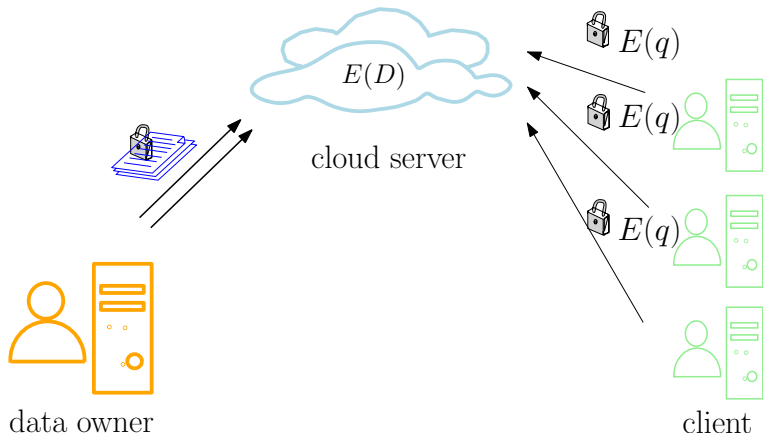
Introduction and Motivation

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 - Secure Nearest Neighbor (SNN)



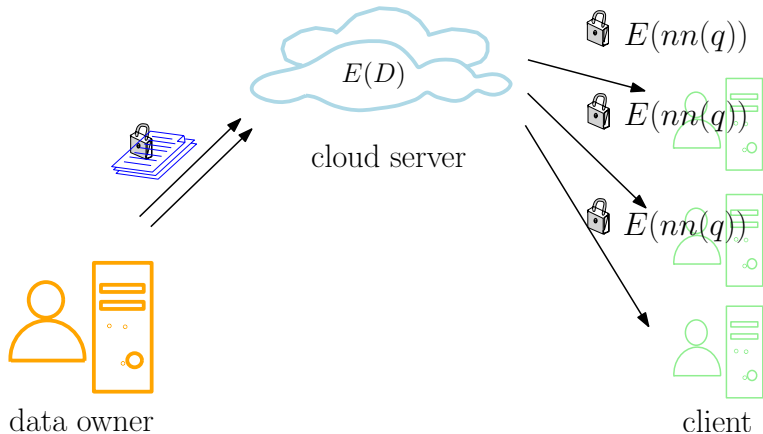
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 - Fully homomorphic encryption encryption due to Craig Gentry, “A Fully Homomorphic Encryption Scheme (Ph.D. thesis)”: mostly of theoretical interest, impractical, and inefficient for large data.
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 - Adversary model: same as whatever model in which E is secure, e.g, IND-CPA, IND-CCA.

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- To appear in ICDE'13.

Insecurity of Existing Methods

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 - Attack we found: after learning only d query points and their encryptions, a linear system of d equations can be formed to decrypt any encrypted $p \in D$.

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 - Attack we found: In the above process, the server learns if q lies to the left or the right of another point, in each dimension, which leads to a binary search to efficiently recover any encrypted point.

Hardness of the Problem: OPE

- Order-preserving encryption (OPE) is a set of functions $\{\mathcal{E}, \mathcal{E}^{-1}, op\}$, such that:
 - $\mathcal{E}(m) = c, \mathcal{E}^{-1}(c) = m$ (here we omit the keys).
 - $op(c_1, c_2) = 1$ if $m_1 < m_2$; $op(c_1, c_2) = -1$ if $m_1 > m_2$.

Rakesh Agrawal, Jerry Kiernan, Ramakrishnan Srikant, Yirong Xu: Order-Preserving Encryption for Numeric Data. SIGMOD 2004

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Theorem

A truly secure OPE does not exist in standard security models, such as IND-CPA. It also does not exist even in much relaxed security models, such as the indistinguishability under ordered chosen-plaintext attack (IND-OCPA).

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Alexandra Boldyreva, Nathan Chenette, Younho Lee, Adam O'Neill: Order-Preserving Symmetric Encryption. EUROCRYPT 2009
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Hardness of the Problem: SNN gives OPE

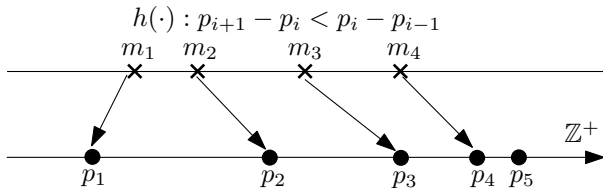
- Given $E(D) = \{E(p_1), \dots, E(p_N)\}$, suppose we have a secure SNN method S such that: $S(E(q), E(D)) \rightarrow E(nn(q, D))$ without the knowledge of E^{-1} .

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- We can construct an OPE, $\{\mathcal{E}, \mathcal{E}^{-1}, op\}$, based on $S(\cdot)$!

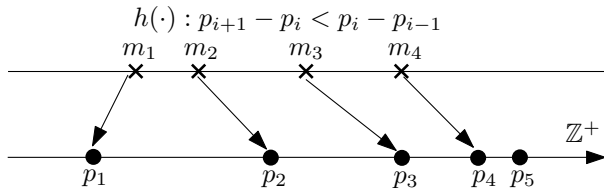
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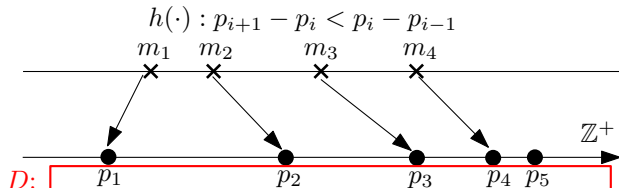
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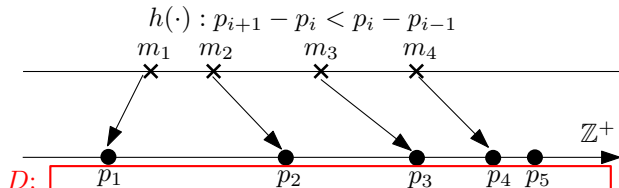


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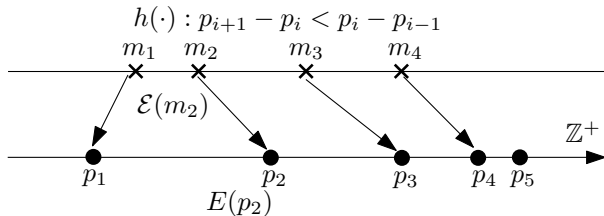
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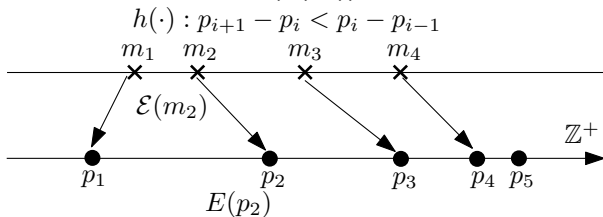
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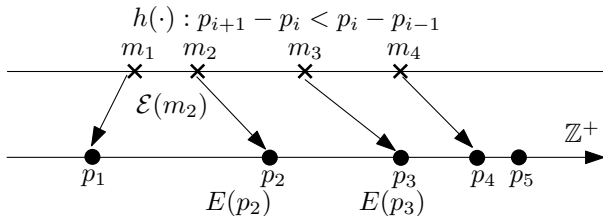
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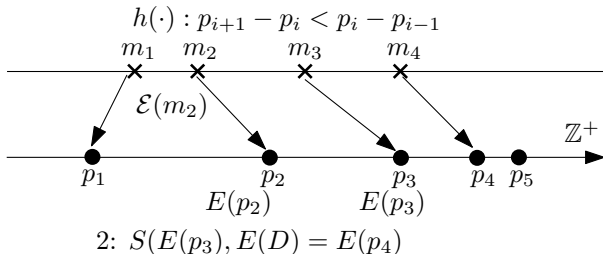
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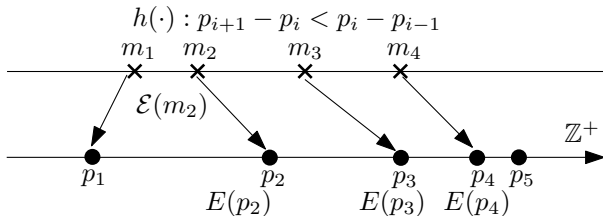
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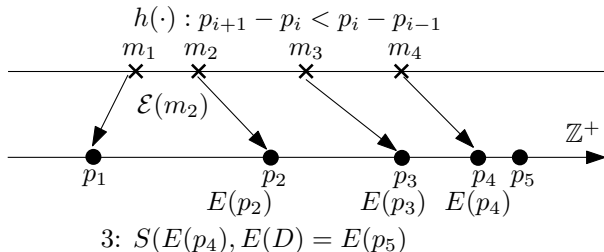
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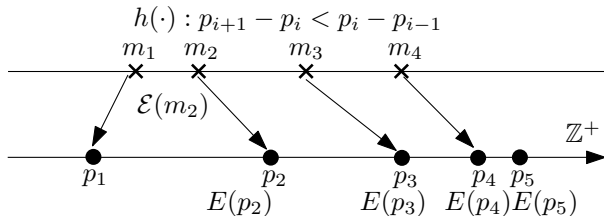
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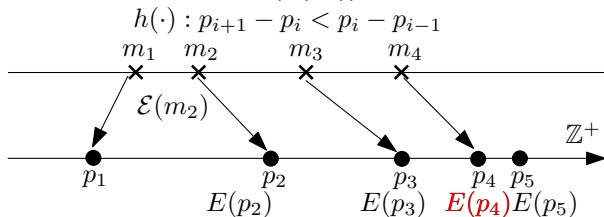
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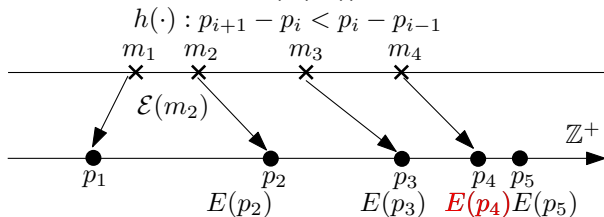
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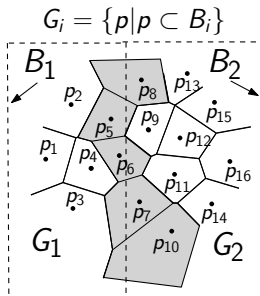
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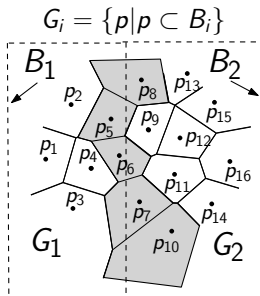
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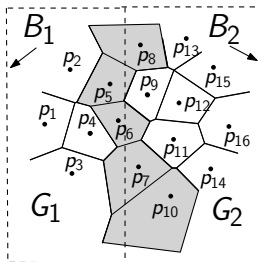
- It only says it is hard to output $E(\text{nn}(q, D))$! What if we relax this restriction and allow something “less precise”?
- Extreme case: just return $E(D)$ and ask client to decrypt and find $\text{nn}(q, D)$. Obviously secure! But expensive!
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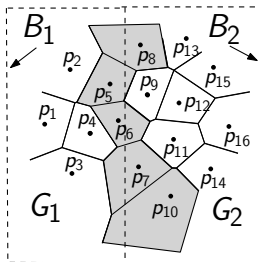
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Challenge:
 $\min \max(|G_i|)$!

Solution Overview

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- Secure Voronoi Diagram (SVD):
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 - Query processing at the client

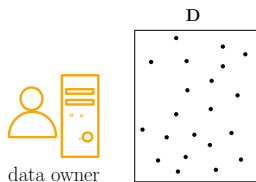
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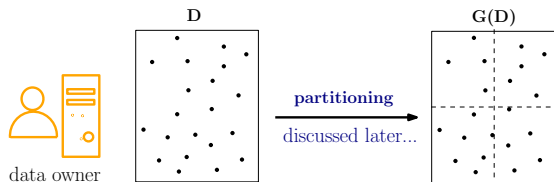
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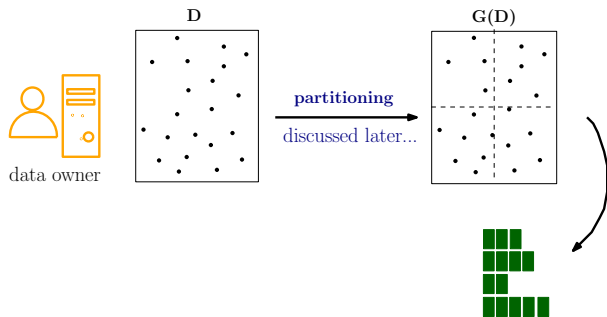
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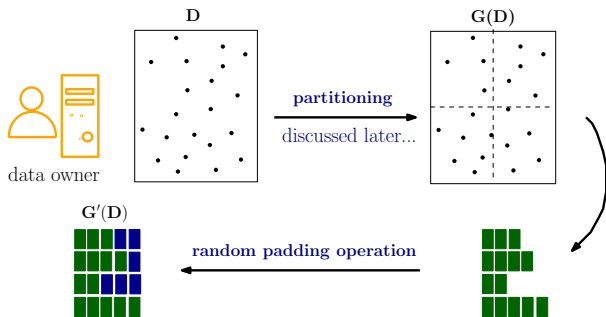
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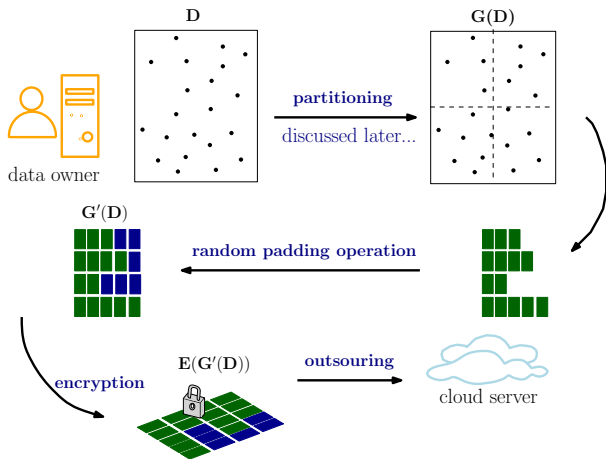
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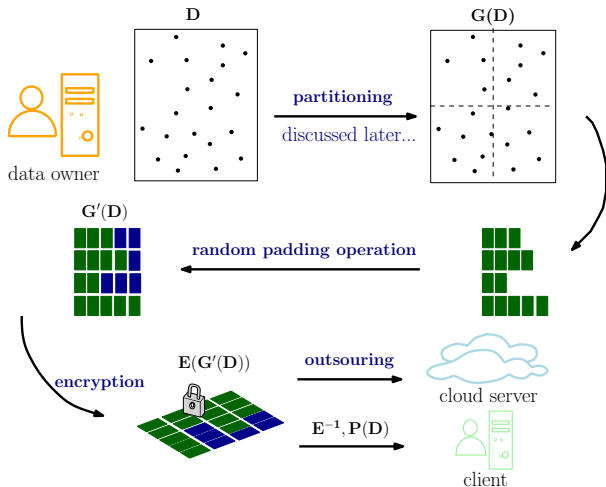
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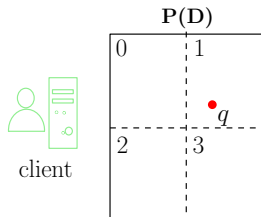
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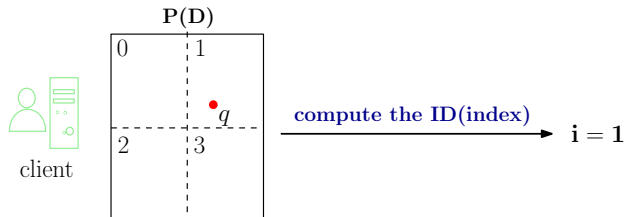
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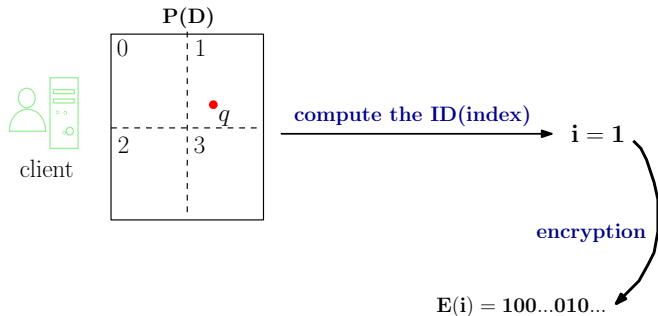
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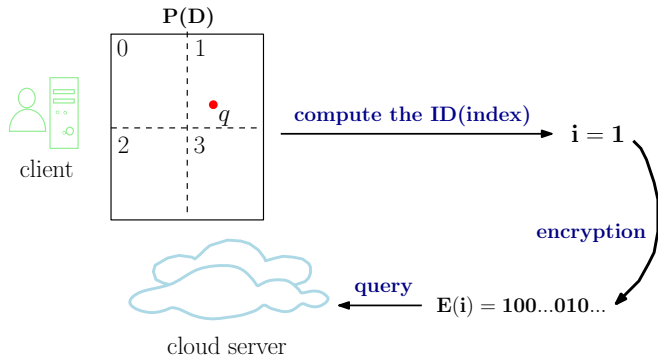
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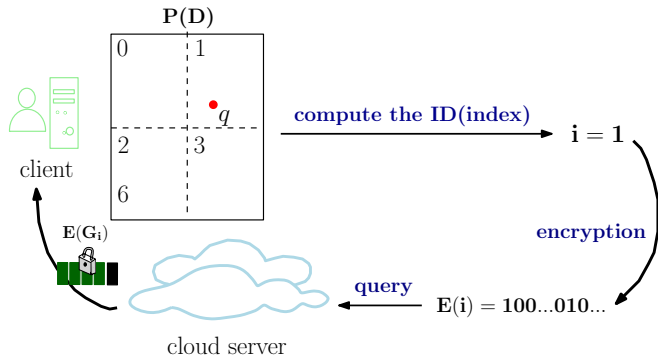
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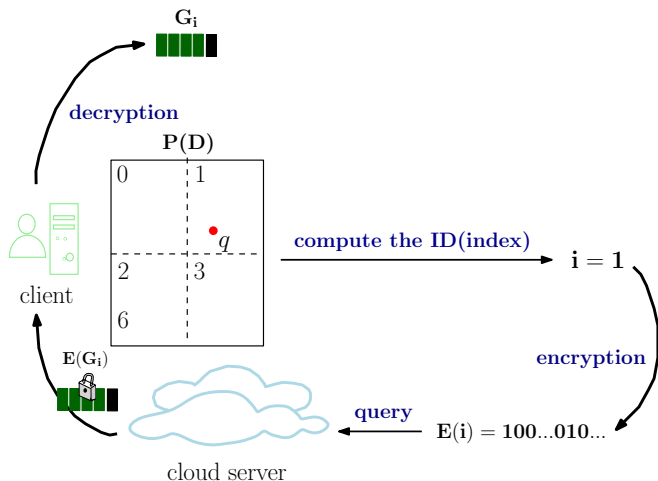
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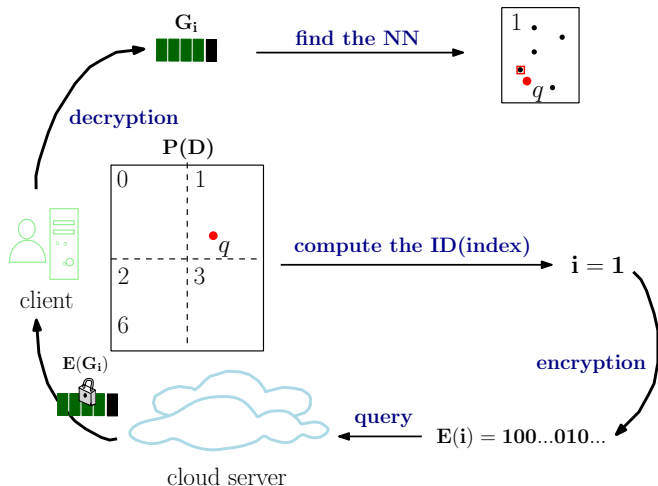
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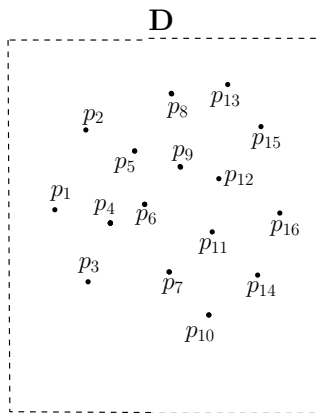
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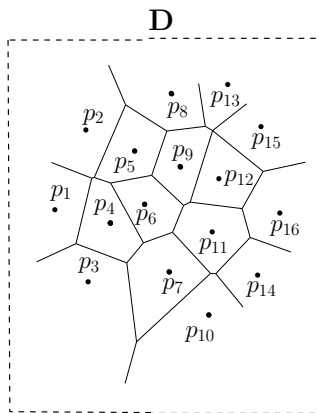


SVD Partitioning Principle

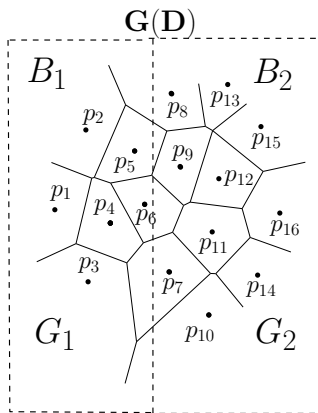
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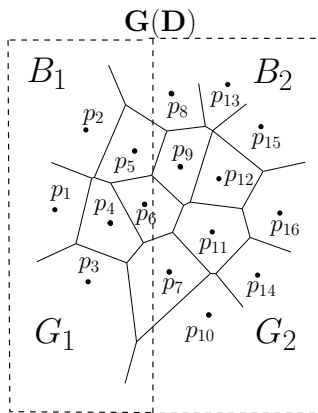
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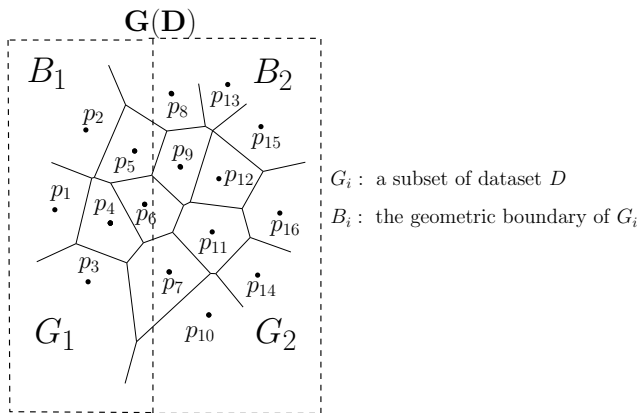
SVD Partitioning Principle



G_i : a subset of dataset D

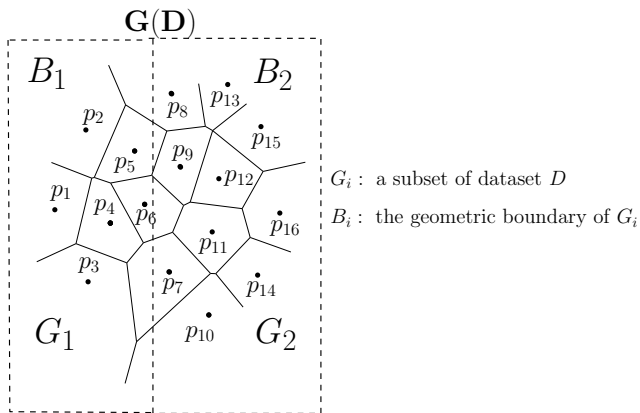
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SVD Partitioning Principle



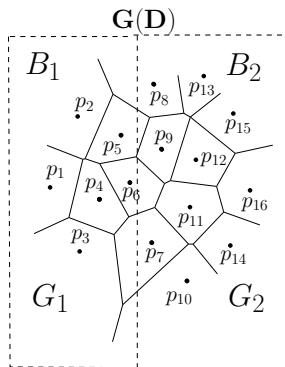
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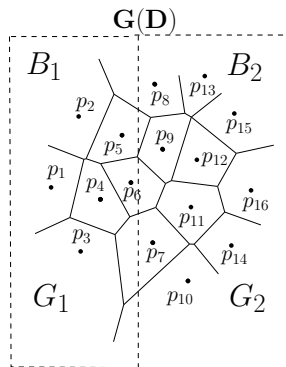
B_i : the geometric boundary of G_i

$$G_1 = \{p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_{10}\}$$

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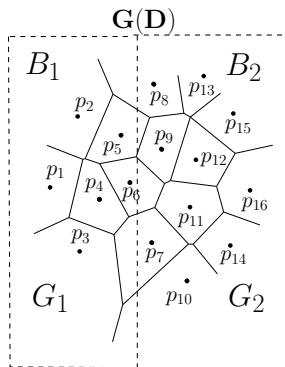
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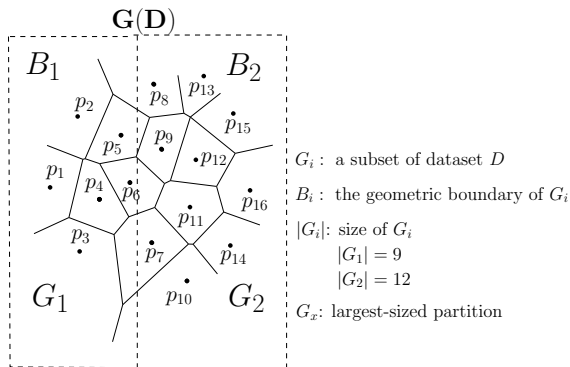
$$|G_2| = 12$$

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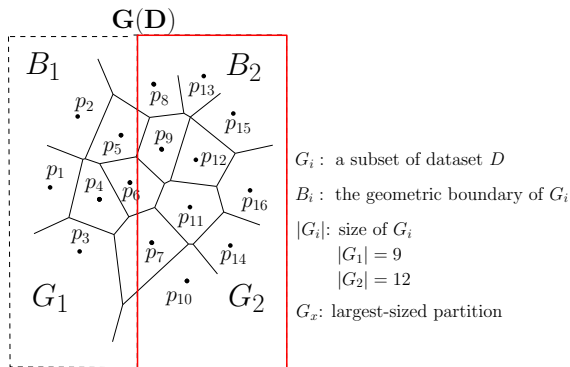


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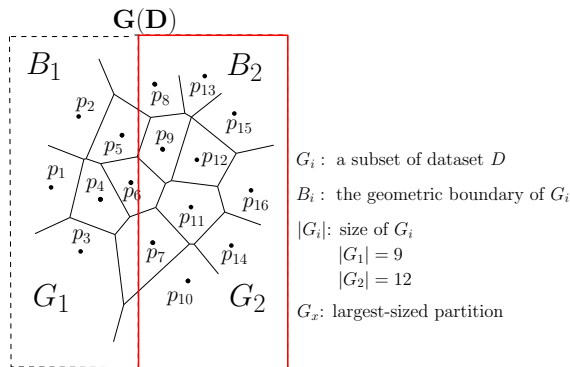


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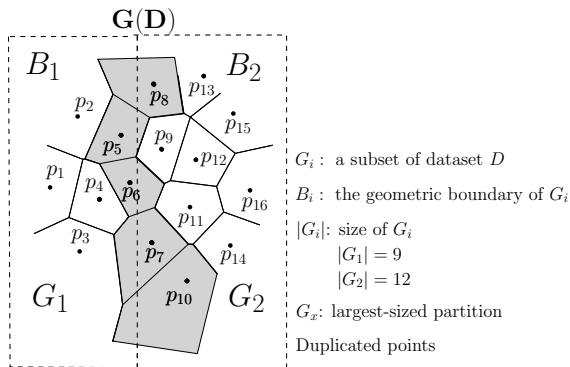


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- 3 minimum $|G_x|$ and minimum $|G_x| - |G_i|$, which means low storage and communication overheads, as well as cheap encryption cost

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SVD Partitioning

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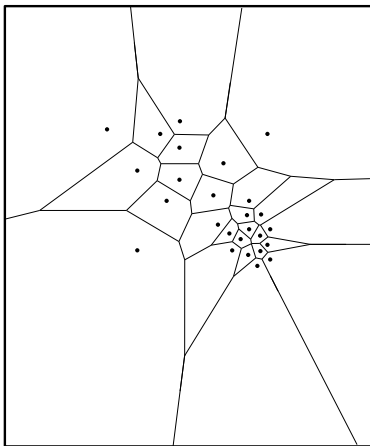
- Square Grid (SG)
- Minimum Space Grid (MinSG)
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SVD Partitioning

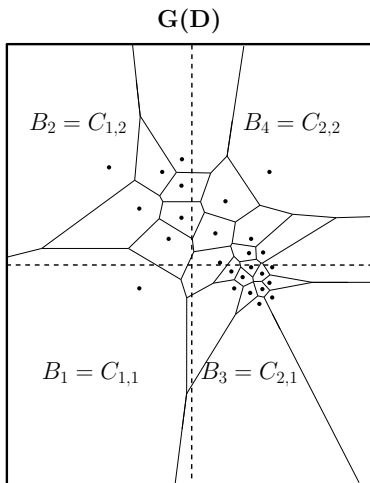
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Square Grid (SG)

D



Square Grid (SG)



Square Grid (SG)

- Merits:

- Demerits:

Square Grid (SG)

- Merits:
 - simple
 - minimum storage cost at client
- Demerits:

Square Grid (SG)

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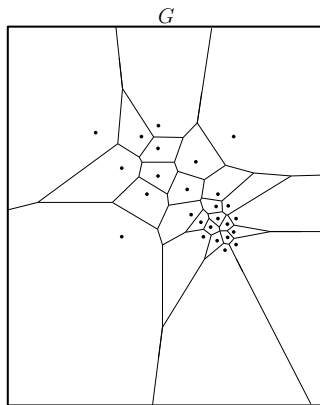
- high storage and communication overheads, as well as expensive encryption cost because of highly unbalanced partitions when the data distribution is skewed

SVD Partitioning

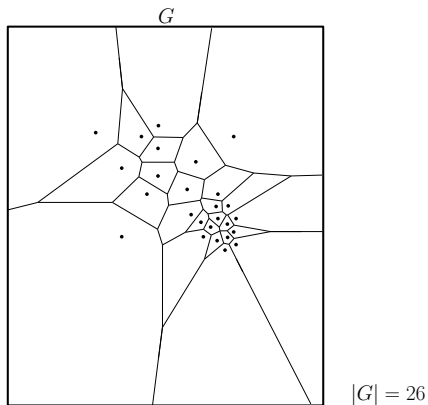
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Minimum Space Grid (MinSG)

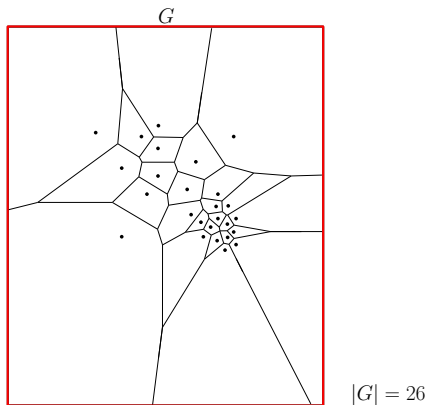
Minimum Space Grid (MinSG)



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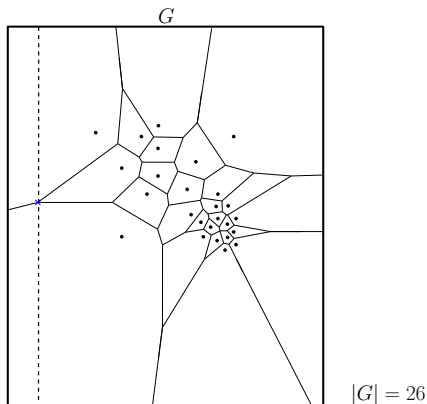


Minimum Space Grid (MinSG)



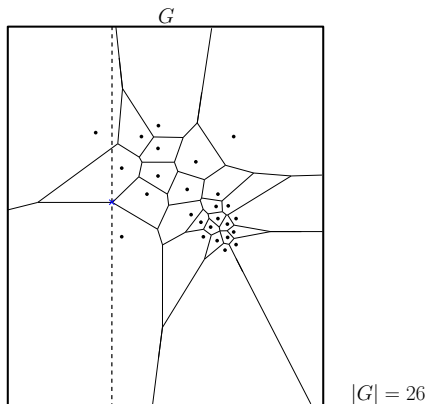
- A greedy algorithm: always split the maximum partition G_x into smaller partitions

Minimum Space Grid (MinSG)



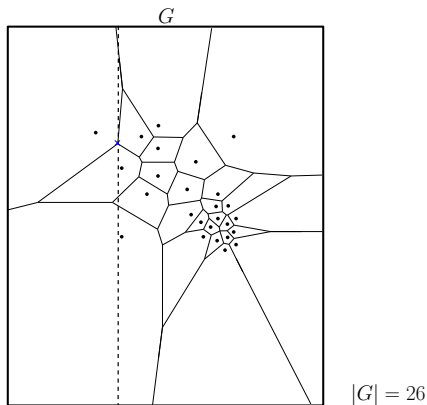
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Minimum Space Grid (MinSG)



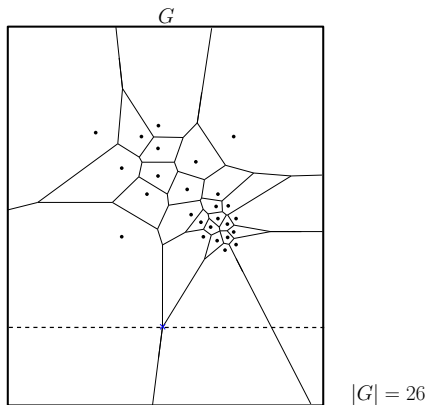
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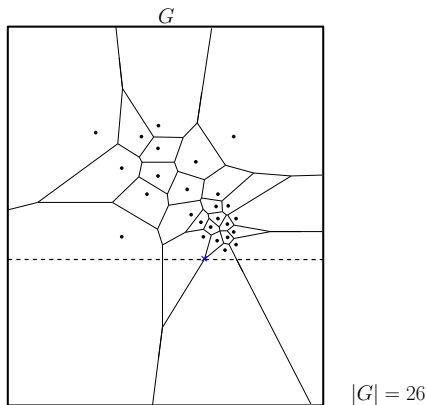
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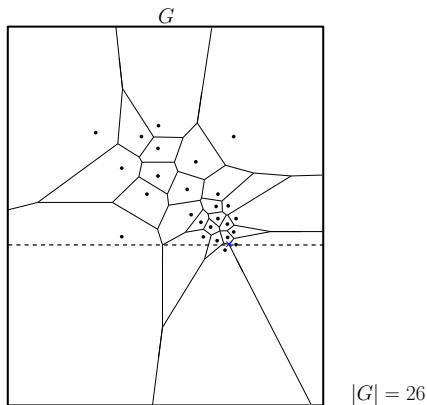
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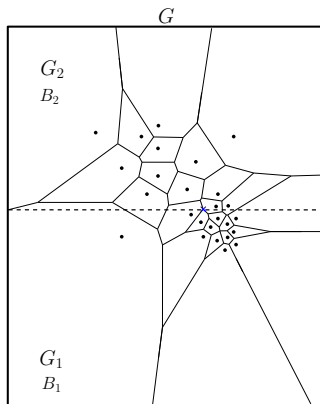
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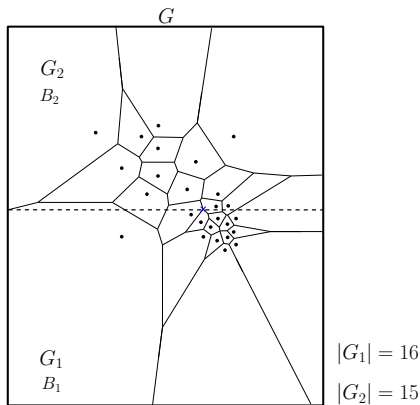
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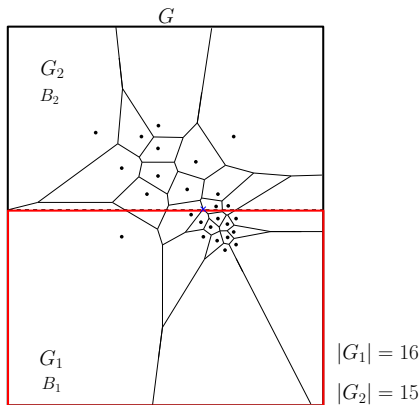
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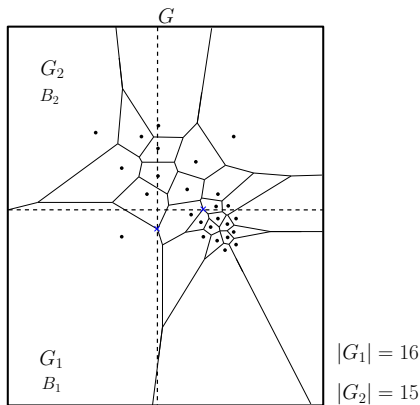
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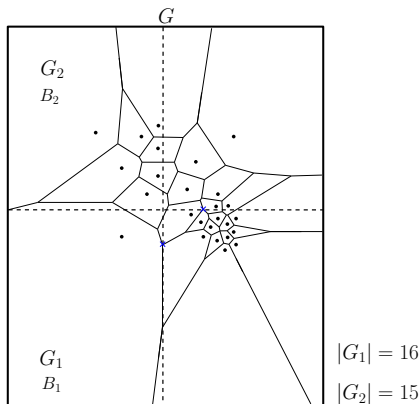
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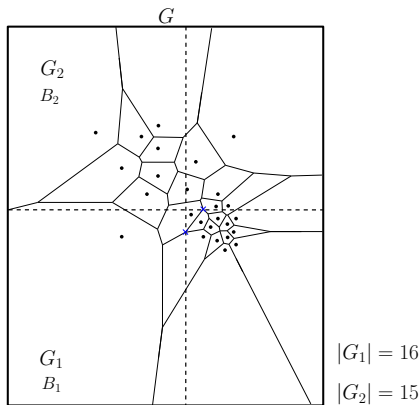
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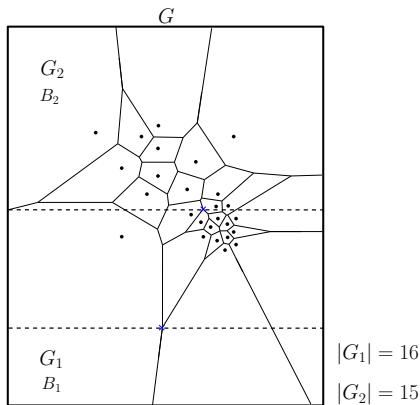
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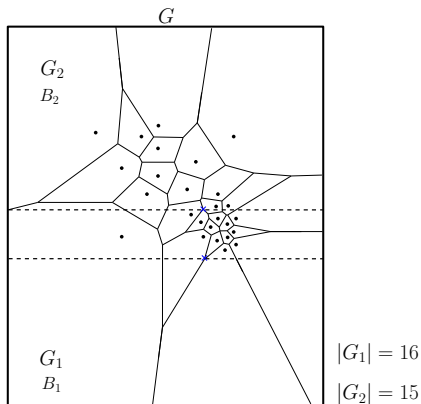
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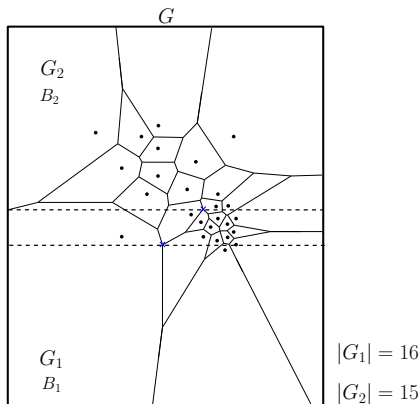
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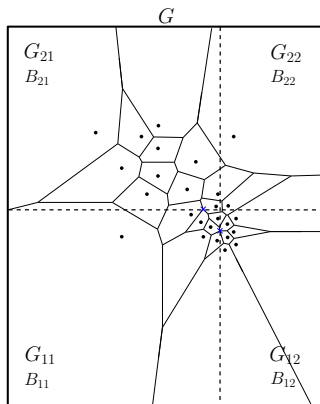
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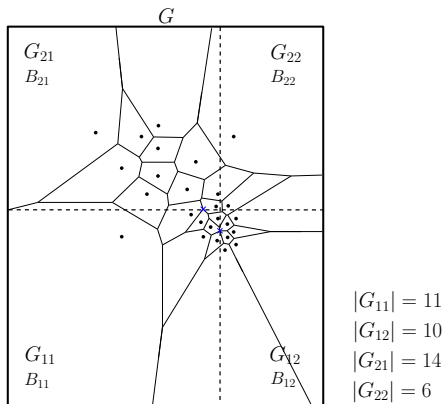
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Minimum Space Grid (MinSG)

- Merits:

- Demerits:

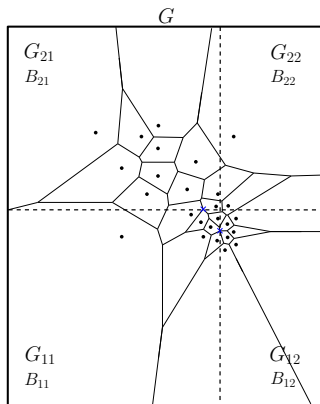
Minimum Space Grid (MinSG)

- Merits:
 - relatively balanced partitions: low storage and communication overheads, as well as cheap encryption cost
- Demerits:

Minimum Space Grid (MinSG)

- Merits:
 - relatively balanced partitions: low storage and communication overheads, as well as cheap encryption cost
- Demerits:
 - complicated partitioning process
 - not most balanced: small-sized partitions introduced by some unnecessary splitting

Minimum Space Grid (MinSG)



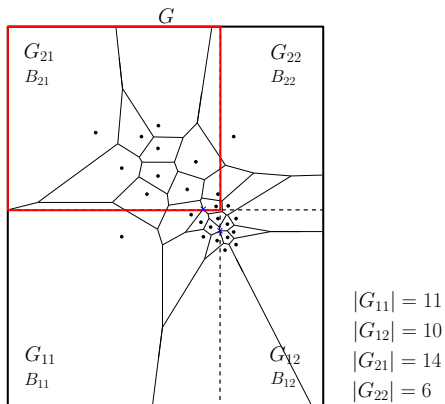
$$|G_{11}| = 11$$

$$|G_{12}| = 10$$

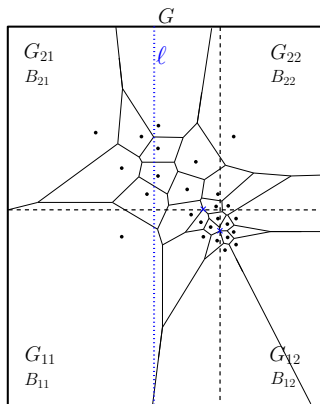
$$|G_{21}| = 14$$

$$|G_{22}| = 6$$

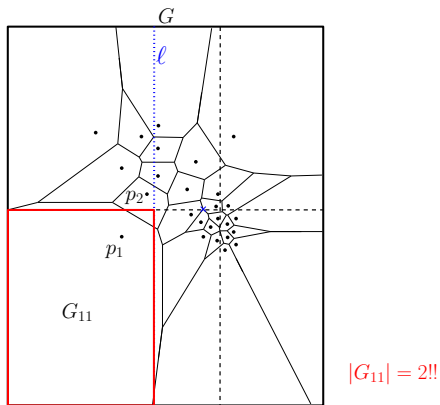
Minimum Space Grid (MinSG)



Minimum Space Grid (MinSG)



Minimum Space Grid (MinSG)



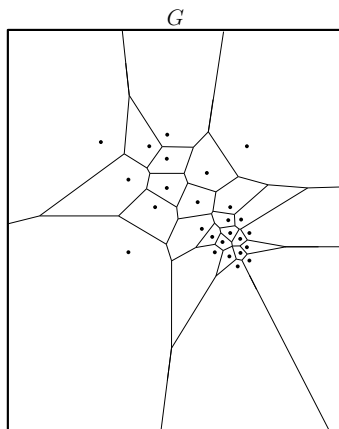
- We need a method that produce more balanced partitions!!

SVD Partitioning

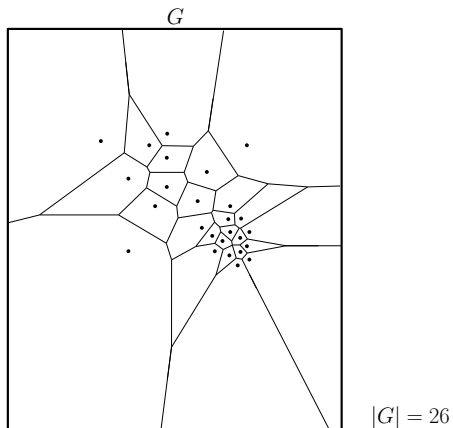
- Square Grid (SG)
- Minimum Space Grid (MinSG)
- Minimum Maximum Partition(MinMax)

Minimum Maximum Partition (MinMax)

Minimum Maximum Partition (MinMax)

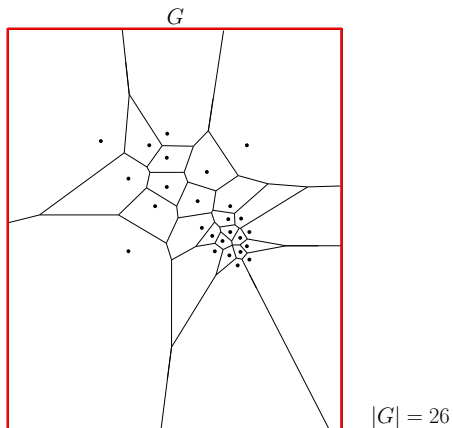


Minimum Maximum Partition (MinMax)



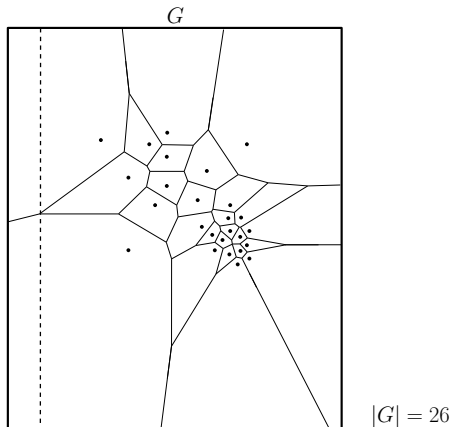
- similar to MinSG in most part

Minimum Maximum Partition (MinMax)



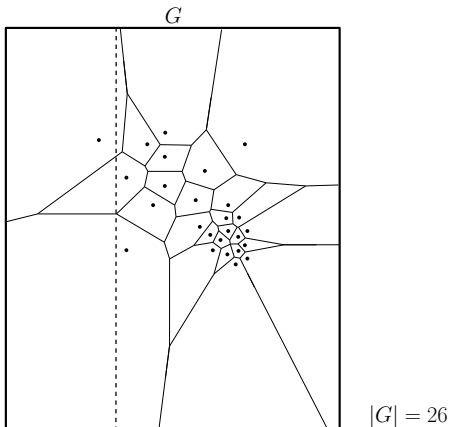
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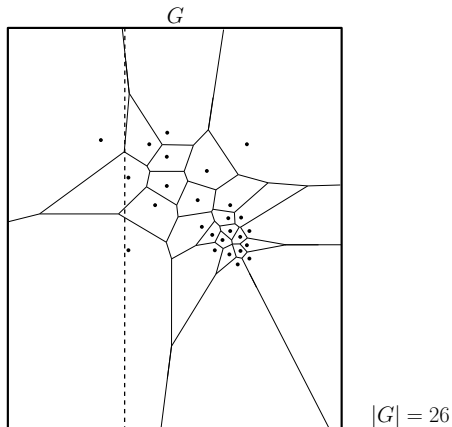
- similar to MinSG in most part
- use **segments** going through the space bounded by B_x instead of lines going through the entire space to split partitions

Minimum Maximum Partition (MinMax)



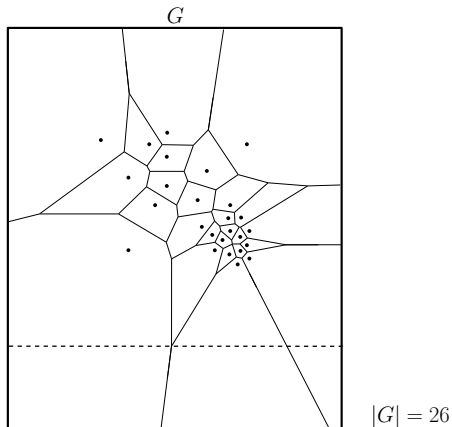
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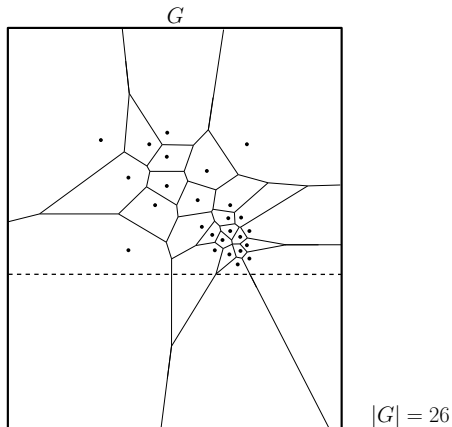
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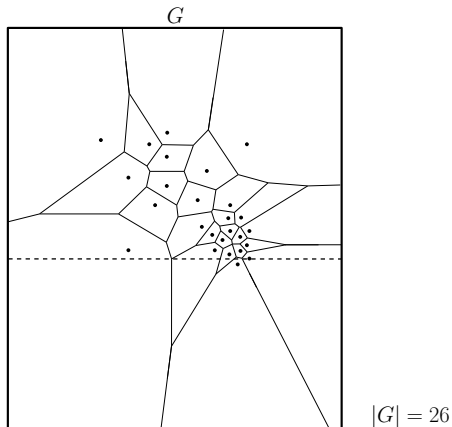
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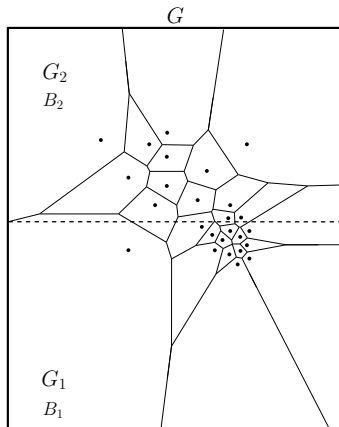
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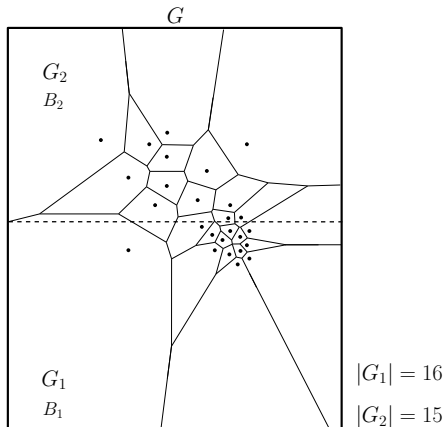
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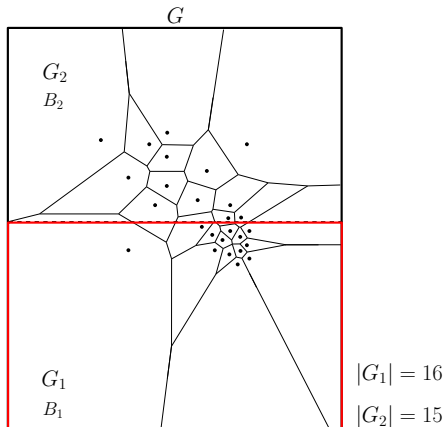
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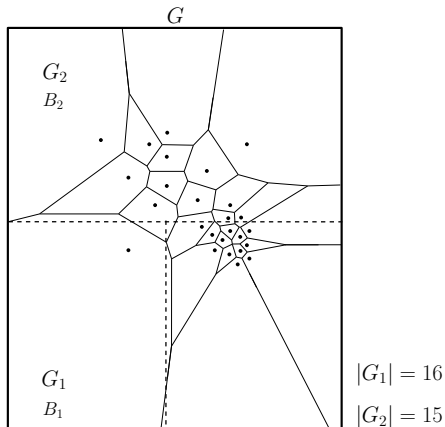
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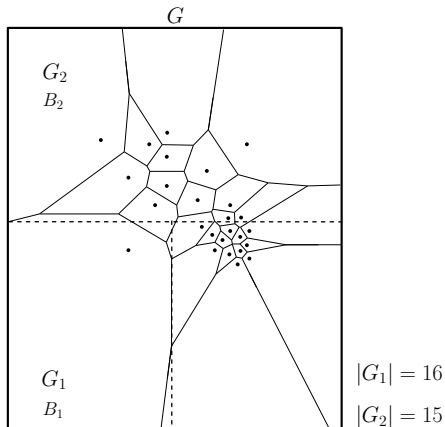
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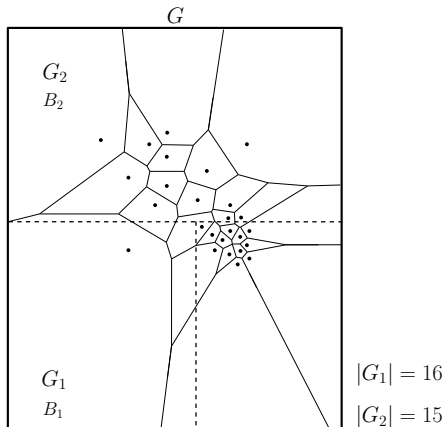
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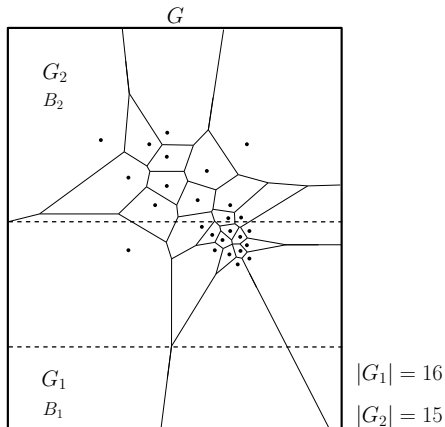
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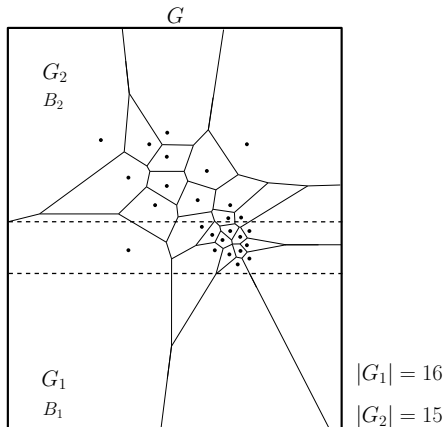
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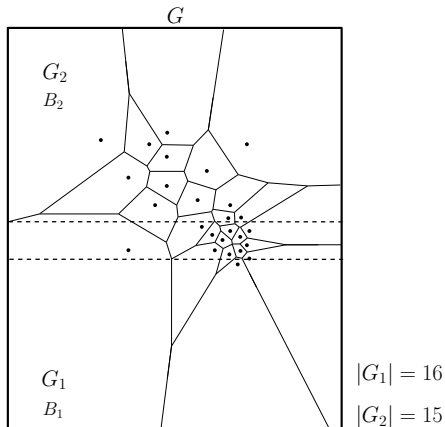
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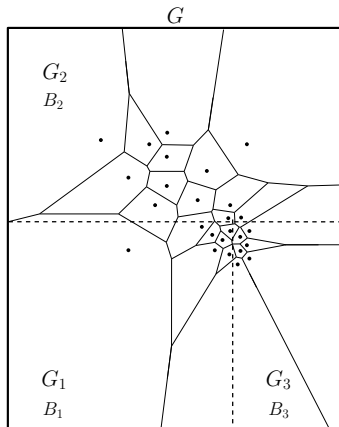
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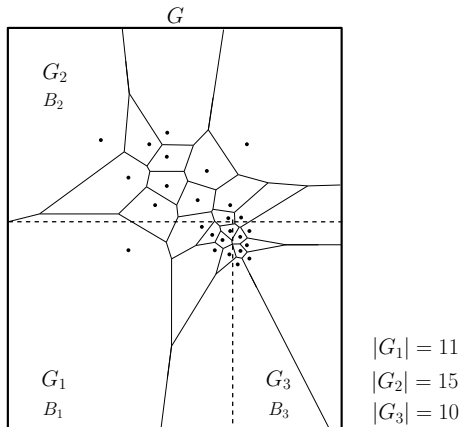
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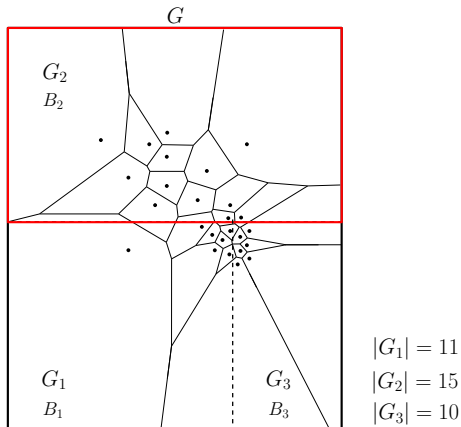
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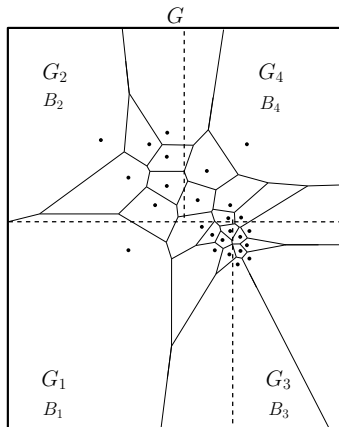
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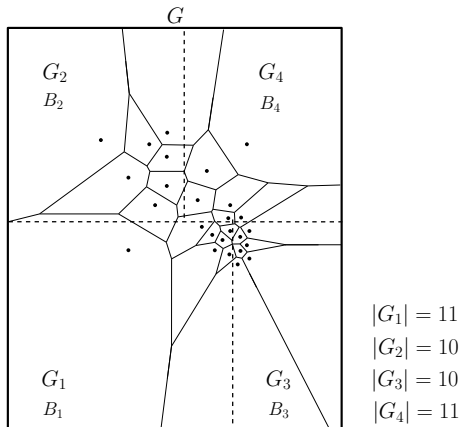
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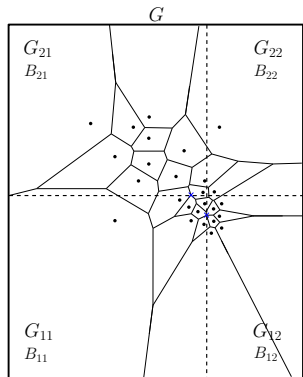
- Merits:
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- Demerits:

Minimum Maximum Partition (MinMax)

- Merits:
 - most balanced partitions: low storage and communication overheads, as well as cheap encryption cost
- Demerits:
 - high storage cost at client

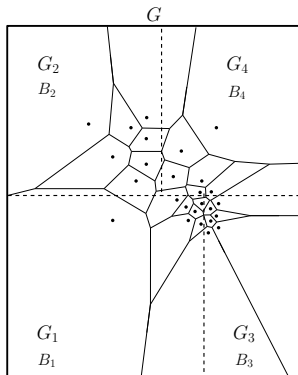
Comparison between MinSG and MinMax

Comparison between MinSG and MinMax



MinSG

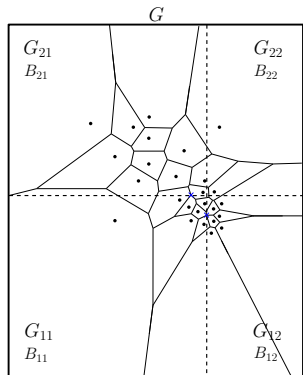
$$\begin{aligned} |G_{11}| &= 11 \\ |G_{12}| &= 10 \\ |G_{21}| &= 14 \\ |G_{22}| &= 6 \end{aligned}$$



MinMax

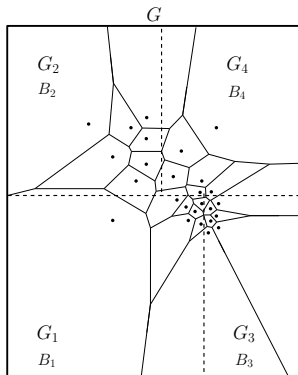
$$\begin{aligned} |G_1| &= 11 \\ |G_2| &= 10 \\ |G_3| &= 10 \\ |G_4| &= 11 \end{aligned}$$

Comparison between MinSG and MinMax



MinSG

$$\begin{aligned} |G_{11}| &= 11 \\ |G_{12}| &= 10 \\ |G_{21}| &= 14 \\ |G_{22}| &= 6 \end{aligned}$$



MinMax

$$\begin{aligned} |G_1| &= 11 \\ |G_2| &= 10 \\ |G_3| &= 10 \\ |G_4| &= 11 \end{aligned}$$

- Clearly, MinMax achieves more balanced partitions than MinSG, which means lower storage and communication overheads, as well as cheaper encryption cost.

Experiment

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 - Points of interest in California(CA) and Texas(TX) from the *OpenStreetMap* project.
 - In each dataset, we randomly select 2 million points to create the largest dataset D_{\max} and form smaller datasets based on D_{\max} .

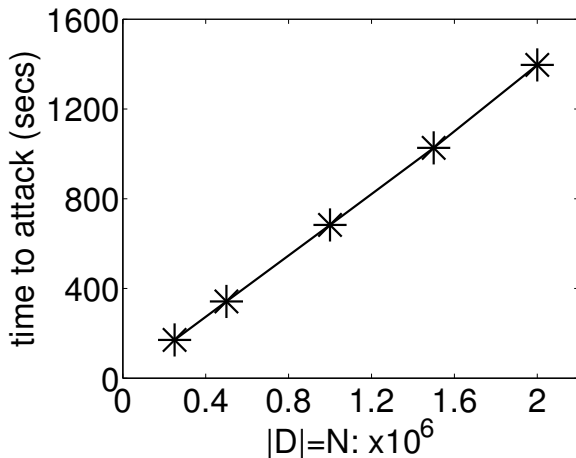
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- Default settings.

Symbol	Definition	Default Value
$ D $	size of the dataset	10^6
k	number of partitions	625
DT	dataset type	CA

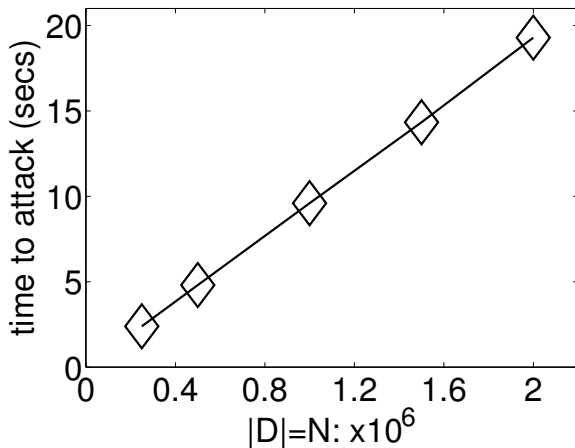
Attack on Existing SNN Methods

- Vary $|D|$: Wai Kit Wong, David Cheung, Ben Kao, Nikos Mamoulis:
Secure kNN computation on encrypted databases. SIGMOD 2009



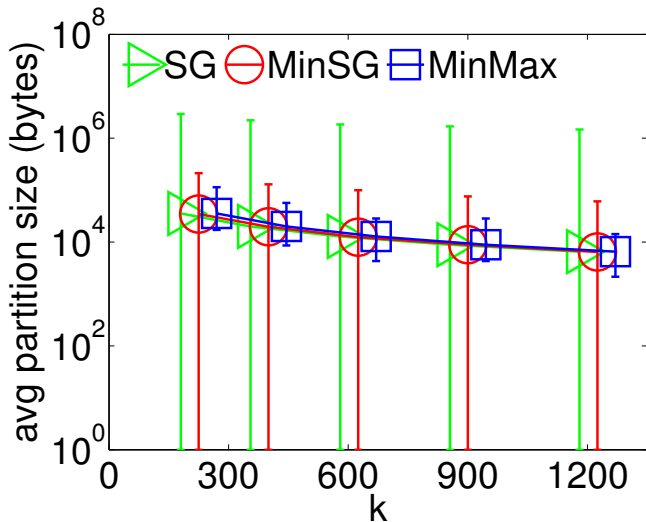
Attack on Existing SNN Methods

- Vary $|D|$: Haibo Hu, Jianliang Xu, Chushi Ren, Byron Choi: Processing private queries over untrusted data cloud through privacy homomorphism. ICDE 2011



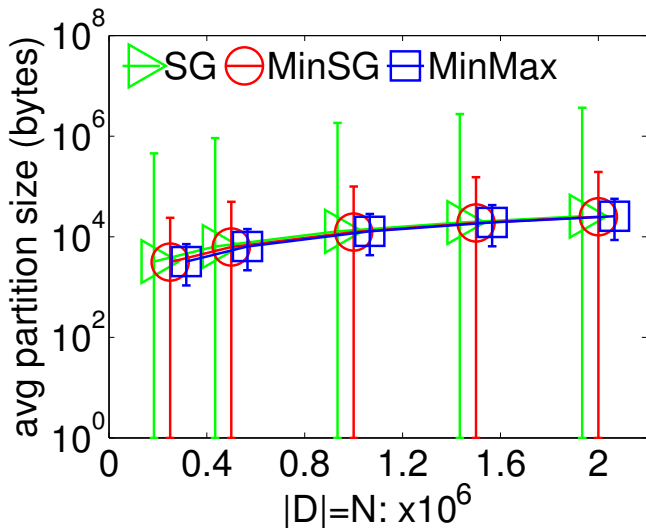
Partition size in different methods

- Vary k



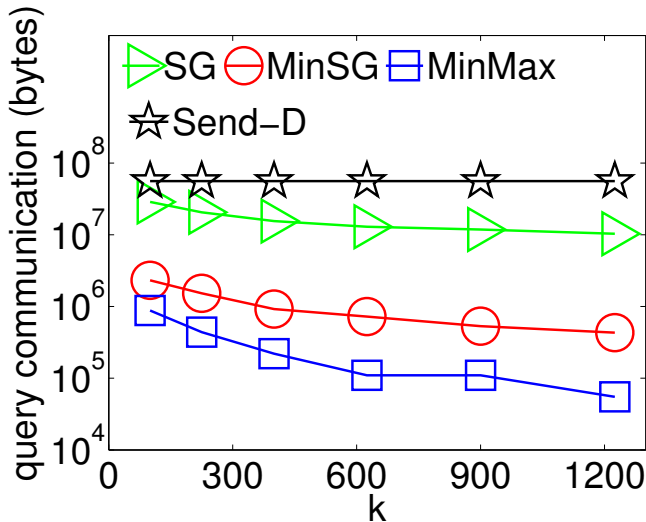
Partition size in different methods

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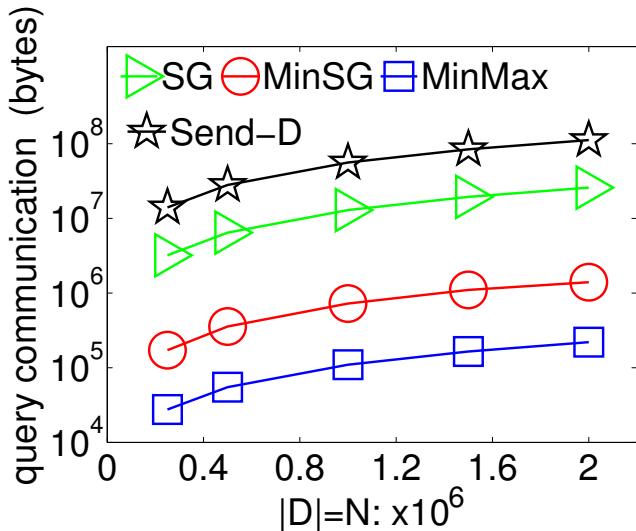
Query communication cost

- Vary k



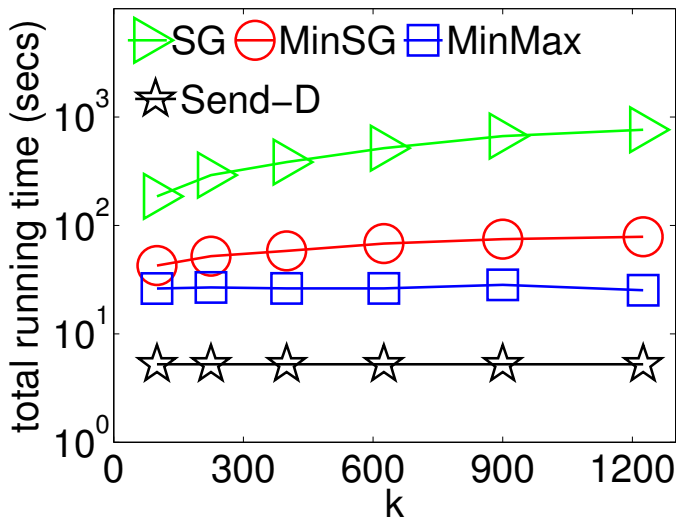
Query communication cost

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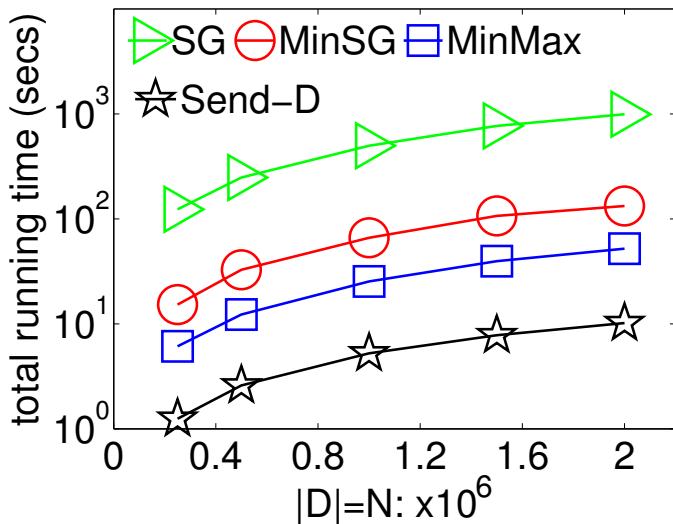
Total running time of the preprocessing step

- Vary k



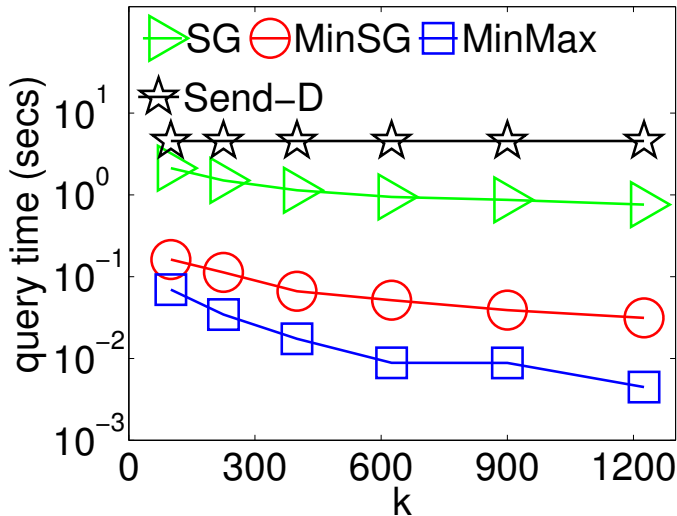
Total running time of the preprocessing step

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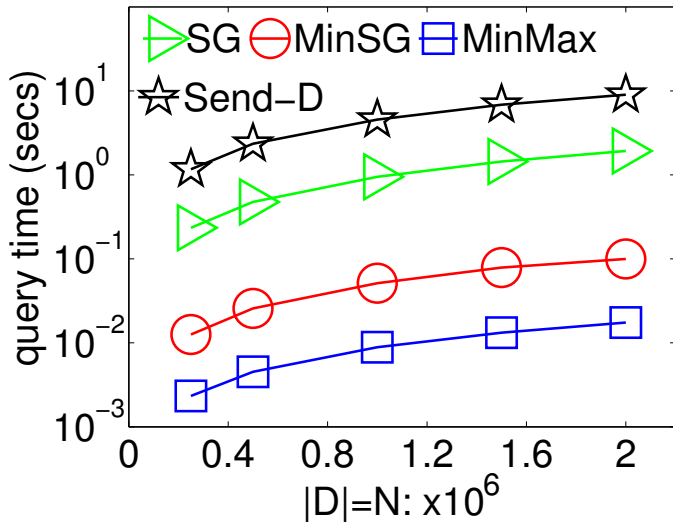
Query time for different methods

- Vary k



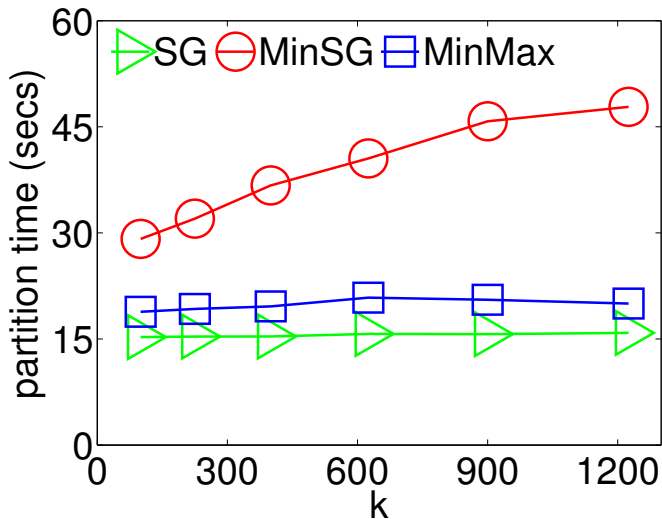
Query time for different methods

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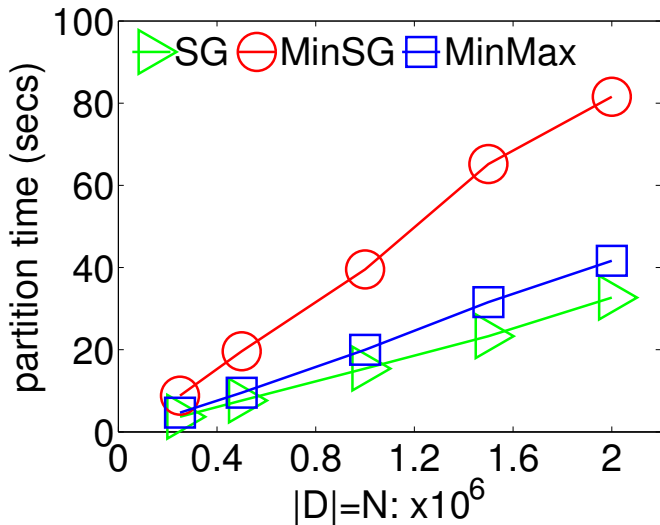
Running time of the partition phase

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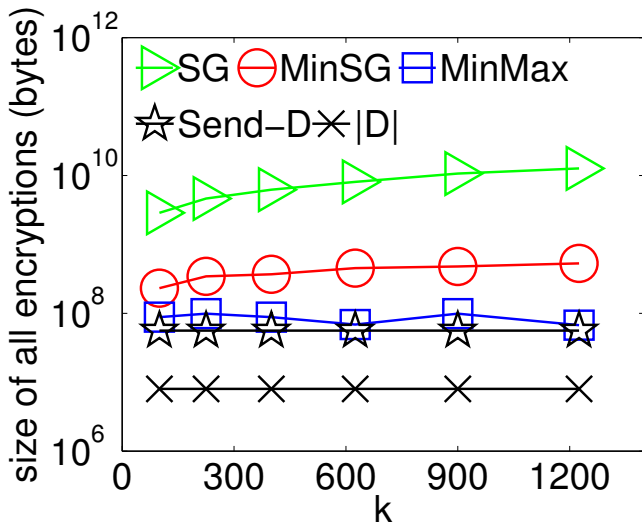
Running time of the partition phase

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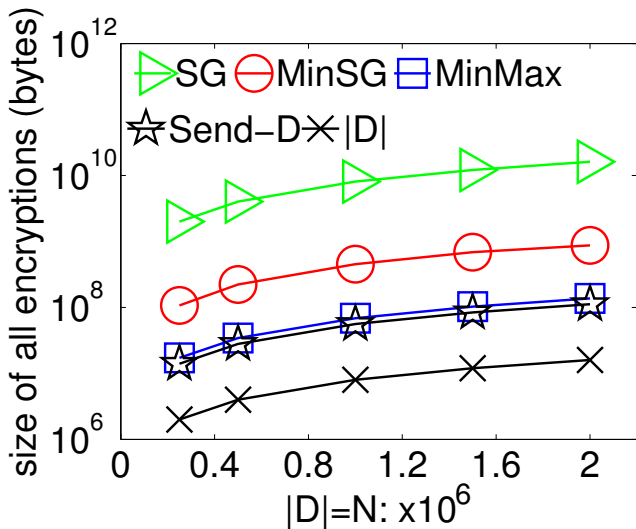
Total size of $E(D)$

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Total size of $E(D)$

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- 6 Variants of similarity search: reverse nearest neighbors, skylines, etc.

- Design a new partition-based secure voronoi diagram (SVD) method.

Conclusion

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- Implement the SVD with three partitioning methods.

Conclusion

- Design a new partition-based secure voronoi diagram (SVD) method.
- Implement the SVD with three partitioning methods.
- Future work
 - extending our investigation to higher dimensions, k nearest neighbors

Thank You

Q and A