Assignment A1: Chapter 0:
Introduction

CS 5100/6100
Spring 2015

Assigned: 13 January 2015

Due: 22 January 2014

For this problem, hand in a problem solution set in pdf format (include name, date, assignment and class number in pdf) which solves the following problems. Show your work.

In addition, please turn in a hardcopy of the report in class before the start of class on January 22, 2014.

1. Give a simple English description of the following sets.
   (a) \{\ldots, -7, -5, -3, -1\}
   (b) \{1, \frac{3}{2}, \frac{5}{4}, \frac{7}{6}, \frac{11}{8}, \frac{13}{8}, \ldots\}
   (c) \{a, aa, aaa, aaaa, \ldots\}
   (d) \{\ldots, -3, -1, 1, 3, \ldots\}, \{\ldots, -4, -2, 0, 2, 4, \ldots\}

2. Give a formal definition of the following sets based on the English description.
   (a) The set of all sets that contain themselves.
   (b) The set of all odd numbers greater than 6.
   (c) The set of all non-empty combinations of any length of the letters a and b.
   (d) The set of all Boolean functions.
   (e) The set of all points on the unit circle.

3. Give the following cross product sets.
4. Give the power set of each of the following.

(a) $\mathcal{P}(\emptyset)$
(b) $\mathcal{P}([a, b])$
(c) $2^S$, where $S = \mathbb{R}$
(d) $\mathcal{P}([0, 1, 2])$
(e) What is the size of the power set of a set of $n$ elements?

5. Languages, strings.

(a) Give the language, $L_a$, of all non-zero length sequences of the letter a.
(b) Is the string 'a' a substring of any member of $L_a$? If so, which ones? Explain.
(c) Give the strings in $L_{ab} = \{\sigma \mid \sigma = \sigma_1\sigma_2\ldots\sigma_n \land \sigma_i \in \{a, b\} \land n \leq 3\}$ in shortlex lexicographic order.
(d) Give a simple example language, $L$, which is prefix free.
(e) What is the length of the longest string in $L = \{a^n \mid n > 0\}$?

6. In a, b and c state if each of the following is a function or not; if not, explain; if yes, give the domain and range in the form $f : S_1 \rightarrow S_2$, where $S_1$ and $S_2$ are sets.

(a) $f^{-1}(y)$ where $f(x) = x^2$.
(b) length($\sigma$) = $|\sigma|$
(c) $f(n) = n + 1$, where $n$ is an integer
(d) Give the complete enumeration of the $\leq$ relation on $\{0, 1, 2\}$.
(e) Given three lines, $l_1, l_2, l_3$, show how to define the relation: "$l_2$ bisects $l_1$ and $l_3$.

7. Consider the graph, $G$, in Figure 1.

(a) Give the formal description of $G$.
(b) What is the longest simple path in $G$?
(c) What is the longest path in $G$?
Figure 1: Graph $G$.

(d) Give the outdegree and indegree of each node.

(e) Is $G$ strongly connected? If so give a constructive proof. Otherwise give counterexample.

8. Show the following are tautologies using truth tables.

(a) $\neg\neg a = a$
(b) $(a \land (a \rightarrow b)) \rightarrow b$
(c) $\neg a \rightarrow a$
(d) $\neg(\neg a \land a)$
(e) $a \rightarrow (a \lor b)$

9. Give an algorithm (pseudo-code) to determine whether a given graph, $S$, is a subgraph of a given graph, $G$. 
CS 6100 Additional Questions

1. A cryptarithmetic problem is one where each digit (0 to 9) is represented by a letter of the alphabet (A to Z), and a sum or difference is represented as words. E.g.:

   \[
   \begin{array}{c}
   \text{C} \\
   + \text{P} \\
   \text{I} \text{S} \\
   + \text{F} \text{U} \text{N} \\
   \hline
   \text{T} \text{R} \text{U} \text{E}
   \end{array}
   \]

   One solution is C=2 P=3 I=7 S=4 F=9 U=6 N=8 T=1 R=0 E=5

   Give three original (from you) cryptarithmetic problems and solutions.

2. A 3-color map coloring problem requires that the regions in a map each be colored using one of three colors (R,G,B) such that no two regions which share an edge have the same color.

   (a) Give the pseudo-code for an algorithm to solve the 3-color problem; it takes a map as input (describe the data types used).
   (b) What is the complexity of your algorithm?
   (c) Give a solution for the following maps:

   ![Map 1](image1)
   ![Map 2](image2)

3. Compare depth-first and breadth-first search on the maps in 2.c; i.e., trace their execution solving the problem.