Summary of General Hypothesis Test Procedure:

1. Define the **null hypothesis**, which is the uninteresting or default explanation.
2. Assume that the null hypothesis is true, and determine the probability rules for the possible outcomes of the experiment.
3. After collecting data, compute the probability of the final outcome or even more extreme outcomes.

Null Hypothesis: Hy that easily allows you to calculate the probability of a particular statistic

Fair Coin Experiment

Exp - flips, $T = T(X_1, \ldots, X_n) = \sum_{i=1}^{n} X_i$

H_0 - coin fair, $p = 0.5$

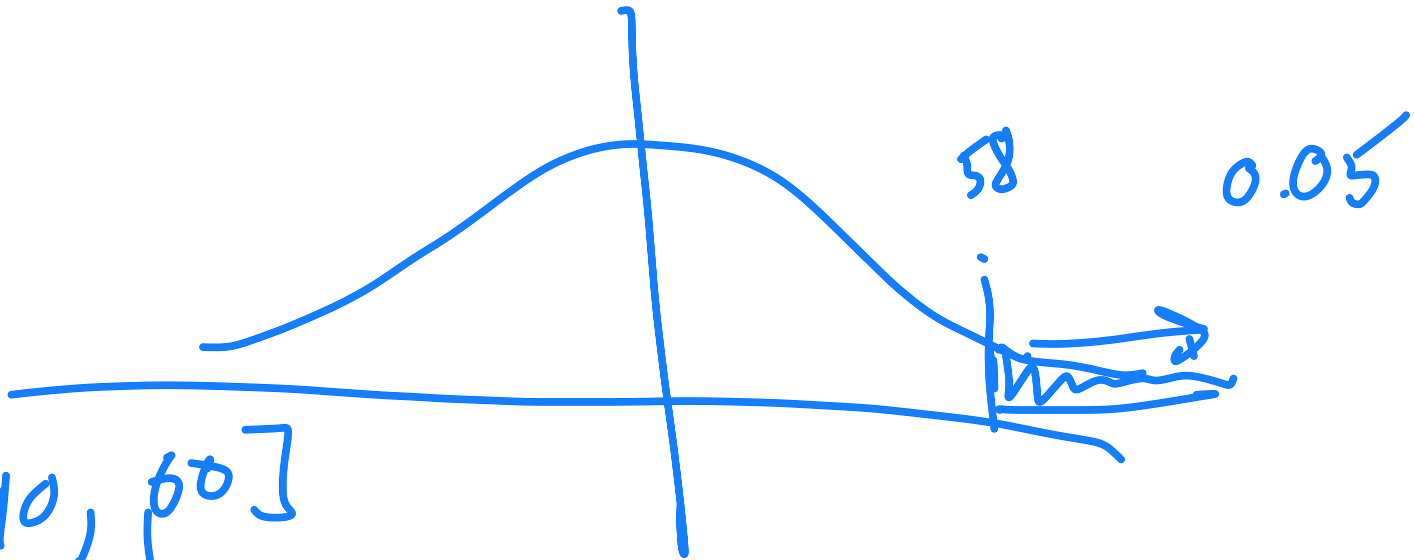
H_1 - $p \neq 0.5$, $p > 0.5$, $p < 0.5$

double sided

$\underbrace{p > 0.5, \quad p < 0.5}_{\text{single sided}}$

$T \sim Bin(n, p) = Bin(n, \frac{1}{2})$ $\quad n = 100$

$t = \sum_i x_i$ $\quad$ qbinom$(.95, 100, \frac{1}{2}) = 58$



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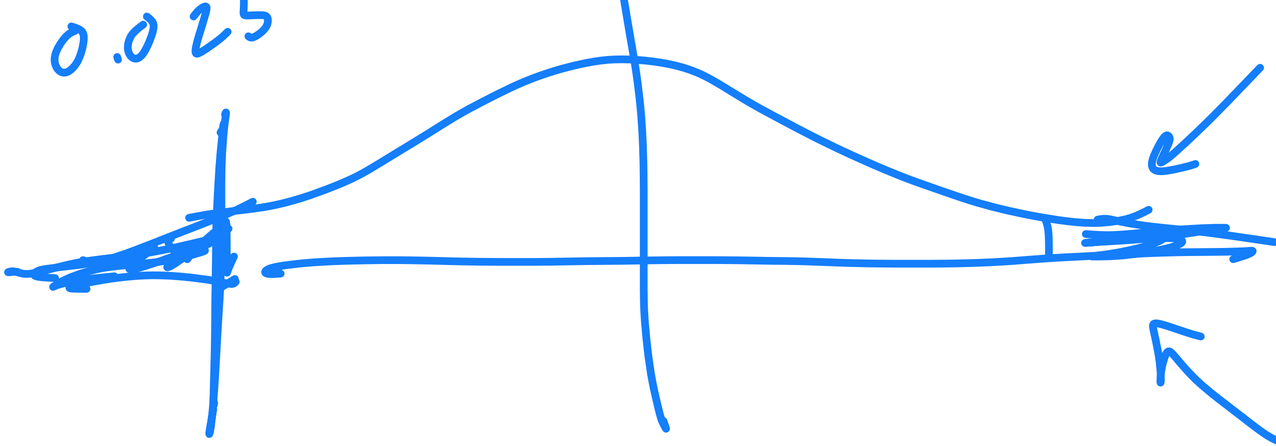


Low ~

$$\begin{cases} \text{qbinom}(0.025, 100, \tfrac{1}{2}) \\ \text{qbinom}(0.975, 100, \tfrac{1}{2}) \end{cases}$$

Approx w/ Normal

$T \sim N(50, 100 \times \frac{1}{4}) = N(50, 25)$

$z_{1-\alpha} = z_{0.95} = 1.64$

$z = \frac{T-\mu}{\sigma}$

$\Updownarrow$

$T = z\sigma + \mu$

$(1.64 \times 5) + 50 = 58.2$

Error

Table of error types		Null hypothesis (H_0) is	
		True	**False**
Decision about null hypothesis (H_0)	**Fail to reject**	Correct inference (true negative) (probability = $1-\alpha$)	Type II error (false negative) (probability = β)
	Reject	Type I error (false positive) (probability = α)	Correct inference (true positive) (probability = $1-\beta$)

$$P(\text{Reject} \mid H_0 \text{ true}) = \alpha$$

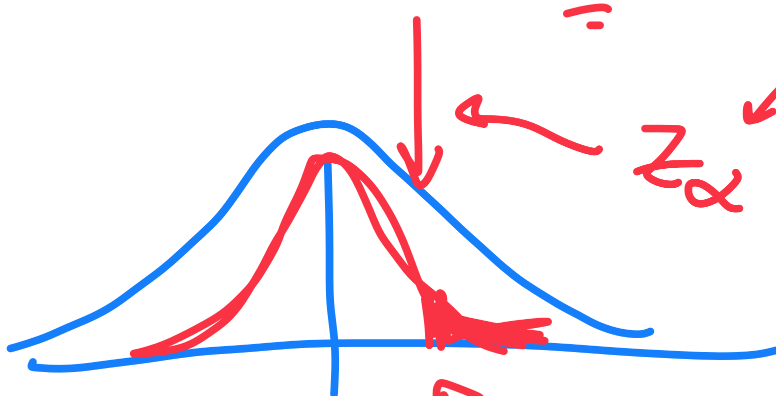
1

Hypothesis Testing of Means

H_0

σ =

$\bar{X}$ - sample mean.



Z_α

μ_0 =

$\bar{X} \sim N(\mu_0, \sigma^2/n)$

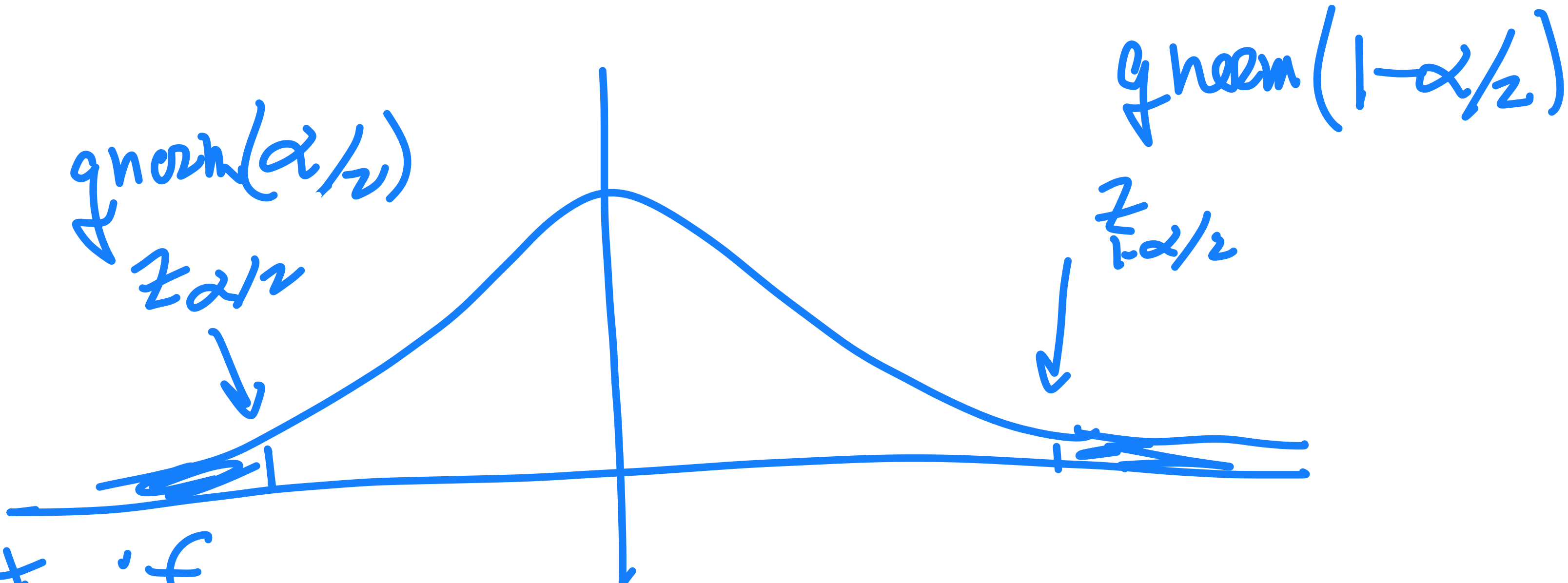
is $Z > Z_\alpha$

critical

$$Z = (\bar{X} - \mu_0)/(\sigma/\sqrt{n}) \sim N(0,1)$$

Hyp of Mean

1b



Reject if

$z < z_{\alpha/2}$

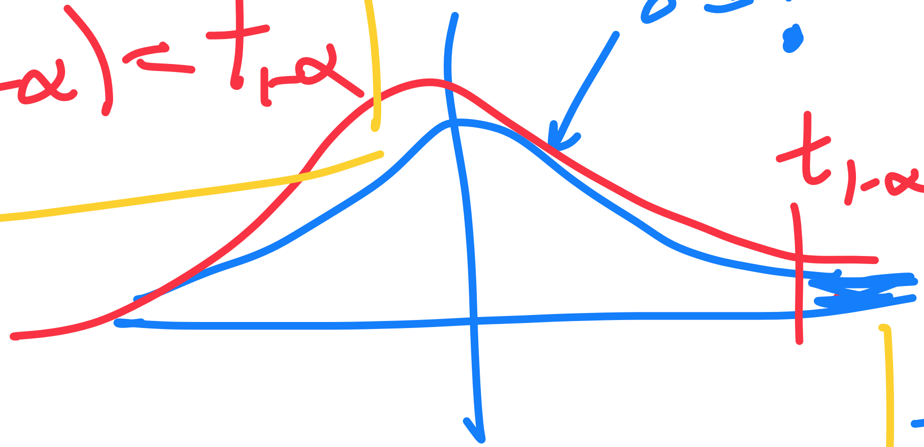
$z > z_{1-\alpha/2}$

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

②

$\boxed{qt(1-\alpha) = t_{1-\alpha}}$

$\sigma = ?$



$(X_1 \ldots X_n)$ - samples

S_n^2 - sample variance

$$\boxed{T = \frac{\bar{X} - \mu_0}{S_n/\sqrt{n}}}$$

$T \sim$ s-t(dof = n-1)

If $T > t_{1-\alpha}$
Reject H_0

Paired Samples - Hypothesis Testing

Same set of specimens two conditions

"before" & "after"

$$\begin{pmatrix} x_1 \\ \\ x_n \end{pmatrix} \quad \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}$$

$$\bar{x} \quad \bar{y}$$

$y_i - x_i = h_i$

$(h_1 \ldots h_n)$

$$\bar{h} = \frac{1}{n} \sum_{i=1}^{n} h_i$$

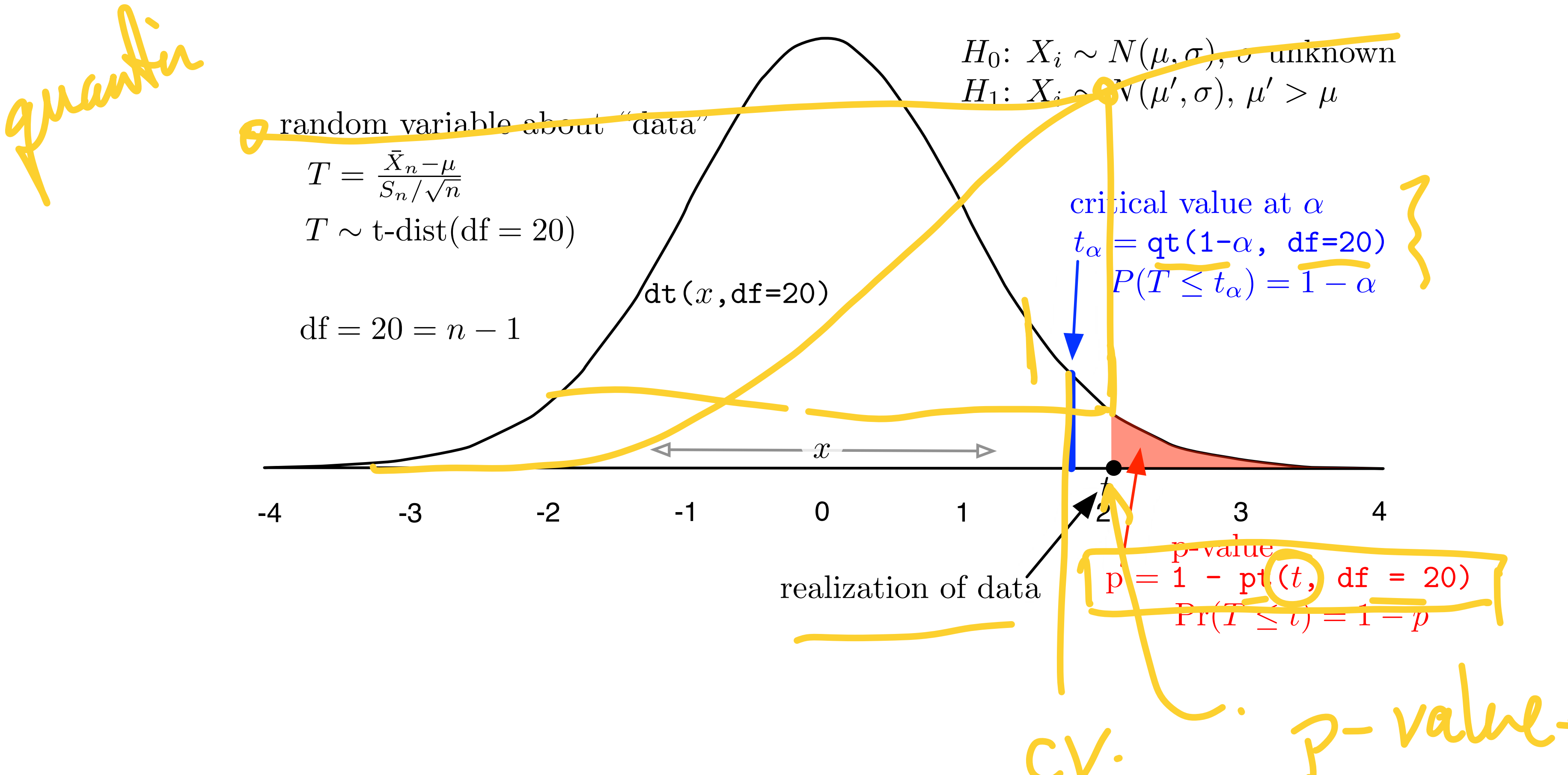
$$\bar{h} \sim N\left(0, \frac{\sigma^2}{n}\right) \simeq N\left(0, \frac{s_n^2}{n}\right)$$

$$T = \frac{\bar{h} - 0}{s_n / \sqrt{n}}$$ ← student-t dist.

H_0 = mean of h is zero

If $T > t_{1-\alpha}$ reject H_0

$t_{1-\alpha}$ $\alpha = 0.05$



quantile
random variable about "data"
$T = \frac{\bar{X}_n - \mu}{S_n/\sqrt{n}}$
$T \sim$ t-dist(df $= 20$)
df $= 20 = n - 1$
dt(x,df=20)
H_0: $X_i \sim N(\mu, \sigma)$, σ unknown
H_1: $X_i \sim N(\mu', \sigma)$, $\mu' > \mu$
critical value at α
t_α = qt(1-α, df=20)
$P(T \leq t_\alpha) = 1 - \alpha$
x
-4
-3
-2
-1
0
1
2
3
4
t
realization of data
p-value
p = 1 - pt(t, df = 20)
$\Pr(T \leq t) = 1 - p$
CV.
p-value.

Two sample hypothesis test equal variances

Scenario: two populations, unknown means, unknown variances. (equal).

$\sigma_x^2 = \sigma_y^2 = \sigma^2$

$X = X_1 \dots X_n$

$Y = Y_1 \dots Y_m$

$\bar{X}_n, \bar{Y}_m$

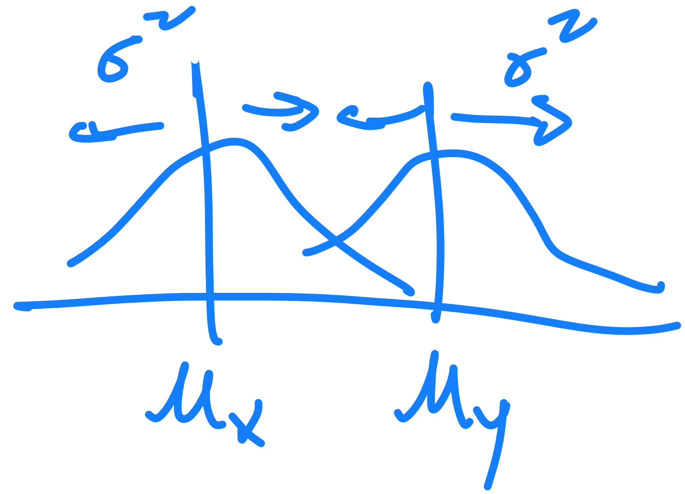
$S_x^2 \quad S_y^2$

$$S_p^2 = \frac{(n-1)S_x^2 + (m-1)S_y^2}{n+m-2}$$

Statistic

$$T = \frac{\bar{X}_n - \bar{Y}_m}{S_p\left(\frac{1}{n}+\frac{1}{m}\right)^{\frac{1}{2}}}$$

student with $n+m-2$ degrees of freedom.



Statistical Simulation

What is Simulation?

Why?

Complex

Predict outcomes. - average

Monte Carlo.

Random #s in Comp.

Pseudo-random #s.

Generate sequences of integers

sequence has memory.

Polynomials → chaotic

n mod

uniform dist

Pseudo-Random

Standard uniform distribution

$u \sim U(0,1)$ ☆

Ex: $Ber(p)$ $u \leftarrow$ runif( )

$$b = \begin{cases} 1 & \text{if } u < p \\ 0 & \text{if } u \geq p \end{cases}$$

u type float