Cont. vs. Discrete RVs.

Discrete - counting, derive or infer mass function

Continuous - mostly cannot be derived from scratch
? assert a distribution (pdf)
Modeling assumptions

Statistics vs. Prob.

Prob - single events.

Statistics - tools for analyzing densities and distributions

Expectation

RV X

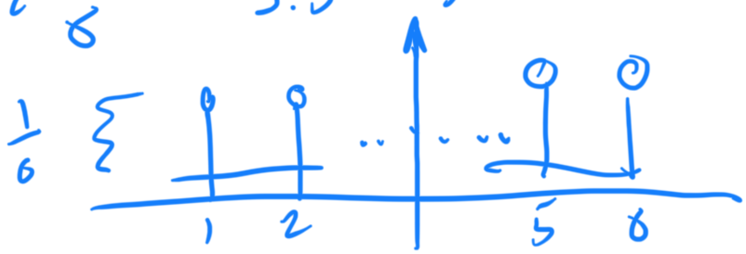
$$E[X] = \sum_i a_i \Pr(X = a_i)$$

$$= \sum_i a_i f(a_i)$$

Roll of die . . .

$$\sum_{i=1}^{6} i \cdot \frac{1}{6} = 3.5$$

3.5



Mean of distribution/density

$f(a)$ $\quad$ $\mu = \sum_i a_i f(a_i)$

?X

Mean of distribution/density

$f(a)$ $\quad$ $\mu = \sum_i a_i f(a_i)$

?

Sum of 3 dice

$$E[X] = E[X_1 + X_2 + X_3] = E[X_1] + E[X_2] + E[X_3]$$

$$= 3 \times 3.5 = 10.5$$

EXPECTATION IS LINEAR.

Difference of 2 dice

$$E[X] = E[X_1 - X_2] = E[X_1] - E[X_2] = 0$$

different RV's.

$\underline{X}$ = Difference of larger die minus smaller die

$X \in \{0, 1, \ldots, 5\}$.

$\text{abs}(X_1 - X_2)$.

$$E[X] = E[\text{abs}(X_1 - X_2)].$$

not linear

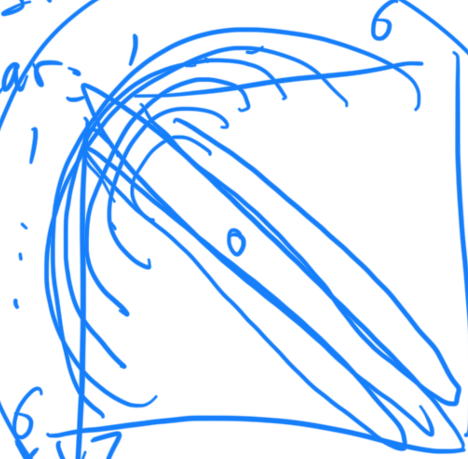
$f(0) = \frac{6}{36}$

$f(1) = \frac{10}{36}$

$f(2) = \frac{8}{36}$

$$\sum_{i=0}^{5} a_i f(a_i)$$

$$= E[\text{abs}(X_1 - X_2)]$$



$0 \times \frac{6}{36} + 1 \times \frac{10}{36} + \ldots$

$$\sum_{i=0}^{5} a_i f(a_i)$$

Binomial

n trials of Bernoulli RV $\{0,1\}$. $\Pr(X=1)=p$

prob of K success's (1's) in n Trials

$$\Pr(X=k) = \binom{n}{k} (p)^k (1-p)^{n-k}$$

prob of $n-k$ failures

# of ways to order k successes in an n-tuple

prob of k successes

$$E[X] = E[\underbrace{X_1 + \dots + X_n}] = E[X_1] + \dots + E[X_n]$$

successes

p p p

$= np$

Variance - "spread" of a dist

$$Var[X] = E[(X - E[X])^2]$$

$E[X]$: mean, μ.

aside: notice μ is not a RV.

$$= \sum_i (a_i - \mu)^2 f(a_i)$$

$|a_i - \mu|$ ← mean deviation

$Var[X]$ denotes σ^2.

$\sigma = \sqrt{Var[X]}$ = std deviation

Cont. form for expectation

$f(a)$ - prob density function

X - cont. RV

$$E[X] = \int_{-\infty}^{\infty} \underset{\text{RV}}{a}\,\underset{\text{pdf at } a}{f(a)}\, da$$

Uniform distribution $U(a,b)$

$$f(x) = \frac{1}{b-a}$$

$$\int_{-\infty}^{\infty} U(a,b)\, dx = \int_a^b \frac{1}{b-a}\, dx = \frac{x^2}{2} \frac{1}{b-a} \Bigg|_a^b$$

$$= \frac{b^2-a^2}{2} \frac{1}{b-a} = \frac{b+a}{2}\left(\frac{b-a}{b-a}\right) = \frac{b+a}{2}$$

Identities for Var

$$\mathrm{Var}[aX+b] = a^2\,\mathrm{Var}[X].$$

RV const.

$X \sim$ Bernoulli $\quad \mu = 0(1-p)+1\cdot p =$

$$\mathrm{Var}[X] = (0-p)^2(1-p) + (1-p)^2\cdot p$$

$(a_1-\mu)^2 \quad f(a_1)$

$$= +p^2 - p^3 + (1-2p+p^2)p$$

$$+p^2 \cancel{- p^3} + p - 2p^2 \cancel{+ p^3}$$

$$= p - p^2 = p(1-p).$$