# Bayes Rule

CS 3130/ECE 3530:
Probability and Statistics for Engineers

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# Bayes Rule

# In-Class Problem:

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You have two urns, one with 4 black balls and 3 white balls, the other with 2 black balls and 2 white balls. You pick one urn at random and then select a ball from the urn. What is the probability the ball is white?
If you picked a black ball, what is the probability that you had picked the first urn (the 4 black, 3 white urn)?

# Bayes Rule — Terminology

likelihood.

prior
Before

Posterior
After

$$Pr(A|B) = \frac{Pr(B|A)Pr(A)}{Pr(B)}$$

marginalization
normalization

# Bayes Rule — Meaning

knowledge want.

observation

state of observations

general understanding of world

$$Pr(A|B) = \frac{Pr(B|A)Pr(A)}{Pr(B)}$$

normalization

# In-Class Problem:

You have a system with a main power supply and auxiliary power supply. The main power supply has a 10% chance of failure. If the main power supply is running, the auxiliary power supply also has a 10% chance of failure. But if the main supply fails, the auxiliary supply is more likely to be overloaded and has a 15% chance to fail. What is the probability that the auxiliary power will fail?

If the auxiliary power fails, what is the probability that the main power also failed?

A ← obs.

M ← know

$$P(M|A) = \frac{\overset{0.15}{P(A|m)}\ \overset{0.1}{p(m)}}{P(A)}$$

P(A) ← total prob 0.105

$$P(A|m)p(m) + P(A|\sim m)p(\sim m)$$

$$= \frac{.15 \times 0.1}{.105} = \frac{.015}{.105} =$$

# In-Class Problem:

You work in a factory that makes sensors for airbags in automobiles. You have three machines that run continuously/simultaneously: A) the old machine makes 10 parts/hour and 10% of parts are faulty, B) a newer machine makes 20 parts/hour and 5% of parts are faulty, and C) the newest machine makes 30 parts/hour and 1% of parts are faulty. The parts get mixed in a bin and you sample one part from that bin. The part is faulty. What is the probability that the part came from machine B?

$P(A) = \frac{1}{6}$ prob of part from machine A

$P(F|A) = .1$

$P(B) = \frac{2}{6}$

$P(F|B) = .05$

$P(C) = \frac{3}{6}$

$P(F|C) = .01$

$$P(B|F) = \frac{\overset{\frac{.05}{3}}{P(F|B)\,P(B)}}{\boxed{.0383}\,P(F)}$$

$P(F|A)P(A)$
$+ P(F|B)P(B)$
$+ P(F|C)P(C)$

$\frac{.23}{6}$

$= .1 \times \frac{1}{6} + .05 \times \frac{2}{6}$
$+ .01 \times \frac{3}{6} =$

$$\frac{\left(\frac{.05}{3}\right)}{.0383} = \boxed{0.435}$$

# In-Class Problem:

$P(T|F)$ $P(F|T)$

prob of T given Fugative is 99.9%

You are given a machine learning, face recognition system for an escaped fugative that ~~has a true positive~~ rate of 99.9%, and a ~~false positive rate of $10^{-6}$~~. $P(T|B)$ Approximately 2 million people pass through airports on a given day, and you need to find where this fugative is. You snap a picture, at random, of a passenger, and the detector says it's the fugative. Is it more likely to be the fugative or a bystander?

$$P(F|T) = \frac{\underbrace{P(T|F)}_{\approx 1}\,P(F)}{P(T)} \leftarrow \frac{1}{2\times10^6} = \boxed{.5\times10^{-6}}$$

$$P(T) = \overset{1}{\cancel{.999}}\,\frac{1}{2\times10^6} + 10^{-6}$$

$$P(T|F)P(F) \times P(T|B)P(B)$$

$$.5\times10^{-6} + 10^{-6} = \boxed{1.5\times10^{-6}}$$

$$= \boxed{\frac{.5}{1.5}} \quad \text{approx} \quad \approx \boxed{.33}$$

$$P(T|B) = 10^{-6} \qquad P(B) = \frac{2\times10^6 - 1}{2\times10^6} \approx 1$$

# Testing and Bayes Rule

# Bayes Rule in Machine Learning

M – models.

prior simplicity

$$P(M|D) = \frac{\boxed{P(D|M)\, P(M)}}{P(D)}$$

ignore

searching for best M

# Discrete Random Variables

"Random Variables" RV

Sample space $\Omega$.

RV function $X : \Omega \rightarrow \mathbb{R}$.

meaning

finite RVs. $a_1, a_2, \dots, a_n$ $|\Omega| = n$

infinite RVs $a_1, a_2, \dots\dots$ $|\Omega| = \inf$

countably

Examples:

Sum of two dice

$\{(1,1), (1,2) \ldots\ldots (6,6)\}$.

$X = i+j \;\forall\; (i,j)$ in $\Omega$.

$Pr(X = a) \quad = Pr(\{(1,2)(2,1)\})$.

$a = 3$

Difference of two dice

$X = i-j \;\forall\; (i,j)$ in $\Omega$.