Independence of Random Events

CS 3130/ECE 3530:
Probability and Statistics for Engineers

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Cond. Prob.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \cap B) = P(A|B)\,P(B)$$

Independence

$$P(A|B) = P(A)$$

$$\Updownarrow$$

$$P(A \cap B) = P(A)P(B)$$

$$\Updownarrow$$

$$P(B|A) = P(B)$$

In-Class Problem:

A fair die is thrown twice. A is the event sum of values is 5. And B is the event that at least one throw is a 2.

- Calculate $P(A \mid B)$. Are events A and B independent?

$P(A|B) = \frac{P(A \cap B)}{P(B)}$ ← $\frac{2}{36}$ / $\frac{11}{36}$

$|B| = 36$

$|B| = 11$

$P(A|B) \stackrel{?}{=} P(A)$

$P(A) = \frac{4}{36}$

No.

Independence and Complements

A & B independent

$$P(B^c|A) = 1 - P(B|A) = 1 - P(B)$$

$$= P(B^c)$$

A & B Ind. $\Leftrightarrow$ A & B^c Ind

$\Leftrightarrow$ A^c & B Ind. $\Leftrightarrow$ A^c & B^c Ind.

$$P(A^c|B) = 1 - P(A|B) = 1 - P(A)$$
$$= P(A^c)$$

In-Class Problem:

You have two urns, one with 4 black stones and 3 white stones, the other with 2 black stones and 2 white stones. You pick one urn at random and then select a stone from the urn. Is the event that I pick urn 1 independent of the event that I pick a white stone? What if I changed the second urn to have 8 black stones and 6 white stones?

$$P(w|u_1) = P(w)$$

$$P(w|u_1) = P(w) = P(w|u_2)$$

In-Class Problem:

You have a system with a main power supply and auxiliary power supply. The main power supply has a 10% chance of failure. If the main power supply is running, the auxiliary power supply also has a 10% chance of failure. But if the main supply fails, the auxiliary supply is more likely to be overloaded and has a 15% chance to fail. Is the auxiliary supply failing independent of main supply failing?

M - main fails

A - aux fails

$P(M) = .1$

$P(A|M^c) = .1$

$P(A|M) = .15$

$\neq$

Total Probability

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Total Probability (Law of)

$B_1 \ldots B_n$ partition of Ω iff

$B_i \cap B_j = \emptyset \quad \forall\, i, j \neq i$

$B_1 \cup \ldots \cup B_n = \Omega$.

$$P(A) = P(A|B_1)P(B_1) + \ldots + P(A|B_n)P(B_n)$$

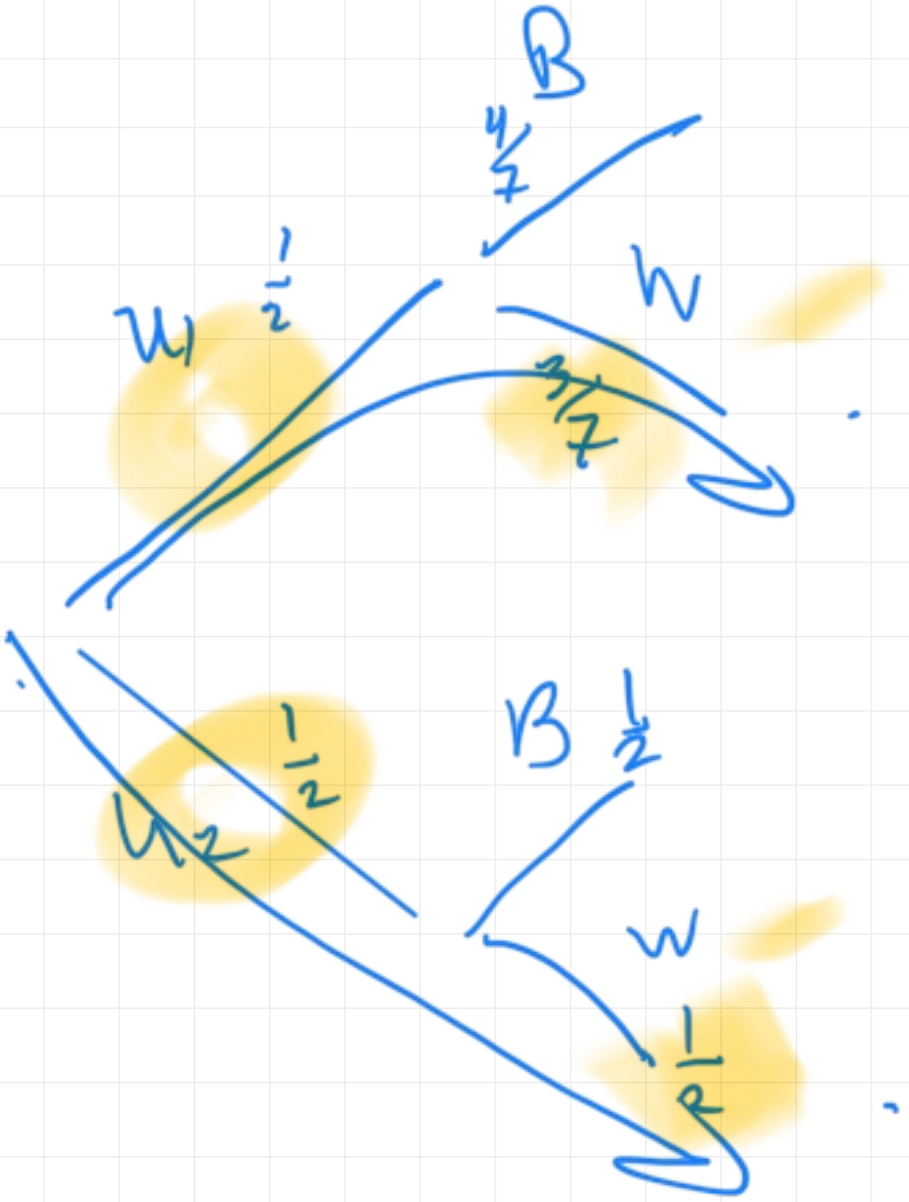
$P(A \cap B_n)$

In-Class Problem:

You have two urns, one with 4 black balls and 3 white balls, the other with 2 black balls and 2 white balls. You pick one urn at random and then select a ball from the urn. What is the probability the ball is white?

$$P(w) = P(w|u_1)P(u_1) + P(w|u_2)P(u_2)$$

$$= \frac{3}{7} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} = \frac{3}{14} + \frac{1}{4} = \frac{13}{28}$$


B
4/7
1/2
3/7
1/2
B 1/2
1/2
W
1/2

In-Class Problem:

You have a system with a main power supply and auxiliary power supply. The main power supply has a 10% chance of failure. If the main power supply is running, the auxiliary power supply also has a 10% chance of failure. But if the main supply fails, the auxiliary supply is more likely to be overlaoded and has a 15% chance to fail. What is the probability that the auxiliary power will fail?

$P(m) = .1$

$P(A|m^c) = .1$

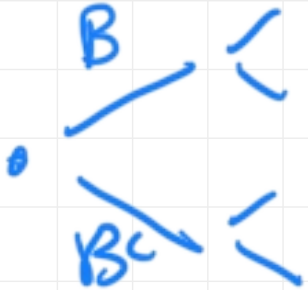
$P(A|m) = .15$

$$P(A) = \underset{.1}{P(A|m^c)}\underset{.9}{P(m^c)} + \underset{.15}{P(A|m)}\underset{.1}{P(m)}$$

$$= .09 + .015$$

$$= .105$$

In-Class Problem (from book):



In 2001 EU implements testing for BSE (mad cow). True positive rate for test is 0.7 and false postiive rate is 0.1. Prevalence of disease is 0.02. What is the probability that a randomly selected cow tests positive?

$P(T|B) = .7$

$P(T|B^c) = .1$

$P(B) = .02$

$P(T) = .7 * .02 + .1 * .98 =$

In-Class Problem:

You work in a factory that makes sensors for airbags in automobiles. You have three machines that run continuously/simultaneously: A) the old machine makes 10 parts/hour and 10% of parts are faulty, B) a newer machine makes 20 parts/hour and 5% of parts are faulty, and C) the newest machine makes 30 parts/hour and 1% of parts are faulty. The parts get mixed in a bin and you sample one part from that bin. What is the probability that the part is faulty?

$P(A) = \frac{1}{6}$ $P(B) = \frac{2}{6}$ $P(C) = \frac{3}{6}$

$P(F|A) = .1$ $P(F|B) = .05$ $P(F|C) = .01$

$P(F) = P(F|A)P(A) + P(F|B)P(B)$

$+ P(F|C)P(C) = .03833\ldots$

$\frac{.1}{6} + \frac{.05 \times 2}{6} + \frac{.03}{6} = \frac{.23}{6}$

Bayes Rule

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Bayes Rule

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

$$P(A \cap B) = P(A|B)\,P(B) = P(B|A)\,P(A)$$

In-Class Problem:

You have two urns, one with 4 black balls and 3 white balls, the other with 2 black balls and 2 white balls. You pick one urn at random and then select a ball from the urn. What is the probability the ball is white?

If you picked a black ball, what is the probability that you had picked the first urn (the 4 black, 3 white urn)?

$$P(u_1|B) = \frac{P(B|u_1)P(u_1)}{P(B)}$$

$P(B|u_1) = 4/7$, $P(u_1) = \frac{1}{2}$, $P(B) = \frac{15}{28}$ (total prob.)

$P(B) = P(B|u_1)P(u_1) + P(B|u_2)P(u_2)$

$P(w) = \frac{13}{28}$

In-Class Problem:

You have a system with a main power supply and auxiliary power supply. The main power supply has a 10% chance of failure. If the main power supply is running, the auxiliary power supply also has a 10% chance of failure. But if the main supply fails, the auxiliary supply is more likely to be overloaded and has a 15% chance to fail. What is the probability that the auxiliary power will fail?
If the auxiliary power fails, what is the probability that the main power also failed?

In-Class Problem:

You work in a factory that makes sensors for airbags in automobiles. You have three machines that run continuously/simultaneously: A) the old machine makes 10 parts/hour and 10% of parts are faulty, B) a newer machine makes 20 parts/hour and 5% of parts are faulty, and C) the newest machine makes 30 parts/hour and 1% of parts are faulty. The parts get mixed in a bin and you sample one part from that bin. The part is faulty. What is the probability that the part came from machine B?

Bayes Rule in Machine Learning