Independence of Random Events

CS 3130/ECE 3530:
Probability and Statistics for Engineers

Jan 18, 2023

Cond Prob.

$$\Pr(A \mid B)$$

Pr A given B
Pr A conditioned on B.

events.

$$\Pr(R_1 \cap R_2 \cap R_3)$$

$$\Pr(R_3 \mid R_1 \cap R_2) P(R_1 \cap R_3)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad ?$$

$$P(A \cap B) = P(A|B)\,P(B) \quad \Longleftarrow \quad ? \quad \text{Trees.}$$

R, G
‖   ‖
20 20

prob of drawing 2
Red stones?

$R \overset{19}{\underset{39}{}} R_2 \to$

$\frac{1}{2}$

$\underset{39}{20} G$

$\frac{1}{2} G$

$P(R_2 | R_1) \times \underline{P(R_1)}$

$\frac{19}{39} \times \frac{1}{2} = \frac{19}{78}$

$P(R_1) \, P(R_2 | R_1)$

Ω

B   A
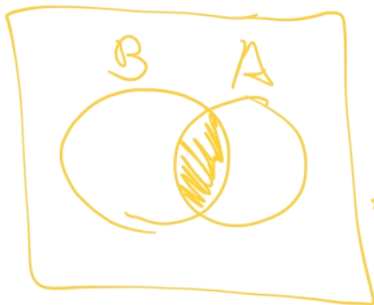
Dice:

A less than 4
B even.

$A = \{1, 2, 3\}$.

$B \{2, 4, 6\}$.

$A \cap B = \{2\}$.

$$Pr(S|Q) \longleftarrow \begin{cases} S = \text{Sum of two dice is 5} \\ Q = \text{one die is "2"} \end{cases}$$

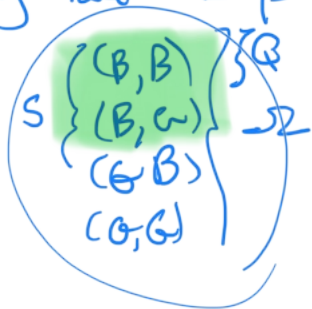$$= \frac{P(S \cap Q)}{P(Q)}$$

You are introduced to a couple

Told: 1) They have 2 children

2) One of children is a boy.

Qu: What is p^r that they have 2 boys.

$$P(Q|S) = \frac{P(Q \cap S) = \frac{1}{4}}{P(S) = \frac{3}{4}}$$

$$= \frac{1}{3}$$

$$S \left\{ \begin{array}{l} (B,B) \\ (B,G) \\ (G,B) \\ (G,G) \end{array} \right. \begin{array}{l} \text{\}} Q \\ \text{\}} \Omega \end{array}$$

Independence

$$P(A \mid B) = P(A) \iff$$

$$\frac{P(A \cap B)}{P(B)} = P(A) \iff$$

$$P(A \cap B) = P(A) P(B) \qquad \underset{3}{\text{Def}}$$

$$\Rightarrow P(B \mid A) = P(B)$$

2 urns

urn 1   4 Red St.
         3 Gr St.

urn 2   2 Red st.
         2 Gr. st.

Ex: 1) Pick urn - random
     2) Select stone.

is the event of urn 1, independent of
                 pick red stone.

$$P(\text{Red} \mid U1) = P(\text{Red})$$

$$\frac{2}{7} + \frac{1}{4} = \frac{15}{28}$$

$$U1 \xrightarrow{\frac{1}{2}} \boxed{\frac{4}{7}} R \cdot \frac{4}{14}$$

$$\xrightarrow{\frac{3}{7}} G$$

$$U2 \xrightarrow{\frac{1}{2}} R \cdot \frac{1}{4}$$

$$\xrightarrow{\frac{1}{2}} G$$

$$P(R \mid U1) = \frac{4}{7}$$

$$P(R \mid U1) = \frac{P(R \cap U1)}{P(U1)}$$

In-Class Problem:

A fair die is thrown twice. A is the event sum of values is 5. And B is the event that at least one throw is a 2. Calculate $P(A \mid B)$. Are events A and B independent?