Prob Stats L09.5

Joint RVs

Discrete RV: Independence

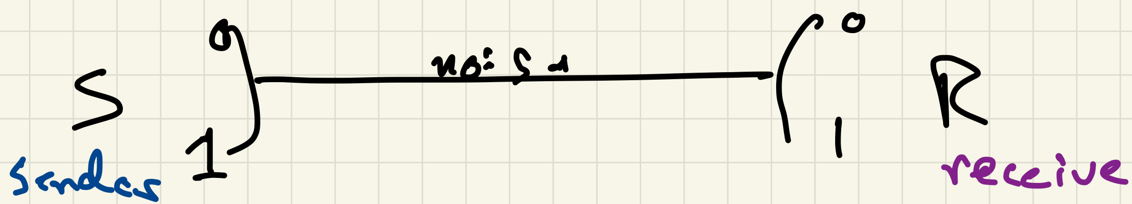
Intro: Continuous RVs

March 2, 2023

Consider Two RVs X, Y (discrete)

joint pdf $f_{X,Y}(a,b) = \Pr(X=a, Y=b)$

$= \Pr(\{X=a\} \cap \{Y=b\})$



R

S \ R	0	1
0	0.45	0.08
1	0.06	0.41

$f_{S,R}(0,1) = 0.08$

$f_S(0) = \Pr(S=0) = 0.53$

$f_S(1) = \Pr(S=1) = 0.47$

marginal distrib

Independence of R.V.s.

Independence of events A, B $\Rightarrow$

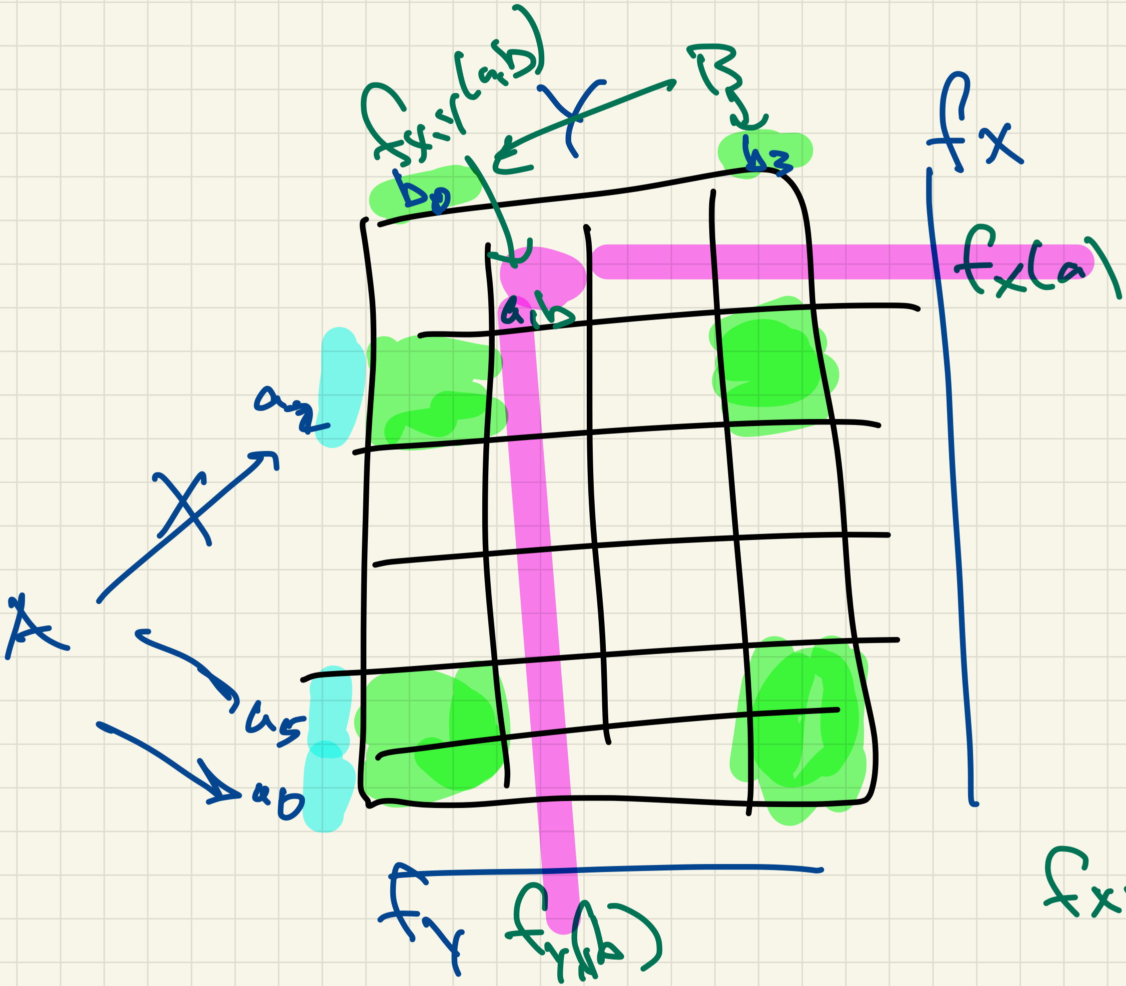
$$\Pr(A \mid B) = \Pr(A)$$
$$\Pr(B \mid A) = \Pr(B)$$
$$\Pr(A \cap B) = \Pr(A) \cdot \Pr(B)$$

$\boxed{\text{R.V. } X, Y}$ independent

f_X, f_Y, $f_{X,Y}$

$$\boxed{\begin{aligned} \Pr(\underbrace{\{X=a, Y=b\}}_{A \cap B}) &= \Pr(\underbrace{\{X=a\}}_{A}) \cdot \Pr(\underbrace{\{Y=b\}}_{B}) \\ f_{X,Y}(a,b) &= f_X(a) \cdot f_Y(b) \end{aligned}} \quad a, b$$

for all values a, b



$$f_{X,Y}(a,b) \overset{?}{=} f_X(a)\,f_Y(b)$$

Independence RV X, Y

implies any two event

$$A = \bigcup_{i \in I} \{X = a_i\} \qquad B = \bigcup_{j \in J} \{Y = b_j\}$$

$\Longrightarrow$ implies events A, B independent

$$\Pr(\{X \in A\}, \{Y \in B\}) = \sum_{i \in I} \Pr(\{X = a_i\}, \{Y \in B\})$$

$$= \sum_{i \in I} \sum_{j \in J} \Pr(X = a_i, Y = b_j) = \sum_{i \in I} \sum_{j \in J} \underline{\Pr(X = a_i)} \cdot \Pr(Y = b_j)$$

$$= \boxed{\sum_{i \in I} \Pr(X = a_i)} \boxed{\sum_{j \in J} \Pr(Y = b_j)} = \Pr(X \in A) \cdot \Pr(Y \in B)$$

$$Pr(S_1=1 \mid R_1=1) = \frac{Pr(S_1=1 \wedge R_1=1)}{Pr(R_1=1)} = \frac{0.41}{0.49} \approx 0.82$$

not independent

$$Pr(S_2=1 \mid R_2=1) = 0.82$$

$S_1=S_2$

$$Pr(S=1 \mid R_1=1, R_2=\cancel{1}0) = 1-(0.18)^2 \approx 1-0.035 \approx .965$$

		R	
S	0	0.45	0.08
	1	0.06	0.41

$f_S(0)=0.53$

$f_S(1)=0.47$

$f_{R,S}(1,1)=(0.47)(0.49$

$f_R(0)=0.51$ $\quad$ $f_R(1)=0.49$

Example

Independent Events but not independent RV

2 Dice

D_1

1	2	3	4	5	6
1/6	1/6	1/3	1/6	1/12	1/12

fair D_2

1	2	3	4	5	6
1/6	1/6	1/6	1/6	1/6	1/6

$A' = \{R_1 = 3\}$

$B' = \{R_2 = 3\}$

$B'' = \{R_2 = 1\}$

Roll one die, roll the other.

R_1 $\rightarrow \{1 = R_1\}$ A

R_2 $\{R_2 = 1\}$ B

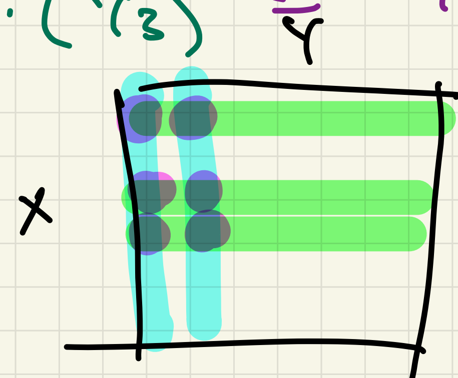
Let X and Y be two fair dice rolls

$$A = \{ X \in \{1,3,4\} \} \qquad B = \{ Y \in \{1,2\} \}$$

are A, B independent events

want check $\Pr(A) \cdot \Pr(B) \overset{?}{=} \Pr(A \cap B)$

$(1/2) \cdot (1/3) = \qquad 6/36 = \frac{1}{6}$



Review for Joint Disc. R.V.

R.V.s M, C

joint pdf $\boxed{f_{M,C}}$

table

	U 0	Y 1	
StS 0	0	0.2	$f_M(S) = 0.2$
DS 1	0.1	0	$f_M(DS) = 0.1$
CS 2	0.5	0.2	$f_M(CS) = 0.7$
	$0.1 + 0.5 = 0.6$	$= 0.2$	
	$f_C(U)$	$f_C(Y)$	

← marginal probabilities

$$\sum_i \sum_j f_{M,C}(a_i, b_j) = 1$$

E

$X = g(M, C)$

Conditional Prob

$f_{M|C=U}(a)$

$= f_{M|C=U}(DS)$

$$= \frac{\Pr(M = DS \cap C = U)}{\Pr(C = U)} \quad f_C$$