Prob.Stats L07b

# Continuous RVs.

## Normal, Exponentials, etc.

Feb 16, 2023

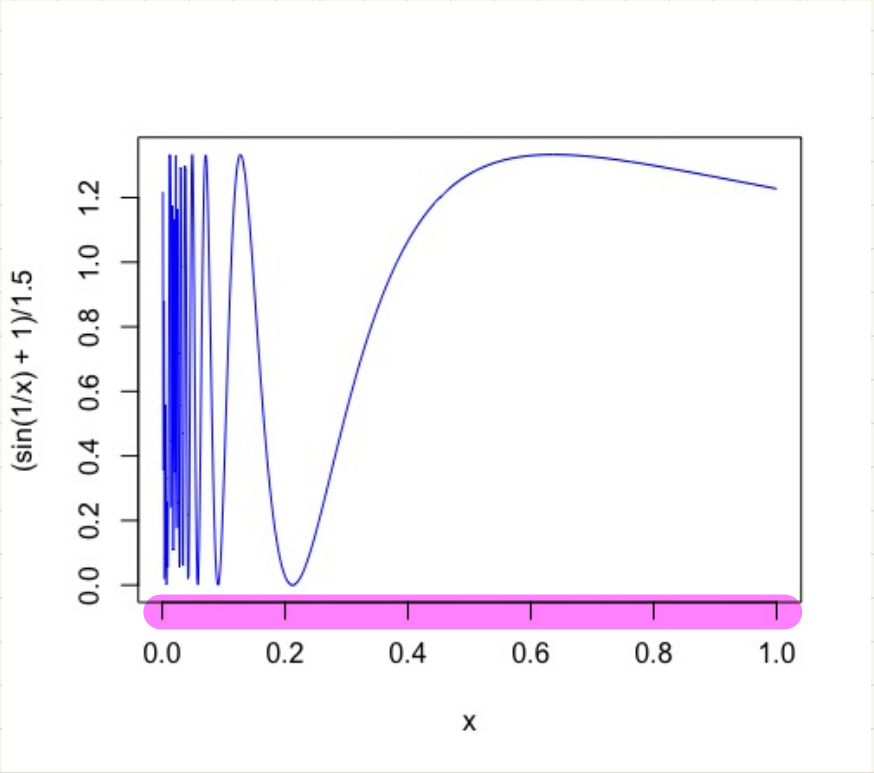
Is $f(x) = \begin{cases} \frac{\sin(1/x)+1}{1.5} & \text{for } x \in (0,1] \\ 0 & \text{for } x \notin (0,1] \end{cases}$

a valid pdf ?

① $f(x) \geq 0$

② $\int_{-\infty}^{\infty} f(a)da = 1$

$P_r(a \leq x \leq b) = \int_a^b f(x)\,dx$

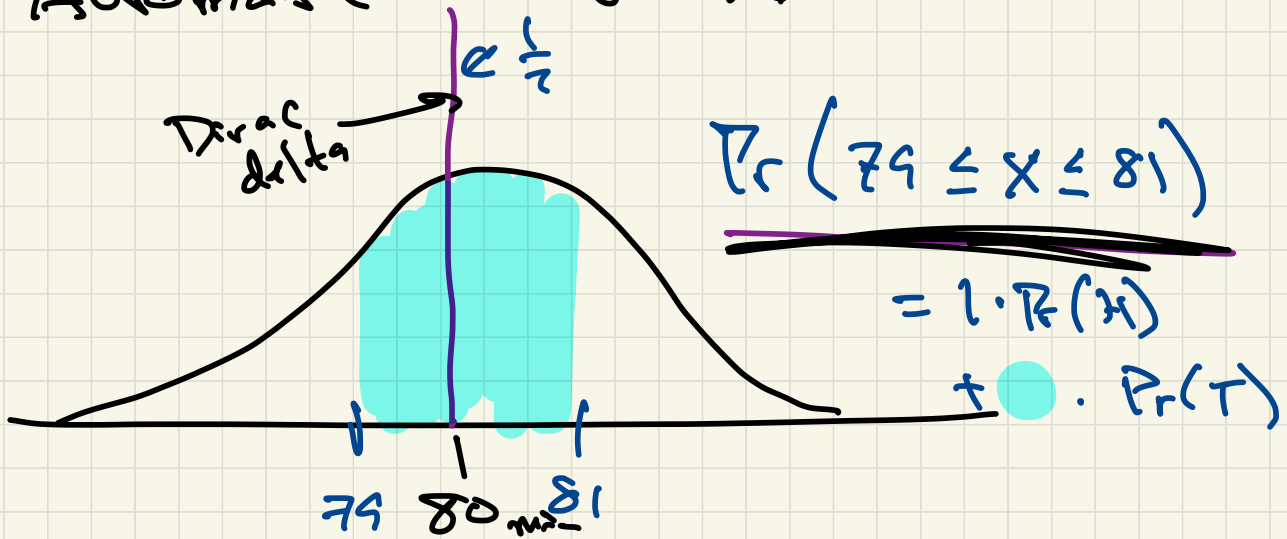


$X$ = length of recorded lecture

Flip $\frac{1}{2}$ T, $\frac{1}{2}$ H

- Press start, Press stop
- Automate 80 min

P




$$Pr(79 \le X \le 81)$$

$$= 1 \cdot Pr(H) + \bigcirc \cdot Pr(T)$$

# (Absolute) Continuous R.V.

$X \sim D$ $\quad D = Unif(\alpha, \beta)$

pdf $\quad f_X(x) \geq 0$

$$\int_{-\infty}^{\infty} f_X(x)\, dx = 1$$

cdf $\quad F_X(a) = P_r(X \leq a)$

$F_X(-\infty) = 0$

$F_X(\infty) = 1$

$e = 2.71\ldots$

$$F(a) = \begin{cases} \exp(a) = e^a & \text{if } a \le 0 \\ 1 & \text{if } a > 0 \end{cases}$$

$$F(\infty) = 1 \qquad F(-\infty) = \lim_{a \to -\infty} e^a$$

$$= \lim_{a \to \infty} \frac{1}{e^a} = 0$$

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$$f(a) = \frac{d}{da} F(a) = \frac{d}{da} e^a = \begin{cases} e^a & \text{if } a \le 0 \\ 0 & \text{if } a > 0 \end{cases}$$

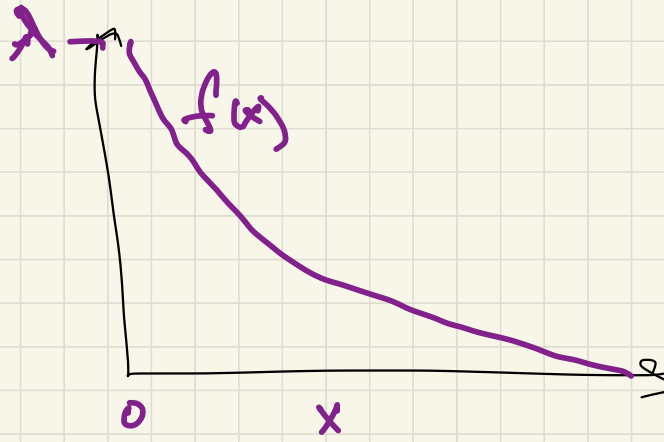
$$F(a) = \int_{-\infty}^{a} f(a)\, da$$

# Exponential Distribution

$$X \sim Exp(\lambda)$$

pdf $$f(x) = \lambda e^{-\lambda x}$$

cdf $$F(a) = 1 - e^{-\lambda a}$$



# Normal Distribution (Gaussian)

$$X \sim N(\mu, \sigma^2)$$

pdf $$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) = \frac{1}{\sqrt{2\pi}\sigma} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

just so $\int f_X(x)\,dx = 1$