ProbStats L06b

# Discrete Random Variables

## Bernoulli, Binomial, Geometric RVs

Feb 7, 2023

Sample Space $\Omega$

Random Variable $X : \Omega \to \mathbb{R}$

Die 

| $\Omega$ | $X(\omega)$ |
|---|---|
| $\omega =$ 1 | 1 |
| 2 | 4 |
| 3 | 9 |
| 4 | 16 |
| 5 | 25 |
| 6 | 36 |

$\mathbb{R}$

$2X + 3.1 = Y$

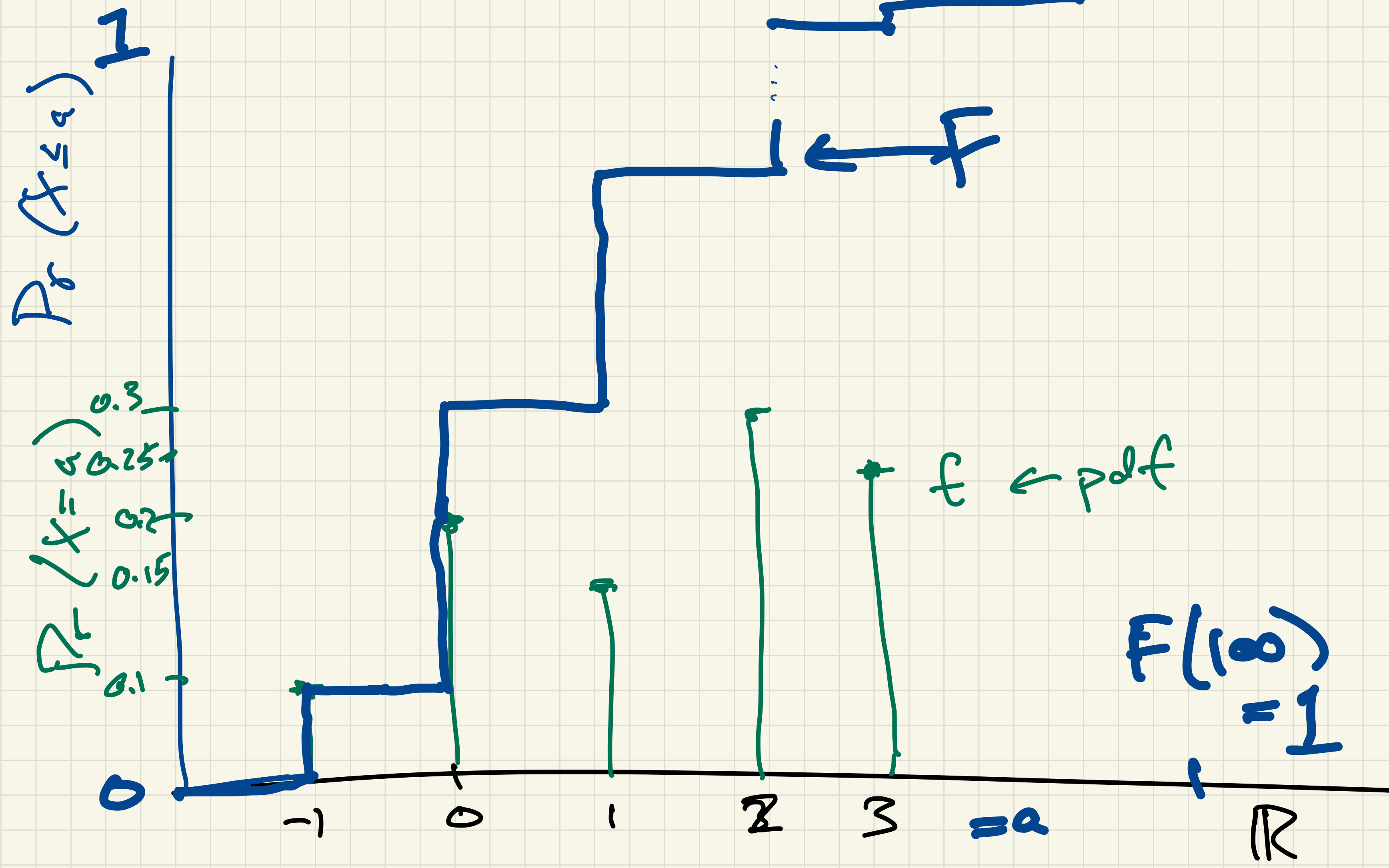
probability density function (pdf)

$$f_X(a) = Pr(X = a) \in [0,1]$$

$$\sum_a f(a) = 1$$

cumulative density function (cdf)

$$F_X(a) = Pr(X \le a) = \sum_{b \le a} f_X(b)$$



1
Pr (X ≤ a)
0.3
0.25
0.2
0.15
0.1
Pr (X = a)
0
F
f ← pdf
F(100) = 1
-1
0
1
2
3
= a
ℝ

# Bernoulli distribution (parameter $p \in (0,1)$)

↳ Bernoulli R.V. $X : \Omega \to \{0, 1\}$

**pdf:**

$$f_X(1) = \Pr(X=1) = p$$
$$f_X(0) = \Pr(X=0) = 1-p$$

$$f_X(2) = 0$$

Fair coin

$\Omega = \{H, T\}$

$X: H \to 1$

$T \to 0$

$p = 1/2$

$$\sum_a f_X(a) = 1 = f_X(1) + f_X(0)$$
$$= p + (1-p) = 1$$

$$f_X(a) \in [0,1]$$

Trick coin

$f_X(1) = f_X(0) \cdot 2 \Rightarrow p = 2/3$

$$X \sim \text{Ber}(p)$$

$$f_X(1) = p \qquad f_X(0) = 1-p$$

$$f_X(1) = f_X(0) \cdot 2$$

$$p = (1-p) \cdot 2$$

$$p = 2 - 2p$$

$$3p = 2$$

$$p = 2/3$$

$$f_X(1) = 2/3$$

$$f_X(0) = 1 - 2/3 = 1/3$$

$$(2/3) = (1/3) \cdot 2$$

# Binomial Random Variable

Param: $p$, $n$

$X \sim Bin(n, p)$

number of times an event repeated $n$ times is true, with each event true w.p. $p$.

$X_1, X_2, \ldots, X_n \sim Ber(p)$

$X \sim Bin(n, p)$ $\quad X = \sum_{i=1}^{n} X_i$

# factorial

$$n! = n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 2 \cdot 1$$

$$= \prod_{i=1}^{n} i$$

"choose" $\binom{n}{k}$

given n distinct balls, how many different subset of size k

$$\binom{5}{3} = \frac{5!}{(2!)(3!)} = \frac{120}{2 \cdot 6} = 10$$

$$\binom{n}{k} = \frac{n!}{(n-k)!\, k!}$$

$X \sim Bin(n, p)$

for $k = \{0, 1, 2, \dots, n\}$

$$f_X(k) = \binom{n}{k} p^k (1-p)^{n-k} \in [0, 1]$$

$f_X(k')$ $\quad k \notin \{0, 1, \dots, n\}$ $\quad f_X(k') = 0$

$p^k$ = prob. $k$ events all true = 1

$(1-p)^{n-k}$ = prob. $(n-k)$ event all false = 0

6 $\quad$ 4

| 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
|---|---|---|---|---|---|---|---|---|---|
| $p$ | $p$ | $p$ | $p$ | $p$ | $p$ | $(1-p)$ | $(1-p)$ | $(1-p)$ | $(1-p)$ |

$$= p^6 (1-p)^4$$

$$= p \cdot p \cdot p \cdot p \cdot p \cdot p \cdot (1-p)(1-p)(1-p)(1-p)$$

# Geometric Random Variables

$X \sim Geo(p)$

$f_X(k) = (1-p)^{k-1} \cdot p$

$F_X(k) = \sum_{k'=1}^{k} f_X(k')$

Event with prob $p$ of success

$k$ chances, 1 success

and success comes at end.

prob. it takes $k$ events until the first success.

# Key parameters

n = number of trials

k = number of successes (Bin) or first success (Geo)

p = Probability of success