Midterm Exam — Feb 29, 2024

CS 3130/ECE 3530: Probability and Statistics for Engineers

Name:

UiD:

- **Do not turn over this sheet until the exam starts at 3:40 pm.**
- **You may use one sheet (front and back) with notes.**
- **Be sure to show all of your work.**
- **You may use scratch sheet. Only work on the exam pages will be graded.**

Instructions: On the final you will be allowed 1 sheet (front and back) that is hand-written or digitally generated for this purpose.
Show all of you work in the space provided for each question. If you run out of space, you may attached another sheet of scratch paper. Attempt to simplify all calculations (seeing the final numbers will be useful for sanity check on answers), but we will try to be generous for small computational mistakes.

1. **(20 pts)** Basic Probability

A game includes a fair 8-sided die with sides labeled from $\Omega = \{1, 2, 3, 4, 5, 6, 7, 8\}$. Three key events occur:
$A = \{3, 6\}$: a multiple of 3
$B = \{2, 3, 5, 7\}$: a prime
$C = \{5, 6, 7, 8\}$: the larger values.

(a) What does $Pr(A \mid B)$ mean in English? What is its value?

(b) Are C and B independent? Explain why or why not.

(c) Given your answer (a) above, write down Bayes' Rule, and use it to solve for $Pr(B \mid A)$.

2. Random Variables and Probability Distributions

We have a quarantined population of sick people. In this population, for an individual, the probability of being affected by a virus is 0.01. We randomly sample n individuals one by one (with replacement, so iid) and check if that person carries the virus.

(a) What distribution models the number of people (among these n samples) having the virus? Write out its name and the values of its parameters.

(b) What is the expectation of the number of people (among these n samples) having the virus?

(c) If $n = 100$, then what is the probability that the number of people having the virus is at least 1?

3. Conditional Probability, Total Probability, Bayes Rule There are three urns with red and green stones. Bag 1 (B_1) has all green stones, bag 2 (B_2) has all red stones, and bag 3 (B_3) 2/3 red stones and 1/3 green stones.

   The random experiment is to pick an urn at random (they are unlabeled) and then pick a stone from the urn and then look at it's color.

   (a) Draw a tree diagram for this experiment.

   (b) If you run the experiment and hold the stone but don't look at it, what is the probability that it is green?

   (c) If you look at the stone and it is red, what is the probability that it came from B_2. Use Bayes rule, show your work.

4. Let X be a random variable with $E[X] = 2$ and $\text{Var[X]} = 4$. Compute the expectation and variance of $3 - 2x$.

5. Consider the random variable X with the density function

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(\frac{(x-\mu)^2}{2\sigma^2}\right)$$

What is the mean and variance of Y, where $Y = 3X + 5$ (expressed in terms of the variables given in the equation).

6. The probability distribution of a discrete random variable X is given by $P(X = -1) = 1/5$,$P(X = 0) = 2/5$,$P(X = 1) = 2/5$

   (a) Compute $E[X]$

   (b) Give the probability distribution of $Y = X^2$ and compute E [Y] using the distribution of Y.

   (c) Compute Var[X]

7. Compute the median of an $\mathrm{Exp}(\lambda)$ distribution.