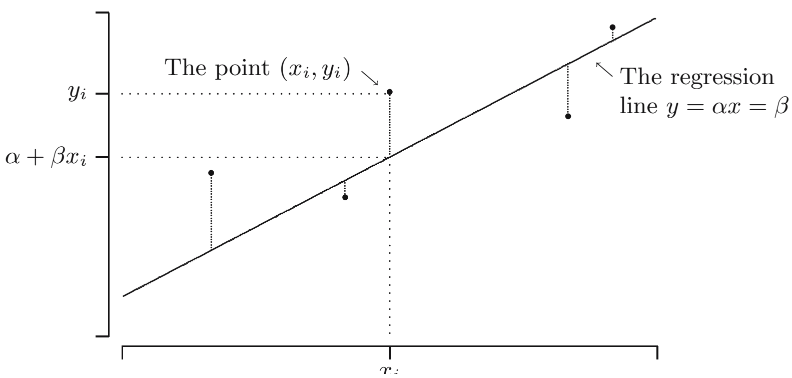
# Linear Regression 4/11/24

## Model of data

$$Y_i = \alpha + \beta x_i + U_i \qquad \text{for} \quad i = 1, 2, \ldots, n,$$




## Method of least squares

$$S(\alpha, \beta) = \sum_{i=1}^{n} (y_i - \alpha - \beta x_i)^2$$

## Minimize least squares:

$$\frac{\partial}{\partial \alpha} S(\alpha, \beta) = 0 \quad \Leftrightarrow \quad \sum_{i=1}^{n} (y_i - \alpha - \beta x_i) = 0$$

$$\frac{\partial}{\partial \beta} S(\alpha, \beta) = 0 \quad \Leftrightarrow \quad \sum_{i=1}^{n} (y_i - \alpha - \beta x_i)\, x_i = 0.$$

## Introduce sums/data:

$$n\alpha + \beta \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} y_i$$

$$\alpha \sum_{i=1}^{n} x_i + \beta \sum_{i=1}^{n} x_i^2 = \sum_{i=1}^{n} x_i y_i.$$

## Solve for alpha, beta:

$$\hat{\beta} = \frac{n \sum x_i y_i - (\sum x_i)(\sum y_i)}{n \sum x_i^2 - (\sum x_i)^2}$$

$$\hat{\alpha} = \bar{y}_n - \hat{\beta} \bar{x}_n.$$

## Unbiased estimators:

$$\mathrm{E}[\hat{\alpha}] = \mathrm{E}\left[\bar{Y}_n\right] - \bar{x}_n \mathrm{E}\left[\hat{\beta}\right] = \frac{1}{n} \sum_{i=1}^{n} \mathrm{E}[Y_i] - \bar{x}_n \beta$$

$$= \frac{1}{n} \sum_{i=1}^{n} (\alpha + \beta x_i) - \bar{x}_n \beta = \alpha + \beta \bar{x}_n - \bar{x}_n \beta$$

$$= \alpha.$$