CS3130/ECE3530: Probability and Statistics for Engineers **Due Tu 2/6**

Name:

uID:

Homework 2: Total Probability, Independence, Bayes' Rule, and Discrete Random Variables

Instructions: Write your answers directly on this pdf (via an editor, iPad, or pen/pencil). The answers should be in the specified place. Students will be responsible for loading their assignments to GradeScope, and identifying what page contains each answer.

The assignment should be uploaded by 11:50pm on the date it is due. There is some slack built into this deadline on GradeScope. Assignments will be marked late if GradeScope marks them late.

If your answers are too hard to read you will lose points (entire questions may be given 0).

Please make sure your name appears at the top of the page.

You may discuss the concepts with your classmates, but write up the answers entirely on your own. **Be sure to show all the work involved in deriving your answers! If you just give a final answer without explanation, you may not receive credit for that question.**

1. Independence: **Show the formula/math you use to check these. No guessing!**

   (a) You roll a six-sided die. Are the following two events independent?

$$A = \text{“the number is greater than 2”},$$
$$B = \text{“the number is odd”}.$$

   (b) You flip two coins. Are the following two events independent?

$$A = \text{“you flipped exactly one heads”},$$
$$B = \text{“you flipped at least one tails”}.$$

2. A doctor sees a patient that may have the flu. Based on the patient's symptoms, the time of year and the other patients they have seen, the doctor knows that 40% of patients with these conditions have the flu. The doctor gives the the patient a diagnostic test. If the patient has the flu, the probability that the test is positive is 0.80. However, if the patient does not have the flu, the probability that the test is positive is 0.05. What is the probability that the patient has the flu given a positive test result? *Hint:* Think about what the two events are, and use Bayes' Rule.

3. You are diagnosed with an uncommon disease. You know that among the general population there is a 3% chance of getting it. Use the letter D for the event "you have the disease" and T for the "the test says so." It is known that the test is imperfect: $\Pr(T|D) = 0.90$ and $\Pr(T^c|D^c) = 0.95$.

   (a) Given that you test positive (the test is positive if you have the disease), what is the probability that you really have the disease?

   (b) Suppose that after testing positive for the disease, you obtain a second opinion: an **independent** repetition of a test with the same overall values for true and false positives. You test positive again. Let S be the event that "you test positive again." Given this, what is the probability that you really have the disease?

4. There is a bucket of 8 red stones and 4 blue stones. I remove one stone from the bucket. I make a note of the color of the stone, but do not tell you the color. Next, I replace the stone I picked along with 4 new stones with the same color as the stone that I selected. After all of this, you select a stone from the bucket.

   (a) Draw a probability tree diagram, where the first level is the color I select, and the second level is the color that you select. **Be sure to give all branches of the tree and their probabilities, as well as the joint probabilities at the end of the branches!**

   (b) Using your diagram, what is the probability that the stone you select is blue?

   (c) Consider the two events $A =$ "I pick red" and $B =$ "you pick blue". Are these events independent? (Don't just answer yes or no, show your work!)

   (d) If you pick a blue stone, what is the probability that I picked a blue stone? Hint: Bayes rule.

5. Let X be a discrete random variable with probability mass function p defined over the set of integers given by:

a	-2	-1	0	1	2
$P(X = a) = p(a)$	$\frac{2}{10}$	$\frac{1}{10}$	$\frac{3}{10}$	$\frac{1}{10}$	$\frac{3}{10}$

and $p(a) = 0$ for all other values of a.

(a) Consider a random variable Y be defined by $Y = X^2$, For example, if $X = 2$, then $Y = 4$. Calculate the probability mass function of Y.

(b) Calculate the value of the distribution functions of X and Y in $a = 1$, $a = 3/4$, and $a = \pi - 1$.