

Reduced Dual-Formulation for Analytical Anisotropic Single Scattering

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Abstract

An analytical approach to solving anisotropic single scattering from point light sources in homogeneous media was recently derived via a dual-formulation of the air-light integral. In this paper, we demonstrate how to reduce the evaluation of the terms involved in the solution and provide an efficient and practical implementation substantially increasing the real-time performance characteristics.

CR Categories: I.3.7 [Computer Graphics]: Three-Dimensional Graphics and Realism—Color, shading, shadowing, and texture

Keywords: participating media, analytical integration

1 Introduction & Related Work

[Pegoraro and Parker 2009] recently derived a closed-form solution to single scattering in homogeneous isotropic media. They subsequently extended the analytical approach to anisotropic phase functions and light distributions via domain partitioning of the air-light integral [Pegoraro et al. 2009]. Solving the resulting dual-formulation however requires multiple evaluations of the computationally costly complex-valued exponential integral function.

In this paper, we reformulate the terms involved in solving the dual-formulation and reduce the computation to only 3 evaluations of the complex-valued exponential integral instead of 4. In addition, we provide an efficient implementation yielding a substantial speed-up of the real-time performance achieved on graphics hardware.

2 Reduced Evaluation & Implementation

Defining u as the variable of integration in the simplified form of the air-light integral as illustrated in figure 1(a), [Pegoraro et al. 2009] proposed the changes of variable $v = u + \sqrt{1 + u^2}$ and $w = u - \sqrt{1 + u^2}$, and formulated a solution requiring 4 evaluations of the complex-valued exponential integral Ei with the parameters v_a , v_h , w_b and w_h involved by the computation of the following terms

$$I_0(-H, v_h, v_a) = i_0(-H, v_a) - i_0(-H, v_h) \quad (1)$$

$$I_1(-H, v_h, v_a) = i_1(-H, v_a) - i_1(-H, v_h) \quad (2)$$

$$J_0(H, w_h, w_b) = j_0(H, w_b) - j_0(H, w_h) \quad (3)$$

$$J_1(H, w_h, w_b) = j_1(H, w_b) - j_1(H, w_h) \quad (4)$$

$$J_e(H, w_h, w_b) = Ei\left(\frac{H}{w_b}\right) - Ei\left(\frac{H}{w_h}\right) \quad (5)$$

with H the optical distance from the light to the ray and where

$$i_0(a, v) = \sin(a)\Re(Ei(av + ia)) - \cos(a)\Im(Ei(av + ia))$$

$$i_1(a, v) = \cos(a)\Re(Ei(av + ia)) + \sin(a)\Im(Ei(av + ia))$$

$$j_0(a, w) = -\sin(a)\Re\left(Ei\left(\frac{a}{w} + ia\right)\right) + \cos(a)\Im\left(Ei\left(\frac{a}{w} + ia\right)\right)$$

$$j_1(a, w) = \cos(a)\Re\left(Ei\left(\frac{a}{w} + ia\right)\right) + \sin(a)\Im\left(Ei\left(\frac{a}{w} + ia\right)\right) - Ei\left(\frac{a}{w}\right).$$

We here highlight that $1/w = -v$, that $\cos(-a) = \cos(a)$ and $\sin(-a) = -\sin(a)$, and that $Ei(\bar{z}) = \bar{Ei}(z)$ yielding the identities

$$j_0(a, w) = i_0(-a, v) \quad (6)$$

$$j_1(a, w) = i_1(-a, v) - Ei(-av) \quad (7)$$

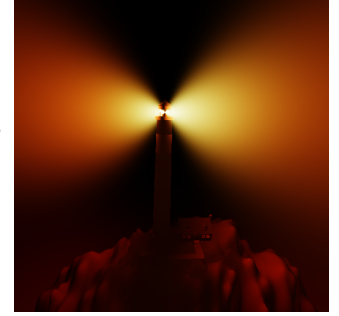
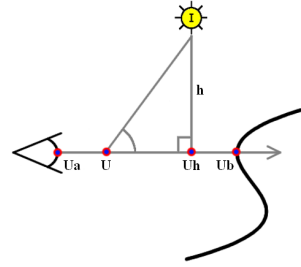


Figure 1: (a) Illustration of the terms involved in the computation of the air-light integral. (b) A lighthouse in thick brume with an anisotropic two-lobed spotlight rendered in real-time.

from which directly follows that the terms involving the complex-valued exponential integral at v_h and w_h are identical. This reduces the computation to only 3 evaluations of the latter function as illustrated in figure 2 which additionally explicitly sets $u_h = 0$.

ComputeIJ(v_a, w_b, H)

1. $co = \cos(-H)$;
2. $si = \sin(-H)$;
3. $\{re, im\} = Ei(-Hv_a, -H)$;
4. $I_0 = si * re - co * im$;
5. $I_1 = co * re + si * im$;
6. $\{re, im\} = Ei(H/w_b, H)$;
7. $J_0 = si * re + co * im$;
8. $J_1 = co * re - si * im$;
9. $\{re, im\} = Ei(-H, -H)$;
10. $I_0- = si * re - co * im$;
11. $J_0- = si * re - co * im$;
12. $I_1- = co * re + si * im$;
13. $J_1- = co * re + si * im$;
14. $J_e = E_1(H) - E_1(-H/w_b)$;
15. $J_1- = J_e$;
16. $return\{I_0, I_1, J_0, J_1, J_e\}$;

Figure 2: Pseudo-code of the reduced evaluation.

3 Results & Conclusion

The method was implemented in a fragment shader using OpenGL and Cg running on an NVIDIA GeForce GTX 280 under Windows Vista 64-bit. The integral was evaluated independently for each color channel, hence 3 times per fragment at a resolution of 512x512 as illustrated in figure 1(b). While the previous method renders at 63 FPS, our new evaluation scheme achieves a frame rate of 77 FPS and consequently yields a speed-up of 1.22X.

In conclusion, we have shown how to reduce the evaluations involved in analytically solving the dual-formulation of the air-light integral for anisotropic single scattering from point light sources in homogeneous media. Moreover, we have provided a practical implementation and demonstrated substantial gains in performance.

References

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