Reaction-Diffusion Computation in Wireless Sensor Networks

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***Abstract*— Biological systems exhibit an amazing array of distributed sensor/actuator systems, and the exploitation of principles and practices found in nature will lead to more effective artificial systems. The retina is an example of a highly tuned sensing organ, and the human skin is comprised of a set of heterogeneous sensor and actuator elements. Moreover, the specific organization and architecture of these systems depends on contextual influences during the developmental stages of the organism. Comparable theoretical and technological methodologies need to be found for wireless sensor networks. We propose the study of reaction-diffusion systems from mathematical biology as a starting point for this endeavor. The main result is the demonstration of the feasibility of computing useful patterns using reaction-diffusion mechanisms in wireless sensor networks; for example, providing clearly delineated stripes which serve as lane markers for robots.**

Section 1. Reaction-Diffusion Fundamentals

Alan Turing introduced a revolutionary reaction-diffusion model as the chemical basis of morphogenesis [17], and this method lends itself particularly well to pattern synthesis in distributed systems. For more detailed explanations, see his original paper (which provides an exemplar of the scientific paper – theory, analysis and numerical solution on the Manchester machine which Turing helped design and build!), as well as the works of Murray [13], Meinhardt [11], etc. Turing's key insight was that diffusion of an inhibitory morphogen could lead to the formation of stable and variegated patterns. This is related to nonlinear far from equilibrium thermodynamics, and dissipative structures (e.g., see Prigogine [16] who received the Nobel prize in chemistry for work in this area). One of the goals of our work is to understand how these principles behave practically in biological sensor systems and how they may be exploited in wireless sensor networks. More recently, reaction-diffusion computers have been proposed. For an in-depth introduction to R-D computers, see Adamatzky et al. [1]. A wide range of issues are discussed there, including: (1) R-D processors, (2) R-D geometrical computation (e.g., Voronoi diagrams and skeletons), (3) R-D logic circuits, (4) R-D robot control, and (5) programming R-D computers.

Due to its two chemical activator-inhibitor system which produces appropriate waves for the patterns required, we use Turing's reaction-diffusion mechanism to generate patterns in wireless sensor networks [6] (we use here some introductory material from that paper); [4] provides an introduction to nonlinear parabolic systems theory and related techniques as they apply to R-D systems. A set of equations forms the basis of this mechanism that mimics the reaction and diffusion aspects of certain chemical kinetics:

$$\frac{\partial \mathbf{c}}{\partial t} = f(\mathbf{c}) + D\nabla^2 \mathbf{c} \tag{1}$$

where $D\nabla^2 c$ expresses the diffusion component, and the reaction is described by $f(\mathbf{c})$. Two *morphogens* or variables form the simplest such systems with of variable acting as the activator and the other as the inhibitor. This can be modeled by:

$$\frac{\partial u}{\partial t} = \gamma f(u,v) + \nabla^2 u, \frac{\partial v}{\partial t} = \gamma g(u,v) + d\nabla^2 v \tag{2}$$

where u and v are the concentrations of the morphogens, d is the diffusion coefficient and γ is a constant measure of scale. The functions $f(u,v)$ and $g(u,v)$ represent the reaction kinetics. We generate spatial patterns using the Turing system of equations:

$$f(u,v) = \beta - uv, g(u,v) = uv - v - \alpha$$

where u and v are the morphogen concentrations, α and β are the decay and growth rates, respectively. Equation (2) becomes linearly unstable to certain spatial disturbances in the domain they define. This domain is called the *Turing space* where the concentrations of the two morphogens are unstable and result in spot patterns.

Section 2. R-D Patterns in Sensor Nets, Computer Vision, and Robotics

We have pointed out [6] several uses for patterns in the sensor nets:

- encoders can be built from stripe, spot or ring patterns to track (physical) distance traveled, or to control the flow of communication packets in order to minimize power cost or to avoid congestion.
- 2D image basis sets (e.g., Haar or Hadamard basis sets) can be provided by sets of R-D patterns; any 2D array (e.g., map, image, etc.) can then be encoded in terms of the coefficients associated with the respective basis images.
- interference patterns (holograms) can be formed from an R-D reference wave.
- the *S-Net* can serve as a signal carrier or modulator by using moving waves produced from R-D mechanisms.

Understanding the precision and reliability of pattern formation is then of high importance.

We introduced the use of Turing's reaction-diffusion pattern formation to support high-level tasks in sensor networks

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Fig. 1. 6 by 7 Experimental Layout of X-bow T-Sky Wireless Sensor Motes.

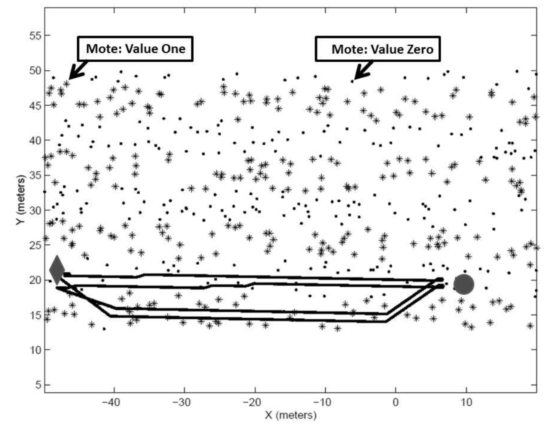

Fig. 2. Robot Path in Reaction Diffusion Pattern (.'s are motes broadcasting value 0; *'s are motes sending value 1; ○ is the fire control point; ◇ is the robot load point). Note that the path from ◇ to ○ runs on *'s while the way back is on .'s.

(*S-Nets*); Figure 1 shows a 42-node sensor network layout (each mote costs about $100) we have used in reaction-diffusion computation experiments. This has led us to explore various biologically motivated mechanisms. We address issues that arise in trying to get reliable, efficient patterns in irregular grids.

Others have explored the use of both reaction-diffusion and more general diffusion methods in computer vision and robotics. For example, Fukuda et al. describe the use of reaction-diffusion techniques in robot motion[3]. Moreover, as described by Peronna et al.[15], multi-scale descriptions of images (i.e., scale-space) can be produced by embedding the original image in a family of images obtained by convolving the original image with a filter; Koenderink[10] showed that this is equivalent to finding the solution of the diffusion equation:

$$I_t = \bigtriangledown^2 I = I_{xx} + I_{yy}$$

We believe that it will be quite useful for *S-Nets* to use similar methods to analyze sensed data of various sorts. More closely related to our work is that of Justh and Krishnaprasad [7] who propose the active coordination of a large array of microactuators by means of diffusive coupling implemented as interconnection templates, and Nagpal [14] who describes methods to create patterns of diverse geometry.

For example, consider a forest fire scenario: sensor devices are dropped into a wide geographic area, establish a network, compute coordinate frames, calculate gradients, and produce a stripe pattern of off-on signals that can be used by fire fighting agents to go to a fire control point by following *on* devices (pattern == 1) and return by following *off* devices (pattern == 0) (see Figure 2 where ○ is the fire control point; ◇ is the robot load point, · is a mote sending a zero value and * is a mote sending a 1 value). Such patterns can be computed by very robust reaction-diffusion systems derived from models of biological pattern formation.

Section 3. Computationally Stable R-D Algorithms

There has been some investigation of how pattern formation is influenced by number of cells, time scale, and initial condition variation. In particular, Bard and Lauder [2] showed that "stable repeating peaks of chemical concentration of periodicity 2-20 cells can be obtained in embryos in periods of time less than an hour. We do find however that these patterns are not reliable. Small variations in initial conditions give small but significant changes in the number and positions of observed peaks." They showed that this method has difficulty producing exact patterns reliably. We have found other difficulties in producing the patterns necessary to support higher-level tasks. We describe these here and propose some solutions.

A more significant issue for us is that the reaction-diffusion pattern formation equations assume that the inter-cell distance is uniform (and usually equal to 1). Our *S-Nets*, however, do not form a uniformly spaced grid in 1D or 2D; in fact, we generally assume that the sensor devices are randomly dropped in the environment. In addition, the diffusion part of the equations uses the inter-node distances in the computation of the second derivative. Two concerns are: (1) these distances are not uniform, and (2) in an actual implementation, there will be some amount of error in the inter-node distance determination. This has led us to investigate the impact of non-uniform spacing on the pattern computation.

Consider the case of Sensor Elements *SEL*s randomly distributed in the square area. It turns out that if all neighbors within a certain distance (e.g., broadcast range) are used in the reaction-diffusion calculation, and the distances are used to compute the Laplacian, then the process generally fails to converge. However, if each *SEL* randomly selects four of its neighbors (e.g., from the broadcast connectivity graph), then the reaction-diffusion process converges; alternatively, one can impose a virtual equi-spaced grid over the mote locations, interpolate values at the grid locations, and run the R-D process on the virtual grid. Figure 3 shows an example of this on a 5×5 square area using 2,000 *SEL*s placed by sampling the x and y coordinates from the uniform distribution.

For the physical mote system, the reaction-diffusion process leads to the formation of spots (i.e., the morphogen concentration $a > 5$) even with the small number of motes. However, spots make take many iterations (1,600) to appear,

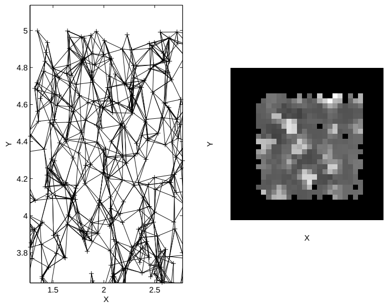

Fig. 3. *S-Net* of Randomly Placed Sensor Elements *SEL*s and the Resulting 2-D Turing Pattern using 4 Randomly Selected Neighbors.

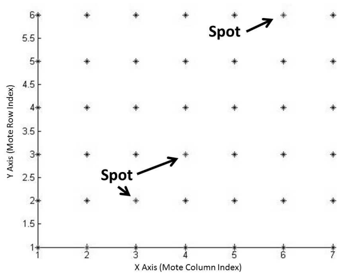

Fig. 4. 2-D Turing Spot Pattern in 42-Node Mote Set.

and the spot pattern is not stable (see Figure 4).

We believe that several issues are at play: the number of nodes, the asynchronous nature of the mote transmissions, as well as the locality and bi-directionality of the connectivity.

Section 4. Future Extensions

We have described techniques for forming patterns in sensor networks based on reaction diffusion equations. Such methods may be used to process data, although that is not discussed here, and to carry sensed signal information. The next focus of our work is on the production of patterns based on sense data analysis (e.g., camouflage synthesis). Such methods may also find application in sensor network security; in this scenario, a deformed pattern will emerge from a distributed computation if there are any nodes which have fallen victim to attack, or if external nodes have managed to get themselves incorporated into the *S-Net*. Of course, resource allocation and exploitation may also be based on patterns, and given the random nature of the patterns, may help conserve resources (e.g., energy) overall.

In addition, we are working on mesh generation in wireless sensor networks, level set calculation, and the creation of active camouflage based on the work Zhu and Mumford [18]; this all fits with our view of Computational Sensor Networks as self-validation systems [5]. They propose (1) a theory for discovering the statistics of a set of natural images, and (2) a framework which allows the definition of reaction-diffusion equations to produce similar natural images, and in particular, they show how to remove conspicuously dissimilar segments from a scene. Specifically, they show that given a learned set of prior models that reproduce the observed statistics, the potentials of the resulting Gibbs distributions have the form:

$$U(I;\Lambda,S)=\sum_{\alpha=1}^{K}\sum_{x,y}\lambda^{\alpha}((F^{(\alpha)}\times I)(x,y))$$

where $S=\{F^{(1)},F^{(2)},\ldots,F^{(K)}\}$ is a set of filters and $\Lambda=\{\lambda^{(1)},\lambda^{(2)},\ldots,\lambda^{(K)}\}$ is the set of potential functions.

Reaction-diffusion equations are found as the gradient descent partial differential equations on $U(I;\Lambda,S)$; diffusion arises from the energy terms while pattern formation reactions are related to the inverted energy terms. These are then used to remove clutter in a scene and to denoise images. We propose that the resulting images can be displayed by LEDs distributed throughout the material of the camouflage canvas, and based upon our previous work in e-textiles [8], [9], [12], we believe that technical solutions exist for the realization of this goal.

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