

Roderic A. Grupen

COINS Department
University of Massachusetts
Amherst, Massachusetts 01003

Thomas C. Henderson

Department of Computer Science
University of Utah
Salt Lake City, Utah 84112

Ian D. McCammon

Department of Mechanical Engineering
University of Utah
Salt Lake City, Utah 84112

A Survey of General-Purpose Manipulation

Abstract

A general-purpose manipulator, in conjunction with other sensory modalities, can be used to measure the three-dimensional character of the environment, support object recognition and localization procedures, and perform alternately as a delicate and powerful manipulator. This paper surveys the technologies which support general-purpose manipulation. While machines are capable of outperforming the human hand in specific applications, there is no machine capable of addressing the variety of tasks accomplished by our hands. We begin, therefore, by discussing the human hand. Some of the "bioware" supporting manipulation and haptics in humans is described. The development of hardware to support general-purpose manipulation is progressing rapidly. We focus on tactile sensing techniques and their role in perception. A variety of mechanical manipulators used in research and the methods employed to control them are presented. Finally, we discuss properties of the task, the manipulator, and the object which constrain the selection of manipulation strategies.

1. The Human Hand

The design of a mechanical manipulator is usually motivated by a particular class of tasks. When the

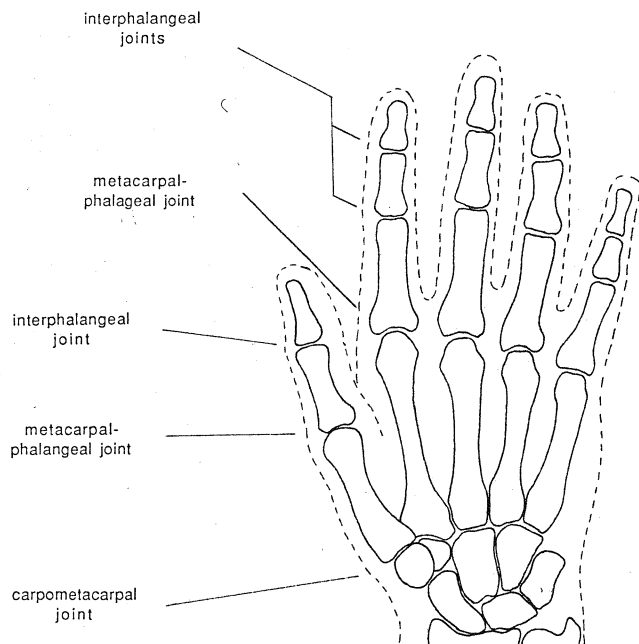
problem domain includes the interaction of the manipulator with relatively large and massive, or small and delicate, objects, the result is what we define here as a general-purpose manipulator. Such a manipulator can be integrated into multimodal sensory systems used to support object recognition procedures or to generate task-defined forces and velocities.

One approach to developing a general-purpose manipulator is to emulate those attributes of the human hand which make it such a versatile end-effector. The practicality of such an approach has been the topic of considerable debate, since the objective is functionality and not geometrical reproduction. The human hand is, however, an existence proof of an extremely versatile manipulator with which every investigator is familiar. For these reasons, this survey of general-purpose manipulation technology begins with a review of the physiology of the human hand and the grasping primitives which this physiology supports.

1.1. Kinematics

Figure 1 presents the skeletal structure of the human hand. The kinematic character of the human hand has been discussed by Lian et al. (1983). Each of the four fingers of the human hand consists of three joints, the most proximal of which is called the metacarpal-phalangeal joint. It has two degrees of freedom and is capable of adduction-abduction over a range of ap-

Fig. 1. The skeletal structure of the human hand [adapted from Jones (1920)].

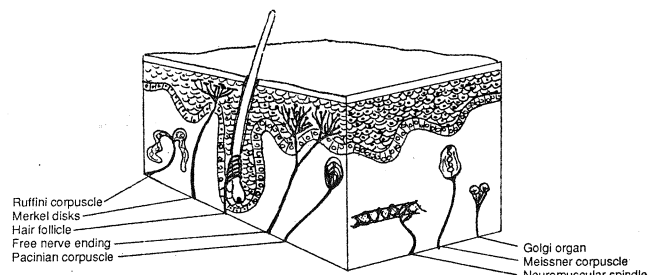


proximately 30° and flexion-extension of approximately 120° . The next two joints of the human finger are the interphalangeal joints. These are hinge joints with only one degree of freedom each and a range of motion of approximately 90° .

The thumb is a more complex mechanism; we will present here only a simplified description. The proximal joint is called the carpometacarpal joint and contains two degrees of freedom. The first degree of freedom is adduction-abduction with a range of motion of approximately 90° . The axis of this rotation is skewed somewhat from the plane defined by the fingers. The second degree of freedom operates the joint in flexion-extension with a range of motion of slightly less than 90° . This motion is entirely within the plane defined by the palm. The next joint of the thumb is the metacarpal-phalangeal joint whose predominant DOF is that of flexion-extension from the plane of the palm toward the palm over a range of about 60° . The last (distal) joint of the thumb is the interphalangeal joint. This joint is a simple hinge joint with one degree of freedom which allows a range of motion of about 90° .

The grasps which these structures support have been described in texts on physiology (Kapandji 1970) and have been observed in detail during the execution of

Fig. 2. Some of the sensors in the skin of the human hand [adapted from Jacobsen (1972)].



representative manufacturing grasps (Cutkosky and Wright 1986). Types of grasps are typically characterized by noting the relative amount of dexterity and power transmission. The human hand accommodates these somewhat independent modes of operation by involving strong forearm muscles in a grasp when power must be transmitted to the environment. The result is a wraparound grip that maximizes contact surface area by involving the palm as well as the fingers (Wright 1983).

1.2. Biological Sensors

Descriptions of the mechanics of human motor and tactile sensing vary slightly according to the source. A variety of specialized structures, which transduce both internal and external signals, and pathways of control have been identified. However, the information content of the resulting composite signal and the amount of processing done en route to the brain is not well understood. The purpose of this section is to expose some of these issues in an effort to develop a perspective on the complexity, redundancy, and distributed nature of the control of the human hand. The following discussion describes some of the sensing structures of the human hand. These structures are presented in Fig. 2.

Internal Receptors

Motor control in humans involves control of antagonistic muscle groups which allows the stiffness of each joint to be modulated. The control of these muscle

groups requires the acquisition of sensory data which describe both positions and forces. Internal receptors contribute to the sensing of the relative position of the parts of the body, a sense termed *proprioception*. Four receptors of this type are described below (Coiffet 1981; Langley et al. 1974).

1. *Neuromuscular spindles* sense the degree of stretch in the muscle fiber. These receptors are to some degree responsible for the reflexive cohesion of the skeleton and provide precise movement control. They come in two varieties: one responds to high frequencies, and another responds to low frequencies and dc traction.
2. The *Golgi organ* exhibits a very slow response and contributes to the control of muscular tension by measuring the degree of stretch of the muscle fiber at the tendon-muscle interface.
3. The *articular surfaces* in the joints of the body produce signals proportional to extreme position, velocity, or ligament tension. These receptors are not analogous to continuous potentiometers, but provide feedback for extreme movement of the joint.
4. *Ruffini corpuscles* represent the final category of internal receptor. This sensor functions as a thermal receptor and contributes to the sense of kinematic forces and accelerations.

Epidermal Receptors

These receptors, located in the outermost layer of the skin, are distinguished from those above in that they respond to external stimuli. Perhaps the most intimate interaction of the human nervous system with its external environment is that of the hair. Hair cells have a variety of roles, including proximity sensing, reporting contacts, and sensing air movement. The contribution of the hair cells to the recognition of the environment is limited and will not be considered further here.

Epidermal receptors provide the most direct indication of general mechanical interactions between the skin surface and an external object. The sensors in the epidermis are characterized by two types of receptors.

1. *Merkel disks* are sensory receptors with a large bandwidth which can respond to both compression and shear stimuli.

2. *Free-ended nerve fibers* consist simply of nerve fibers with free endings which respond to a variety of stimuli, including temperature and pain. These receptors are classified by their diameter, conduction speed, sensitivity or threshold level, and the presence or absence of a myelin sheath. The performance of these sensors is proportional to the diameter of the nerve fiber. As the diameter increases, the threshold stimulation decreases, the amplitude of the output signal increases, the duration of each signal impulse increases, and the velocity of the signal increases. The presence of a myelin sheath induces a somewhat different conduction path in the nerve fiber which increases the velocity of propagation. The types prevalent in the epidermis are the *A-* and *C-type* free-ended receptors. Type *A* has a relatively large diameter and is myelinated while type *C* is smaller, more numerous, and unmyelinated.

The signal output of any one of these epidermal neural receptors consists of a generated potential, which is proportional to the stimulus level. When the generated potential exceeds a threshold value, an action potential is induced. Action potentials form a series of constant amplitude spikes whose frequency is proportional to the level of stimulation. When the threshold value of the receptor is exceeded, the input stimulation is encoded in the frequency domain. The threshold varies from receptor to receptor so that increased stimulation corresponds to more numerous signal responses.

Dermal Receptors

Beneath the epidermal layer of the skin, the dermis incorporates two types of receptors (Coiffet 1981):

1. *Meissner corpuscles* respond to light touch; they are located beneath the convoluted dermal papillary layer. These structures are ovoid with the major axis perpendicular to the skin surface and contain neural fibers running normal to and tangential to the skin surface. The Meissner corpuscles are specialized high-frequency transducers.
2. *Pacinian corpuscles* (as distinguished from an

intramuscular pacinian-type receptor) respond best to accelerating mechanical displacement rather than constant velocity deformation. These sensors are quite sensitive to pressure, but not to direction, and respond primarily to vibration stimuli.

1.3. Biological Performance Specifications

Properties of the various sensory receptors are commonly described in relative terms. From the perspective of mechanical design, it is instructive to consider the range of values for these parameters. Table 1 has been assembled from a variety of sources (Coiffet 1981; Gordon 1978; Heimer 1983; Kandel and Schwartz 1981; Langley et al. 1974) and presents some performance specifications for the sensory apparatus in the human hand.

When considering the diversity, specialization, and redundancy of the sensory information provided by the skin and muscle, it is apparent that the implementation of an active, touch feedback, dextrous manipulator in humans requires a complex integration of "bioware." An indication of the emphasis on touch in humans is the fraction of the somatosensory cortex that is devoted to it. Figure 3 is a graphical depiction of just this. Note the amount of the cortex that is dedicated to the hands and feet (Kandel and Schwartz 1981).

2. Technologies for Tactile Sensing

A mechanical manipulation system which performs as well as the human system presumably requires similar sensory information. Proprioceptive data, such as joint position, velocity, and force, is necessary for the low-level control of the manipulator. Exteroceptive data is essential to higher-level manipulation algorithms. The emphasis here will be on the acquisition of tactile data resulting from an interaction with the environment. The technology associated with tactile feedback is extremely diverse, ranging from simple sensors that

Table 1. Sensory Specifications on the Fingertip

Frequency response	0 to 400 Hz (+ very high freq)
Response range	0 to 100 g/mm ²
Sensitivity	Approx. 0.2 g/mm ²
Spatial resolution	1.8 mm
Signal propagation	Motor neurons 100 m/s Sensory neurons 2 to 80 m/s Autonomic neurons 0.5 to 15 m/s

signal the presence of a contact, to artificial skins that attempt to provide information about the mechanical strain present at a contact and the thermal conductivity of the environment. This section will discuss some of the methods that have been employed to provide this sensory information, and the problem of object recognition from tactile data.

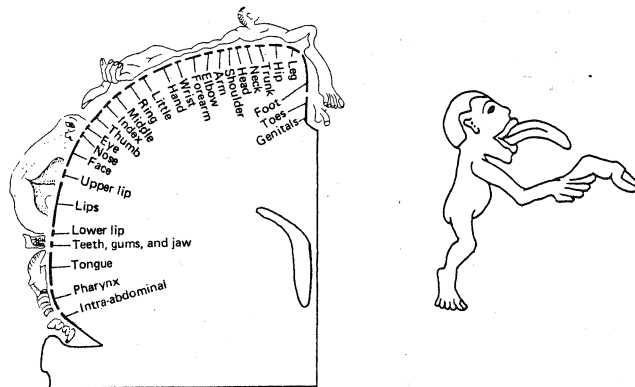
2.1. Methods of Tactile Sensing

This section provides an overview of tactile technologies and considers representative designs. For a more extensive reference list pertaining to tactile sensing see Hollerbach (1987) or Pennywitt (1986).

First, we establish criteria with which to discriminate among tactile sensors. A primary issue in the selection of appropriate technology is the distinction between contour sensing and the sensing of interface forces (Fearing and Hollerbach 1984). These approaches suggest distinct goals and methods.

Contour sensing produces information concerning the contact profile or contour of the object. The incentive to develop such sensing techniques came from a desire to increase the utility of parallel jaw grippers. These grippers might, in the presence of uncertainty, when presented with curved objects, or by design, produce point or line contacts with an object. A conformable surface on the gripper allows it to make a surface contact with the object and, therefore, provides a more stable grasp configuration. Contour sensing is useful in recognition tasks (McCammon 1984; Page and Pugh 1976) but is not directly useful in the determination of interface forces. Moreover, the performance of contour sensing mechanisms typically degrades

Fig. 3. The Sensory Homunculus [reprinted by permission from Kandel and Schwartz (1981) © Elsevier].



rapidly when tangential forces are present. For these reasons, when the objective is a system capable of grasping, recognition, and manipulation, it is preferable to sense the contact forces directly.

Force sensing provides more pertinent information for grasping and manipulation control. As mentioned above, a compliant covering improves the nature of the contact by distributing forces over the surface of the object. Furthermore, the covering serves to protect the force sensing hardware underneath. The state of stress beneath an elastic medium is linearly superimposable, but the deformation is not. These observations allow the state of stress beneath a compliant skin to be modeled. The ability to distinguish the difference between planar surface contact and vertex contact has been demonstrated (Fearing and Hollerbach 1984).

Other parameters of contact have been acknowledged to be useful in the control of manipulation systems (Harmon 1982). The maximal *spatial resolution* of a particular sensor is a function of the maximum spatial density permitted in its fabrication. The spatial frequency measurable in tactile features is defined by the resolution of the sensor. While there are applications for sensors with very high spatial resolution, typical manipulation tasks call for resolution on the order of 1–2 mm, or roughly that of the human skin. The *sensitivity* and *dynamic range* must be selected appropriately for each task. Ideally, touch sensors should be stable, monotonic, and repeatable. *Hysteresis* in a sensor implies that its output is not only a function of the mechanical input but also of the recent history of inputs. The human touch apparatus is fairly hysteretic, but remains quite useful. The engineering

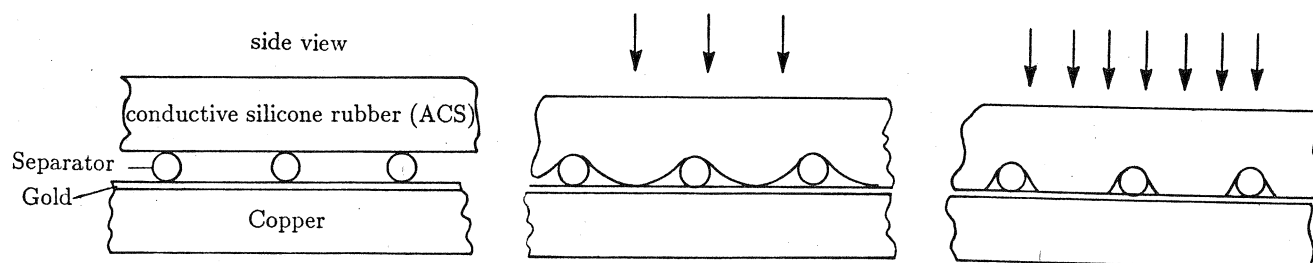
approach to integrating tactile feedback into a manipulation controller decidedly prefers nonhysteretic sensors. The *frequency response* of the sensor may be critical in the design of a tactile system. Some contact parameters require relatively high frequency signal content (e.g., slip detection or active texture analysis), but the lower limit on frequency response is often the access loop time in the control software. Finally, the designer must consider the means of *addressing* the tactile patches and make room for the electrical connection from sensor to controller. The *number of wires* needed to access an array of sensors varies as a function of the technology used and the scheme employed to address the sensor patch (Jacobsen et al. 1987).

The most straightforward tactile sensor is the binary contact switch. This sensor can be configured as an array to provide simple binary contact images, but does not provide useful force information. It is more commonly used to signal the controller that the manipulator has reached a known set point or is approaching the extent of its safe workspace.

Since we are interested in measuring interface forces, it is natural to consider strain gauge technology to measure orthogonal strains and to reconstruct the 3-D state of stress. This idea has been used to measure forces in parallel jaw grippers (Zalucky 1984) and inside hemispherical fingertips to determine the point of contact with the object (Salisbury 1984). The approach is of little use in tactile arrays, however, since the resolution achievable is insufficient for most applications.

Conductive elastomers have been used extensively in the design of tactile sensors (Dario et al. 1984; Harmon 1980; 1982; Hillis 1982; Raibert 1984). Since it is desirable to cover the tactile system with some type of compliant layer, it might initially appear advantageous to use a conductive rubber, a doped rubber, or a conductive foam to act both as protective skin and pressure transducer. These materials rely on the property that predictable changes in the resistivity of the material result from local deformations. However, the currently available materials are often hysteretic and doped materials are generally not very rugged. Moreover, low sensitivity, noise, drift, long time constants, and low fatigue life pose problems in these transducers. Nevertheless, commercial tactile systems exist which employ this technology (Barry Wright Corp.

Fig. 4. Hillis' elastic contact resistance sensor [reprinted by permission from Hillis (1982) © MIT Press].



1984) and have been used in manipulation research and industrial applications.

Hillis (1982) addressed these limitations by proposing the sensor illustrated in Fig. 4. The sensor employs a conductive silicone rubber which deforms around the separator and contacts an electrode under an applied pressure. Increased pressure loading causes increased surface contact between the silicone and the electrode. The contact resistance is then inversely proportional to the contact area and thus to the applied pressure. The signal output of the sensor is proportional to the contact resistance rather than the point-to-point resistance in the silicone material. The separator can be designed to produce the correct sensitivity, resolution, and dynamic range for the specific application. Hillis constructed an array of 256 tactile sensors with a spatial resolution of about 1 mm using this technology. The number of wires needed to address the array is reduced to a manageable level by accessing only rows and columns as in a keyboard. An array of 256 tactile cells so addressed produces a cable of 32 wires resulting in a cable diameter of less than 3 mm.

Raibert (1984) developed a tactile sensor by incorporating VLSI local processing and tapered separating spaces to produce a smart sensor. A representation of the separator cell is shown in Fig. 5. A prototype chip using 48 tactile cells in a 6×8 array was constructed with 15 electrodes per cell to produce 4 bits of pressure output per cell. Using serialized input and output data, the full scale prototype incorporates 200 tactile cells with a 1 mm spacing driven by only five wires: power, ground, clock, data-in, and data-out.

Optical technology has been found to be useful in the construction of tactile sensors. One such application involves the shuttering of a light beam by the contact deformation (Coiffet 1981; Rebman and Trull

1983). The sensitivity and dynamic range of such a sensor can be changed without altering the optical transduction. Commercial systems of this type are available (Lord Corp. 1984) and have been used in tactile image research (Muthukrishnan et al. 1987).

Another optical tactile sensor has been proposed by Begej (1984). Here, a tactile image is generated using the frustration of total internal reflection. The effect is illustrated by touching the sides of a glass of water while looking down into the glass. The firmer the grasp on the glass, the more prominent the image of the contacts on the otherwise silvery glass surface. A schematic of Begej's transducer is presented in Fig. 6. Pressure applied to the transduction membrane causes more of the textured surface to contact the transparent medium and creates an intensity image of the state of compression in the material. Begej's implementation conducts this image away from the contact site by way of optical fibers and then displays it with a video system. Although this approach does not produce information about tangential contact forces, the sensor has been developed into a fingertip-shaped sensor (Begej Corp. 1986).

Capacitive tactile sensors have been developed on the property that capacitance is a function of the thickness of the dielectric material. With this in mind, a sensor can be visualized which produces a capacitive impedance output in response to a contact deformation (Boie 1984). Boie suggested a sensor which is presented in Fig. 7. The concept shows advantages over conductive elastomers in speed and noise immunity. Investigators at MIT (Siegel et al. 1986) have shown promising results, and arrays of high spatial density appear to be possible.

Magnetostrictive materials may similarly be exploited to transduce contact. These materials are characterized by the change in their magnetic fields in

Fig. 5. Raibert's VLSI tapered tactile cell [reprinted by permission from Raibert (1984) © IEEE].

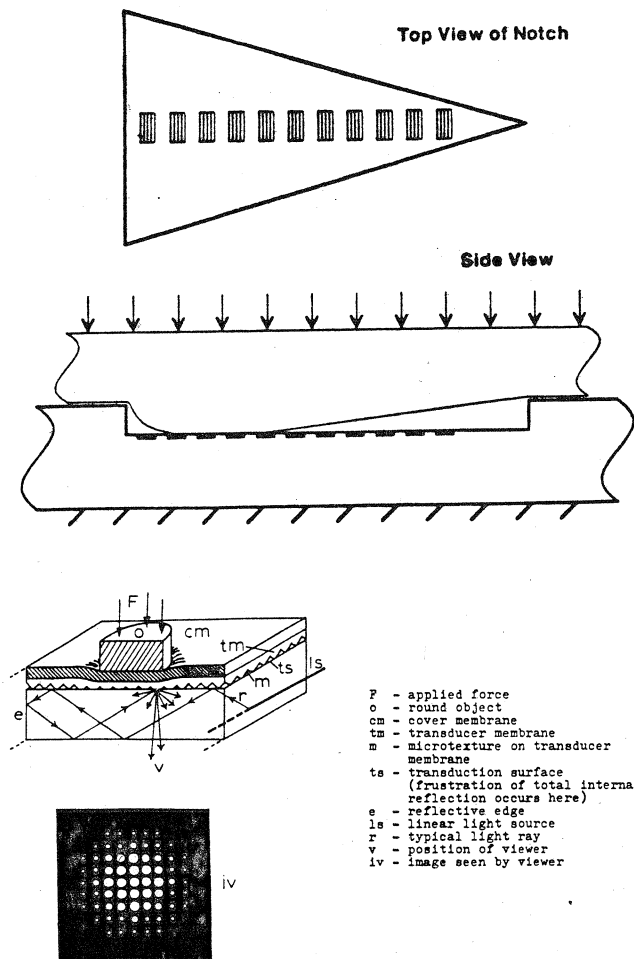


Fig. 6. Begej's optical tactile sensor [reprinted by permission from Begej (1984) © SPIE].

Fig. 7. Boie's capacitive tactile sensor [reprinted by permission from Boie (1984) © IEEE].

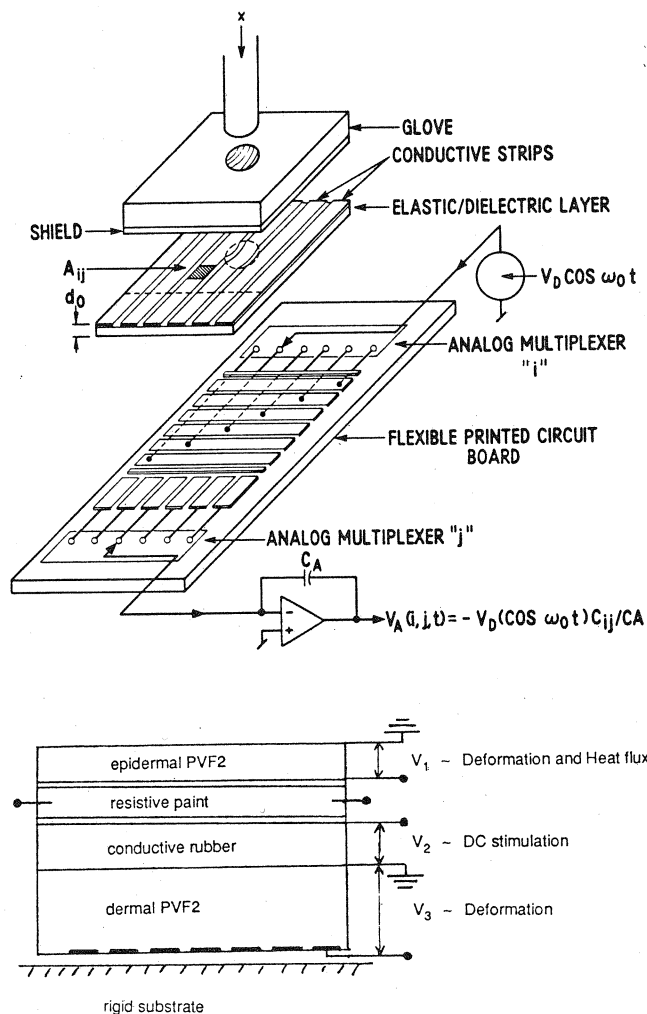


Fig. 8. The Dario et al. artificial skin [adapted from Dario (1984) © IEEE].

response to an applied load. The application of cleverly oriented induction coils may then transduce an applied force as an induced voltage. Luo et al. (1984) and Vranish (1986) have demonstrated touch sensors based on these principles. Luo et al. fabricated an array of 256 tactile cells with a spacing of 2.5 mm that demonstrated good linearity, low hysteresis, good dynamic range, small thermal drift effects, and good sensitivity. The sensors are, however, sensitive to external fields as are the capacitive sensors.

Piezo- and pyroelectric transduction have been shown to be potentially useful in many tactile sensing applications. Piezoelectric materials respond to mechanical deformation by producing output voltage

potentials. Pyroelectric materials respond to heat fluxes by generating induced voltages. A sensor was constructed that utilizes both of these properties in an ambitious attempt to emulate human skin (Bardelli et al. 1983; Dario et al. 1984). The design is interesting in that it addresses many of the components of the signal content mentioned earlier in our discussion of the human tactile system. The system is presented in Fig. 8. The material PVF₂ (polyvinylidene fluoride) possesses both piezo- and pyroelectric characteristics. Figure 8 shows a composite structure which is intended to provide an extremely large bandwidth response as well as to measure the relative thermal con-

ductivity of the object being touched. Three dependent signals are produced by the sensor. The dermal layer of PVF₂ is used to transduce contact forces. The structure responds to frequencies up to 500 Hz and was limited by the resonant frequency of the composite structure; it does not, however, respond well to dc forces. The layer of conductive rubber on top of the dermal PVF₂ is included to add this dc capability. The resistive paint in the structure is used to provide a reference temperature (98.6°F) to the epidermal PVF₂ layer. The signal output by this layer is then a function of both the heat flux and deformation due to a contact with the environment. The conductive rubber beneath the resistive paint effectively insulates the dermal PVF₂ layer so that the signal may be (approximately) separated. In practice the sensor produces signals that are quite cryptic due in part to the difficulty of separating the thermal and deformation induced potentials.

2.2. Machine Perception with Tactile Information

The perceptual capabilities of the tactile system in human beings are familiar to everyone; the extent that these contact stimuli influence our models of the environment; however, has been the object of much discussion. The tactile sense of space is a result of not only tactile stimulus but it involves the integration of tactile, visual, and kinesthetic signals to the brain. The degree to which a sufficient, spatially coherent model of the environment can be constructed in the absence of one of these sensory modes is compromised (Benton 1982). It is clear, however, that humans can model the immediate environment quite well in the absence of visual input and can recognize and discriminate between familiar objects.

The relative contribution of the various sensor signals in the human hand produces a very rich and diverse array of tactile sensation. The *touch blend* (Schiff and Foulke 1982) of temperature, pressure, and vibration signals permits the distinction between wet, slimy, greasy, syrupy, mushy, doughy, gummy, spongy, or dry elements of the environment.

The role of movement, or active exploratory strate-

gies in touch has been acknowledged to be critical to the efficient utilization of tactile information (Allen 1984; Dario et al. 1984; Ellis 1984; Gordon 1978; Harmon 1984a and b; Hillis 1982; Schiff and Foulke 1982). Hardness, texture, compliance, and surface features such as contour, vertexes, and edges are available to the tactile sensor which is capable of scanning its local surroundings. There has been, and continues to be, some interest in treating the analysis of tactile information in a manner analogous to vision processing, where tactile images are processed to yield tactile discriminators. These techniques are useful only for objects of a scale similar to or smaller than the tactile array. The discussion here focuses on the collection of spatially distributed low-level tactile features by active exploratory strategies.

The first step in an active tactile search involves a gentle rapid probing contact with the environment (Gordon 1978). After potentially harmful features are identified, a more deliberate exploration of the target object is begun. The shape of the object is examined by determining the spatial distribution of surfaces, corners, edges, and holes. The object may be grasped and moved, providing perhaps relative weight, center of mass, hardness, and thermal conductivity parameters. More geometrical information can be ascertained by excursions of the fingertips with variable contact pressures to determine surface details such as texture, size, shape, and notable areas of curvature (Ellis 1984; Stansfield 1986).

Ellis (1984) cites a simple application of active tactile exploration involving pressure modulation. Here, it was noted that estimates of edge radii and object compliance can be obtained by modulating the pressure applied to the object's surface. Ellis (1986) also describes the use of passive tactile information to discern the texture of various surfaces. This approach requires high spatial resolution, special packaging, and cabling, whereas an active texture analysis requires high bandwidth.

Hillis (1982) describes the difference between an analytic or bottom-up approach, such as that employed by vision algorithms, and the top-down or knowledge driven approach which seems more appropriate in tactile recognition systems. Vision systems acquire an image and then process this data. The technology which supports vision demands this type of an

approach. Tactile systems, however, acquire small amounts of data at a time and are more appropriately used in an active, knowledge-driven manner. The active approach interacts well with an evolving representation of the environment and is an efficient means of identifying objects given a model base.

Schneider (1986) demonstrates the active approach to recognition and localization in a 2-D, planar domain using an implementation of the interpretation trees proposed by Grimson (1986a,b) and Grimson and Lozano-Pérez (1986). The procedure involves a representation of the object which consists of tables of constraints for distances, normals, and directions between faces. Models of the object(s) are used to create a tree of all consistent interpretations of the data already collected. The resulting interpretation tree is pruned by generating tactile sensor trajectories which most efficiently discriminate between the competing interpretations.

Bolles and Cain (1982) and Bolles and Horaud (1986) and recent work by Hansen and Henderson (1987) demonstrate systems which hypothesize and subsequently verify an object's orientation by comparing the topology of local sensed features to an extended CAD model. Such methods permit the system to acquire additional features of the object in order to discriminate among competing hypotheses. The promising results of such systems, along with the development of systems which optimally combine information from different sensory modalities (Bolles and Cooper 1986), suggests that the dynamic world models required for manipulation are not unrealistic.

It was noted earlier that the tactile model or sense of the environment in human beings is a function not only of tactile and kinesthetic information, but is also integrated into a spatial representation involving visual information. Allen (1984; 1986) makes use of this anthropomorphic example by designing a recognition/localization system which combines a passive vision system with an active tactile system to produce surface and feature representations of objects. The system he describes consists of a stereo pair of CCD cameras and a tactile finger. The tactile sensor contains 133 conductive rubber tactile patches distributed over a structure roughly the size and shape of the human index finger but with no articulation. The finger is positioned

within the workspace by a six degree-of-freedom PUMA robot arm.

The vision system uses a stereo pair of cameras to grow a sparse set of 3-D points located on the boundary contours of closed regions in the image. Rather than push the computationally intensive vision system, the tactile system is employed at this point to ascertain the object's surface character.

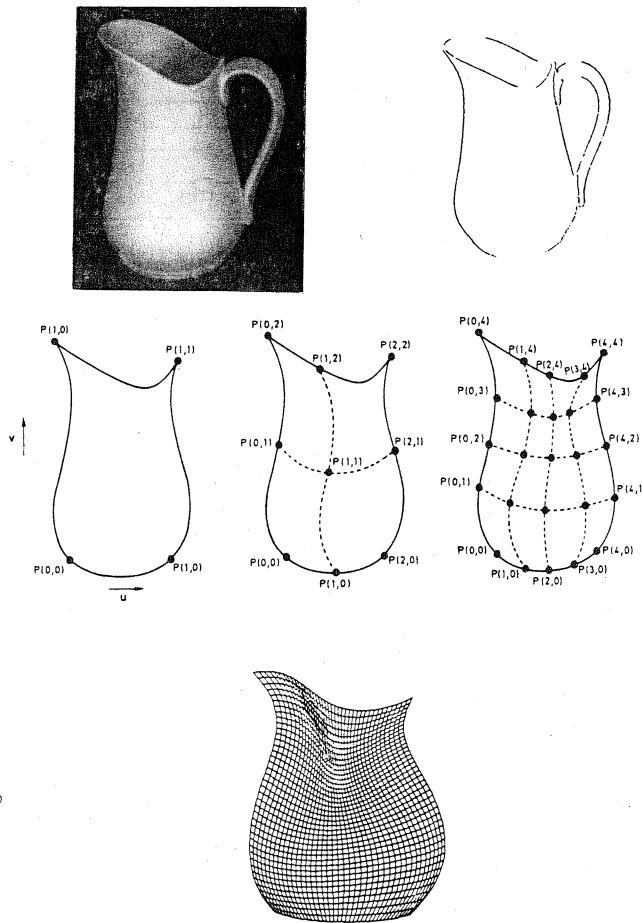
The tactile system determines from the visual image which areas of the object merit further attention. When a region is identified, the controller initiates an exploratory procedure. The inputs to this process are a least squares plane and normal derived from the 3-D boundary contours provided by the vision system. The intermediate level of the controller then determines an appropriate approach vector and orientation for the tactile sensor. Subsequent contact with the object fills in additional surface information or instigates the tracing of another boundary contour if a cavity or hole is encountered. The discrete 3-D data is combined into a surface description using a bicubic interpolation creating a composite surface which is curvature continuous (C^2 continuity). Figure 9 is a sequence reprinted from Allen (1986), representing the evolution of a surface description for a pitcher. The final figure in the sequence is the approximate composite surface resulting from four surface patches (it is derived from the center figure above it). This approach allows task constraints to further refine the surface only in the regions of interest. Such approaches provide a framework for the efficient, knowledge-driven perception of the environment.

3. Technologies for Mechanical Manipulators

Studies indicate that many industrial processes could be entirely supported by mechanical manipulators with two or three fingers equipped with tactile sensors. In fact, there are estimates that 80% of industrial assembly tasks could be accomplished using these relatively simple configurations (Harmon 1984).

The workhorse of the mechanical end-effectors is the parallel jaw gripper. Given an appropriate set of objects these grippers are capable of manipulation and

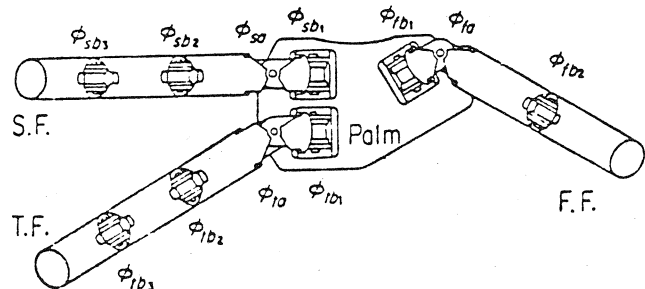
Fig. 9. Allen's passive vision/active tactile recognition procedure [reprinted by permission from Allen (1986) © IEEE].



may be used to support object recognition. Given the flat geometry of the gripper, it is natural to apply conventional image processing techniques to extract tactile features such as vertexes or line contacts.

It has been suggested that three fingers may be necessary to reproduce the predominant grasps of the human hand (Harmon 1984a and b); others point out that four fingers allow a grasping redundancy and, therefore, exchange between fingers. Why human beings possess five fingers is matter of much conjecture. The pectoral fins of fish and the forelimbs of many lower animals contain five "finger" bones, which implies that there is not necessarily a logical connection between grasping and the number five. The control system for the human manipulation machinery does, however, make use of this redundant finger. For example, a finger not involved in a grasp might per-

Fig. 10. Structure of the Okada manipulator [reprinted by permission from Okada (1979) © IEEE].



form an active search of the object or the environment; assembly compliance can be addressed by actively sensing the relative orientation of the part and the subassembly with which it is to be mated.

3.1. Representative Manipulators

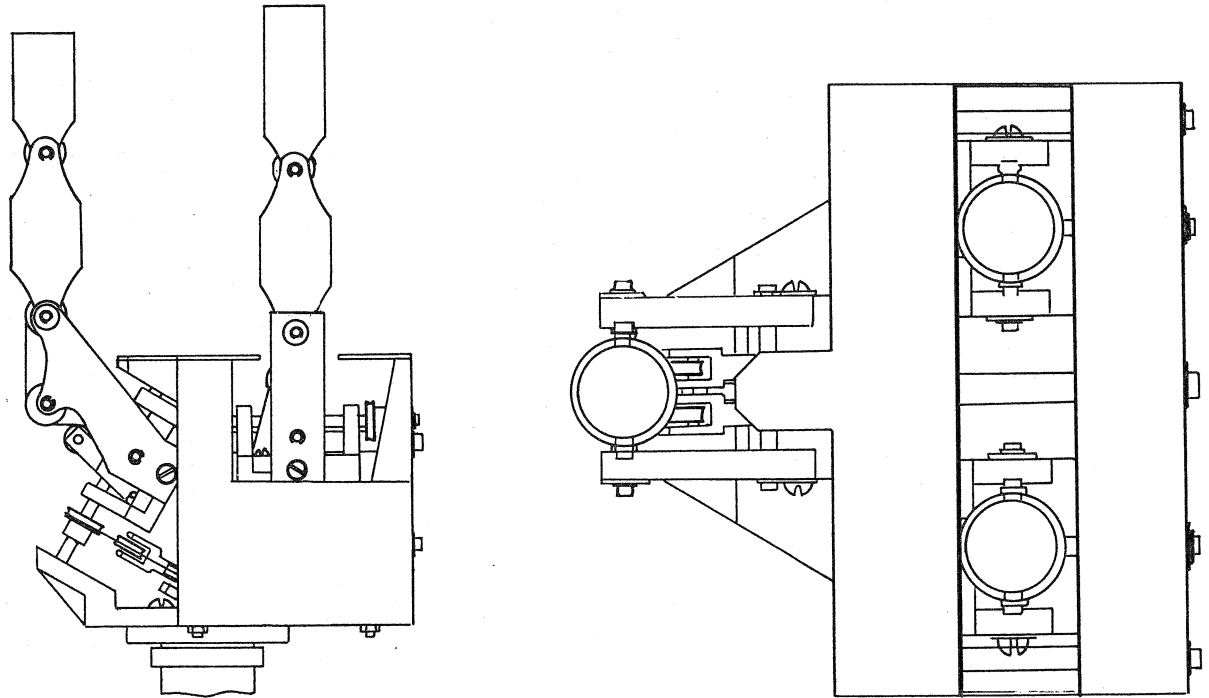
This section presents representative manipulators that have been developed for use in research. For additional references see Hollerbach (1987) and Cutkosky (1985).

The Belgrade hand (Tomovic et al. 1987) is being developed in Yugoslavia. This hand is designed to support a reflex control paradigm wherein a functional configuration for the hand is selected and used to control the corresponding *grasp mode* in the hand. Reflex control is discussed further in Section 3.2. The hand has five fingers which are used in one of two modes: a three-finger mode or a five-finger mode. There are only two drive motors, which control multiple fingers and permit the hand to be inherently shape adaptive. This hardware abstraction of the human hand should prove to be easily controllable and useful over a subset of the tasks involved in general-purpose manipulation.

Okada (1979; 1982) developed an industrial object-handling system which consisted of a 5-degree-of-freedom arm/wrist coupled with a three-fingered, 11-degree-of-freedom hand. The hand itself is a rough model of the human hand, addressing such grasping primitives as wrapping, pinching, picking, gripping, and searching. Each finger has four degrees of freedom, and the thumb has three degrees of freedom, as shown in Fig. 10. The arm was included to produce a large

Fig. 11. The Lian et al. manipulator [reprinted courtesy of the Society of Manufacturing Engineers. Copyright © 1983, from the

Robots 7/13th ISIR Proceedings (Lian, Peterson, and Donath 1983)].



workspace for the manipulator and to improve the possible pregrasp orientations. To minimize the weight of the hand and increase its maximum payload, the motors used to drive the hand are located in the trunk of the device and transmitted through wire-sheathed cables approximately 1.7 m long. As a result, there are significant frictional and elastic effects. The finger control consists of a hybrid position/torque servo implemented in hardware. The control scheme is selected by observing the movement of the finger in response to a command input. The transition between position and torque control is smoothed somewhat by varying the command voltage so as to maintain the error signal (the difference between the command and the feedback) at a constant value. The hand has demonstrated the ability to stably grasp simple objects by defining one side of the opposing grip as position control and the other as force control. It is reported to have the ability to successfully grasp and hold objects up to 500 g in weight.

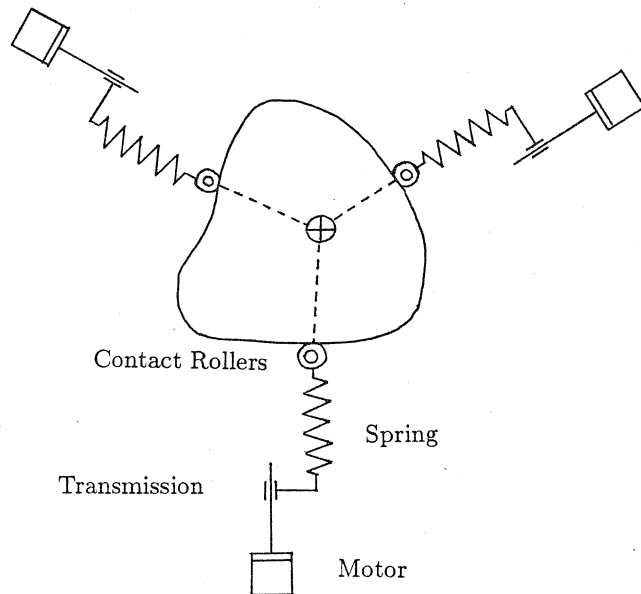
Lian et al. (1983) developed an anthropomorphically motivated hand which was intended to reproduce several important human grasps, among which are

- a. Tip opposition—tip of index finger to tip of thumb
- b. Lateral opposition—side of index finger to tip of thumb
- c. Palmar prehension—fingers and thumb wrap around a cylindrical shape
- d. Spherical prehension—fingers and thumb wrap around a spherical shape
- e. Digitopalmar opposition—fingers and palm form an opposition pair

The design of the manipulator involved the kinematic modeling of the mechanism and the graphical simulation of the above grasps. The resulting three-fingered design is a model of the human thumb, index, and middle fingers. Each finger has three degrees of freedom. The proximal joint of the fingers (the metacarpal-phalangeal joint) operates only in adduction-abduction about an axis perpendicular to the axis of the two remaining interphalangeal joints. The thumb is modeled similarly except that the proximal joint is inclined at 60° to the palm, while the fingers are perpendicular to the palm. A schematic of this configuration appears in Fig. 11. The fingertips of the manipu-

Fig. 12. Schematic of the Hanafusa et al. manipulator [reprinted courtesy of the Society of Manufacturing

Engineers. Copyright © 1977, from the Robots 7th ISIR Proceedings (Hanafusa and Asada 1977)].



lators are fitted with replaceable hemispherical friction tips which approximate point friction contacts. The fingers are actuated using dc motors driving stainless steel cables inside of wirewound sheaths. This means of power transmission introduces considerable elasticity and nonlinear friction. The design was not reported to incorporate tactile sensing, and thus requires that grasping forces be determined by measuring cable tensions and joint positions. This limits the grasping options to categories (a) and (b). This type of grasping force transduction leads to ambiguous results if the object contacts the finger in more than one position.

In contrast to the anthropomorphic approach to manipulator design, Hanafusa and Asada (1977a and b) developed a planar, three-fingered hand capable of implementing an analytical grasping algorithm. The resulting design is not particularly dexterous, but is interesting because it introduces the concept of grasp stability. A schematic of the hand gripping an object is presented in Fig. 12. Each single-degree-of-freedom finger is positioned 120° from the adjacent fingers and is actuated by a step motor through a coil spring. The finger force is measured by a potentiometer inside the coil spring. The displacement of the fingertip is measured by a second potentiometer. This arrangement allows the control of the normal gripping force of each

Table 2. Salisbury's Contact Types

Remaining DOF	Type of contact
0	Glue, planar contact with friction
1	Line contact with friction
2	Soft finger
3	Point contact with friction, plane contact without friction
4	Line contact without friction
5	Point contact without friction

Adapted from Salisbury (1982).

finger. The contact point is on a roller to eliminate any tangential components of the grip. The potential function consists of the sum over all fingers of the product of the finger force and the differential movement integrated over the path from the initial state to the stable prehension state plus the gravitational potential. The potential is also the elastic strain energy of the finger's coil spring. When the prehension state energy reaches a local minimum, it is stable; that is, an external force which displaces the object from its command position can be overcome by the elastic field produced by the fingers. Since the deflection of the tip of the finger is measured, a control system can modulate the apparent stiffness of the manipulator by driving the finger motors synchronously with fingertip movements. In practice, the hand requires the integration of a vision system capable of extracting the object silhouette (Boissonnant 1982) and positioning the hand close to a local minimum. The hand's dexterity is limited; however, it has demonstrated an ability to perform many industrial manipulation tasks.

The Stanford/JPL hand was motivated by anthropomorphic considerations and kinematic and control issues (Mason and Salisbury 1985; Salisbury 1982; Salisbury and Craig 1982). The parameters used to evaluate the effectiveness of the manipulator were the number of fingers, the number of links per finger, and the type of contact between finger and object. Contact types considered included those presented in Table 2. The analysis involves the classification of proposed finger combinations based on their ability to

1. Exert arbitrary forces or impress arbitrarily small motions

Fig. 13. The Stanford/JPL manipulator [adapted from Salisbury (1982)].

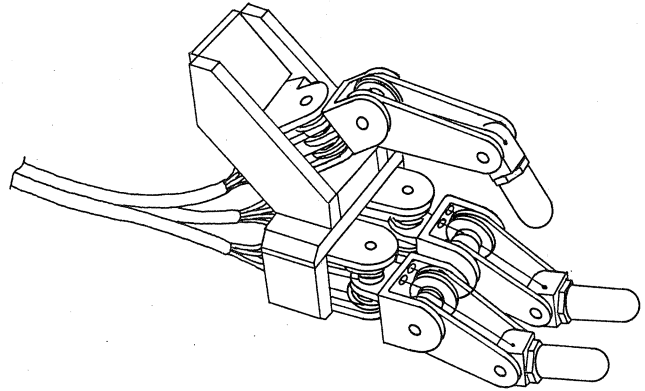
2. Entirely constrain the object by fixing all the active joints

Salisbury evaluates a proposed mechanism by employing a number synthesis (Paul 1979) to speak about the mobility and connectivity of the hand/object system.

Mobility is the number of independent parameters needed to completely specify the state or configuration of the mechanism. Connectivity is the number of independent parameters needed to specify the relative position of two bodies. In this case, the bodies considered are the palm and an object. Connectivity is determined by fixing the two bodies and computing the difference between the system mobility and the mobility of all subchains connecting them. In order for the hand to impart arbitrary motions and forces, the connectivity of the active mechanism must be six (for general 3-D motion). For the hand to completely constrain objects, the connectivity of the fixed mechanism must be less than or equal to zero.

Salisbury considered 600 candidates with one, two, or three fingers with three links per finger, where all the contact types for a given hand are the same. More complex hands (with additional links or fingers) were not considered, and it was noted that several acceptable designs were thus dismissed. Immediately discarded were designs that required five degrees of freedom per contact, since this corresponds to a frictionless point contact and severely limits the usefulness of the resulting manipulation system. By these criteria, the best resulting design was one with three fingers, three links per finger, and three-degree-of-freedom contacts. This type of contact is achieved by point contacts with friction or planar contacts without friction.

The final design was based on a number of analytical and intuitive procedures. A simulation of the hand was used to optimize over a user-selected parameter space. Among these are the relative finger locations, link lengths, and joint range of motion. The model computed a performance index based on three-fingered grasps using fingertip prehension. While power grasps were not explicitly evaluated, the final design was imbued with features thought to be desirable in this respect, among which are relative finger placement and palm area. In addition, the relative orientation and placement of the fingers was influenced by the lo-

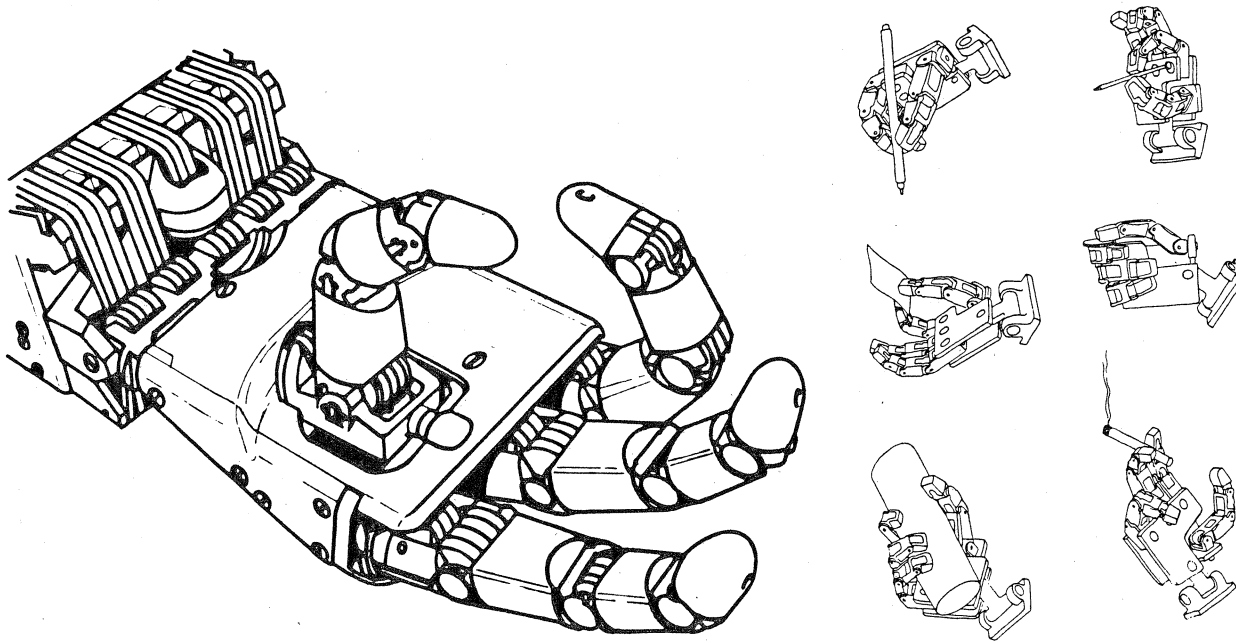


cation of so-called isotropic points in each finger's workspace and superimposing these loci in an area over the palm. Isotropic points are locations in the workspace that minimize the condition number of the Jacobian matrix and thus minimize the error propagation from input torques to output forces. The hand design superimposes the loci of the isotropic points of the three fingers in a suitable area over the palm. The resulting hand design is presented in Fig. 13. The Stanford/JPL hand is actuated by a tendon scheme using cables with tension sensors. Four antagonistic tendons control the three joints in each finger. Teflon-coated tendons are routed through conduits and are driven by 12 motors and gear trains mounted in the forearm. A servo loop senses the cable tension near the fingertip, motor position and velocity, and estimates the joint angles and velocities. The effects of friction and elasticity can be mitigated by sensing cable tensions close to the point of application. The hand has been fitted with hemispherical force transducing fingertips which allow the controller to modulate contact force interaction and accurately measure the position of the contact.

The Utah/MIT dexterous hand (Jacobsen et al. 1983; 1984; 1986) is perhaps the most ambitious effort towards a truly anthropomorphic manipulator. The hand has three four-degree-of-freedom fingers and one four-degree-of-freedom thumb, but deviates from human phalange lengths and joint positions in order to route the tendons.

One of the distinguishing features of the design paradigm of this hand is that, from the outset, it was intended to be a model of the human hand. The reasons

Fig. 14. The Utah/MIT dexterous hand [reprinted by permission from Jacobsen et al. (1984) © MIT Press].



for this approach are the following:

1. The human hand is an existence proof that a hand with this geometry, given a suitable control scheme, is a versatile manipulation device.
2. Research into the nature of grasping and manipulation with such a mechanism would permit the researcher to correlate the performance of the mechanical hand with its human counterpart.
3. Such a design should be more easily adapted to teleoperation.

The hand is actuated by extremely fast pneumatic cylinders through polymeric tendons. The resulting pneumatic actuators are fast, low friction, and can generate relatively high forces. They incorporate a pressure control valve so as to minimize the effects of compressibility in the working fluid. The resulting system acts as a mechanical force source with no spring constant and very little mass and damping.

The 16-degree-of-freedom hand is actuated antagonistically using a 2N tendon approach, which requires a system of 32 independent tendons and actuators. The hand is actuated remotely through the tendons which reduces the payload weight of the hand, and makes space for peripheral sensor systems.

The resulting actuation system is actually faster

than the human hand, while providing about the same forces. Since each joint is actuated by two pneumatic actuators in an antagonistic arrangement, each joint's stiffness is controllable. In addition to these human-like mechanical qualities, the hand was also imbued with certain anthropomorphic reflex qualities that will hopefully increase its effectiveness as a manipulation device. The stiffness or the configuration of the hand is modified based on the interaction of the hand with the environment. Specifically, the hand implements two reflex motions which are observed in the human hand:

1. *Proximal stiffening*: a contact with the environment causes the proximal joints to stiffen.
2. *Distal curling*: a contact causes all distal joints to curl about the already established contact point.

The Utah/MIT dexterous hand and some of the anthropomorphic grasps that motivated its development are presented in Fig. 14.

3.2. The Development of Grasping Control Strategies

We will begin by describing briefly the historical background of robotic control. For an extensive review of

the history of force control, see Whitney (1987). Perhaps the first gripper operated by a computer under a feedback control strategy was demonstrated by Ernst (1961). It was capable of performing simple manipulation tasks using tactile feedback to verify the presence or absence of an object. However, position control alone proved unsatisfactory in the vast majority of assembly operations.

Whitney (1977) implemented an admittance matrix model to predict the manipulator velocity in response to external forces. The remote center of compliance (RCC) was suggested to facilitate assembly operations such as peg-in-hole insertions (Drake 1977). Force, position, and velocity sensors were used to make small corrections in the trajectories of contacting parts. The system permitted sensed contact forces and geometrical information about the task to drive small relative reorientations of mating parts.

Similar efforts to imbue the manipulator with a compliant character have defined orthogonal subspaces within which either position or force may be controlled. Paul and Shimeno (1976) implemented this strategy by selecting the manipulator joints that are most closely aligned with a command force vector and imposing the appropriate input force with those joints. The joints not involved with the force command were position-controlled. Raibert and Craig (1981) combined force and position trajectory constraints by specifying a position-controlled space and a complementary force-controlled space. The components of the error signals in both force and position which map into a joint's workspace contribute to the feedback control of that joint. Mason (1981) used the notion of natural and artificial constraints. Tasks are modeled by a C-surface defined in the configuration space of the manipulator. Position control is defined along C-surface tangents, while force control is possible along C-surface normals. Lozano-Pérez (1981) suggested a means of manipulation strategy synthesis using this task description.

Salisbury (1982) suggested that task-defined stiffnesses be established (Mason and Salisbury 1985). Task constraints are established using the position and force measurements so that system stiffnesses can be adjusted accordingly. The environment is actively sampled along six orthogonal axes to measure contact forces resulting from motion. Knowledge of the nomi-

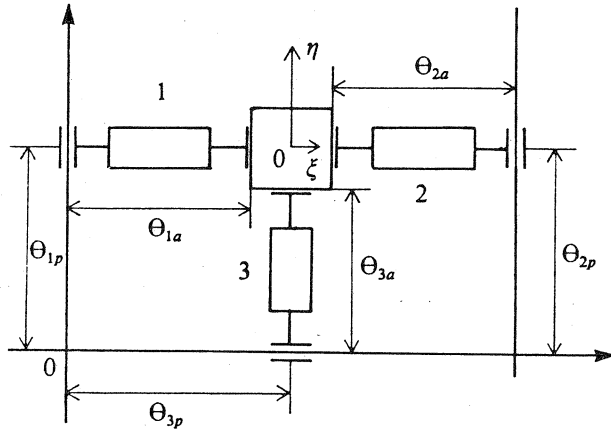
nal control stiffness and the contact force are used to evaluate the environmental stiffness. His analysis included the grip Jacobian which expressed the transformation from a set of contact forces or velocities to a net object frame force or velocity (Mason and Salisbury 1985). These results were expanded upon by Kerr and Roth (1986) when the selection of internal forces for the application of external forces was accomplished using linear programming techniques. An approach similar to this was described by Gruben et al. (1988), which computes the minimal internal wrenches required to generate an external wrench.

Geschke (1983) demonstrated a unique implementation of the position/force controller in the form of his *robot servo system* or RSS. A single instruction in RSS initiates a servo process which actively seeks its goal until either canceled or redefined. Compliant subspaces may be defined which change continually as the task geometry changes.

To complement the work on force synthesis for grasping, researchers have developed position synthesis algorithms. Jameson and Leifer (1987) described a grasp goal function which is maximized by varying the location of the hand and the position of the contacts. The result is a prediction of the most stable grasp. Gruben and Henderson (1988) developed a means of planning the geometry of the hand/object system which is suited to the task. The task is expressed as the sum of stability and external force components. The positions of the contact sites are defined which address the task and satisfy the constraints imposed by the object and the hand.

Bekey and Tomovic (1986) suggested a markedly different approach to robot control, termed *reflex control*. This nonnumeric approach to control hypothesizes that control in humans is based on sensory data and uses stored patterns of response. Control then consists of properly interpreting the pattern of input data and then defining a response which is kinematically *natural* for the manipulator. The paradigm has been applied to grasping (Tomovic et al. 1987) and has motivated the development of the Belgrade hand. The method of control seems to be limited, however, to that class of manipulation tasks for which humans employ reflexive control: shape adaptation, contact slip recovery, and damage avoidance.

Fig. 15. A demonstration of stable grasp analysis [reprinted by permission from Kobayashi (1985) © MIT Press].



3.3. The Stable Grasp

An interesting approach to the problem of stability in the hand/object system is the enumeration of all the mechanical impedances imposed on the object by virtue of its contact with each finger. To introduce the concept, consider the following problem posed by Kobayashi (1985; 1986) and illustrated in Fig. 15. Each finger in this 2-D gripper is active in one direction and passive (compliant) in the other. In order to maintain a stable grasp while the actuators move the object, Kobayashi developed the idea of the manipulation and free subspaces:

S_m —Space in which all fingers remain in contact with the object and may move without violating the mode of contact of another finger

S_f —Space or degrees of freedom remaining for the object when all actuators are fixed.

These spaces are constructed by intersecting the spaces associated with each finger. The grasp is entirely constrained if the rank of the S_f space is zero and may be manipulated in the space defined by S_m . These spaces can be directly deduced for the simple geometry presented; note that the object coordinates are related to the finger coordinates as follows:

$$\text{Finger 1} \quad \begin{Bmatrix} \partial \xi \\ \partial \eta \end{Bmatrix} = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \partial \theta_{1a} + \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \partial \theta_{1p}. \quad (1)$$

$$\text{Finger 2} \quad \begin{Bmatrix} \partial \xi \\ \partial \eta \end{Bmatrix} = \begin{Bmatrix} -1 \\ 0 \end{Bmatrix} \partial \theta_{2a} + \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \partial \theta_{2p}. \quad (2)$$

$$\text{Finger 3} \quad \begin{Bmatrix} \partial \xi \\ \partial \eta \end{Bmatrix} = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \partial \theta_{3a} + \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \partial \theta_{3p}. \quad (3)$$

The free space depends only on the passive degrees of freedom, and it is possible to examine finger combinations to find a stable grasp. With the first two fingers,

$$S_f = \cap S_{fi} = R \left[\begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \cap \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \right] = R \left[\begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \right]. \quad (4)$$

Therefore, fingers 1 and 2 allow unconstrained movement in one direction (the y direction). Similarly, for three fingers:

$$S_f = \cap S_{fi} = R \left[\begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \cap \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \cap \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \right] = \{\emptyset\}. \quad (5)$$

The above equation suggests that when three fingers remain in contact with the object, the object is fully constrained. Furthermore,

$$S_m = \cap S_{mi} = R^2. \quad (6)$$

This analysis extends directly into three dimensions; the objective then is a free space with rank zero and a manipulation space of rank 6. An extension of the concepts presented here allowed Kobayashi to define grasping conditions which permit the manipulation of the object using only the active degrees of freedom of the manipulator (i.e., fixed wrist and arm).

The constrained space approach is used by others to quantify stability (Hanafusa and Asada 1977a and b; Salisbury 1982) and to predict and control instability (Fearing 1983a and b; 1986). The stability criteria can be decomposed into three requirements:

1. The object is in static equilibrium, there is no net force or moment.
2. All forces are applied within the cone of friction so that there is no slippage.
3. An externally applied force can be resisted by finger forces with a finite and controllable deflection.

To satisfy the no-slippage condition, each contact must transmit forces within its cone of friction. An initial grab which cannot satisfy this constraint will cause the object to slide or roll relative to the fingers until the grasp fails or is successful. During this process, either the object or the hand must comply. An object priority grasp produces a stable grasp without disturbing the object. To accomplish this grasp the contact must consist of low magnitude forces and high compliances.

The final stability criterion requires that the grasp can resist an externally applied displacement force. The analysis involves superimposing the grip force and an externally applied displacement force. The criterion is satisfied if the grip can satisfy static equilibrium conditions without violating the cone of friction of either finger. The deflection of the hand/object in response to this disturbance can be regulated by controlling the stiffness of the fingers.

This analysis is directly extendable to a three-fingered (or more) hand and suggests an approach to manipulation (Fearing 1983a; 1986). The baton-twirling problem can be viewed as a succession of stable grasps. For example, two fingers in a stable grasp can be perturbed by a third, producing a controllable deflection (in this case a rotation). One of the fingers involved in the original grasp is now released and repositioned in order to perturb the grasp produced by the remaining fingers. The result is either the twirling of a baton, as in the hand priority grasp, or the twirling of a mechanical gripper in the case of the object priority grasp.

Nguyen (1986) develops the concept of a stable grasp by analyzing the simplified world of planar polygons constrained by multiple-point contacts without friction. The fingers are modeled as virtual springs with controllable stiffnesses. The stability criterion employed is based on the system potential energy: an equilibrium position is one for which the potential energy of the system is minimized, and is stable if the energy of the perturbed system increases. These properties are embodied quite elegantly in the scalar potential function describing the strain energy of the virtual springs. The simplicity of this formulation has been noted by several investigators (Hogan 1985; Liapunov 1966; Lyons 1986; Nguyen 1986). Nguyen's approach locates local minima by identifying positions where

the gradient of the potential function vanishes and employs the Hessian matrix to describe the behavior of the potential in the vicinity of this minimum.

The issue of dynamic stability has been approached in this manner by Hemami and Chen (1984) when simulating the two-link planar biped system influenced by holonomic (connection) constraints. The Liapunov stability of the system is considered to suggest methods of controlling the dynamic stability. Liapunov stability requires that the potential function describing the system be positive definite while the time derivative of the potential function must be negative semidefinite. Hemami suggests the form of a Liapunov function which is positive definite (for suitable control inputs) while the derivative of this function is negative semidefinite. Therefore, the system can be stabilized provided that it is near a unique equilibrium operating point.

4. Toward a Manipulation Paradigm

The evolution of a manipulation strategy can be described conceptually by rules that depend on the conditioning of the hand/object system. A well-conditioned system is stable (redundantly), is manipulable, and projects well onto the task. System velocities, however, cause the conditioning of the system to degrade as it seeks the goal. If the state of the system approaches the envelope specified in the task, then the planner must improve the state. The prioritized rules are enumerated below.

1. *Geometry synthesis*—The system must be capable of directing incremental improvements in a hand/object interaction that no longer addresses the task. This might occur, for instance, if an object movement requires the finger to go outside its workspace. If a contact system has been established, stability of the object must be maintained using a subset of the contact system while individual redundant contact elements are moved to improve the system state. This rule must always result in a

hand/object configuration which is well conditioned for the task.

2. *Force or velocity synthesis*—When the hand/object geometry is properly conditioned for the task, the system is prepared to accept force and velocity commands pursuant to the task command. If an object velocity results, then the system will eventually violate the conditioning envelope and cause rule 1 to fire.

If we assume that the manipulation process can be described by a vector space in which object stability and manipulability constraints may be expressed, then the manipulation process may be viewed as the migration of a state vector within this vector space. The task requirements serve to delimit a volume of this space within which a solution must be found. The final state reduces the error between the task specification and the system state to zero and optimizes locally for stability and manipulability. If no solution is eventually produced, then the task specification of the goal is incompatible with the stability and manipulability requirements and the task must be relaxed if a solution is to be found.

This section discusses issues surrounding the directed movement of general-purpose manipulators toward a goal state while the object remains stably constrained and the manipulator remains adequately conditioned. Primary in the discussion is the definition of the task; it must allow the expression of a goal state for each of these objectives. In this section, we express the task as a vector in the six-dimensional wrench space and examine the implications of such an approach. The discussion then focuses on the definition of constraints and their role in manipulation.

4.1. The Task and the Grasp State Error

Screw systems are a natural means of expressing the six degrees of freedom of an object (Salisbury 1982). We present the stability and external force components of a task as the sum of two wrench (generalized force) commands.

$$\bar{T} = [\Omega_{R1} \quad \Omega_{R2} \quad \Omega_{R3} \quad \Omega_{R4} \quad \Omega_{R5} \quad \Omega_{R6}] \\ + [\Omega_{T1} \quad \Omega_{T2} \quad \Omega_{T3} \quad \Omega_{T4} \quad \Omega_{T5} \quad \Omega_{T6}] \quad (7)$$

where Ω_{Ri} is a magnitude goal for forces in the i th degree of freedom, and Ω_{Ti} are terms used to anticipate the task, inertial or external loads. These task descriptions represent the force magnitude goals for the contact system in each of the object's six degrees of freedom. The system is, therefore, not permitted to be stable only with respect to infinitesimal perturbations, but is compelled to seek contact topologies which are predictably robust with respect to expected perturbations.

A system is quantified by computing the error of the current state with respect to the task. If we represent the wrench space spanned by a contact system using a set of n orthonormal six-dimensional basis vectors, then we may express the error of this system relative to the task as follows:

$$E = T - \sum_1^n \text{mag}_i \times \hat{b}_i \quad (8)$$

where

E = contact system error relative to the task

T = task vector

\hat{b}_i = an orthonormal basis vector for the contact wrench space

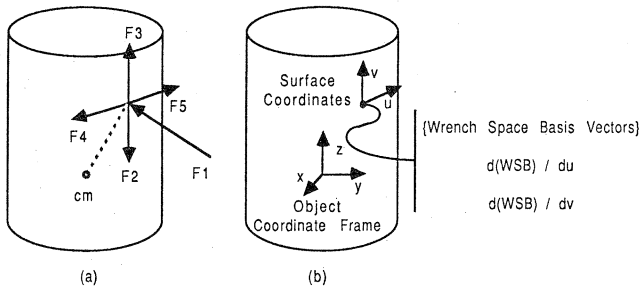
mag_i = min(the contact system capability along \hat{b}_i , the projection of T onto \hat{b}_i)

This procedure removes the components of the task which project onto the contact wrench space and are within the magnitude limitations of the contact system. The residual vector is, therefore, the deficiency of the current contact system relative to the task. This procedure can be used to define an error vector for a contact system relative to a task.

4.2. Properties of the Object

Given a set of positions on the object's surface through which interactions may take place, we attempt to produce an incremental improvement in the system state

Fig. 16. An object model representing a point contact with friction [reprinted by permission from Grupen (1988) © IEEE].



by defining migrations of these positions over the object's surface.

An object model is useful in this process to describe a family of wrenches that may be transmitted through a surface element. The construction of such a model requires the identification of the contact type and the friction coefficients. Figure 16A depicts a set of contact forces that may be transmitted to the object by a point contact with friction. We model the force system using a unit normal force and scaled (by the static coefficient of friction) tangential forces.

The proximity of the contact to the object's center of mass allows the force system to be replaced by a set of wrench space basis vectors. The wrench space described by the force system in Fig. 16A is spanned by the wrenches produced by forces F_1 through F_5 . This data abstraction includes local surface properties (surface normal), contact friction properties and global object properties (surface position relative to the center of mass) in a six-dimensional subspace representing the degree of constraint due to this contact from the perspective of the object. The object model proposed to support manipulation consists of an orthonormal basis for this wrench space and the net change in the contact wrench with respect to changes in the surface coordinates u and v as illustrated in Fig. 16B.

Unions over sets of contact wrench systems span a subspace of the six degrees of freedom of the object. We require as our criterion of stability that the constraint span all six degrees of freedom. That is, a basis for the union of the wrench subspaces over all contacts must be of rank six. This condition ensures that infinitesimal perturbations of the object may be controllably constrained in six degrees of freedom; however, a robust grasp should meet magnitude thresholds in each degree of freedom. If we describe the entire

contact system by a wrench space vector, then we may compute an error with respect to a task as in the previous section.

In order to reduce the object state error, the planner interrogates the object model to determine the value of the derivative wrench systems with respect to orthogonal surface migrations. If the error is expressed as a linear combination of the derivative wrenches, a migration of the interaction sites in surface coordinates can be defined which incrementally improves the state of the system.

4.3. Properties of the Manipulator Jacobian

Several metrics designed to characterize the conditioning of a manipulator have been suggested (Klein and Blaho 1987; Li and Sastry 1987; Salisbury 1982). The inverse kinematic models are intended to suggest configurations of the finger which condition it effectively for the generation of grasp forces and allow sufficient workspace to comply to the geometry of the object.

To characterize the conditioning of each finger within its workspace, consider a system of linear equations of the form (Golub and Van Loan 1983)

$$Ax = b, \quad (9)$$

where

$$\begin{aligned} A &= N \times M \text{ transform} \\ x &= M \times 1 \text{ output space vector} \\ b &= N \times 1 \text{ input space vector} \end{aligned}$$

The input and output space nomenclature is used here to describe the (input) joint angle space of the manipulator and the (output) Cartesian forces or velocities. For the case of the four DOF finger, the input space is the four DOF joint space ($N = 4$), and the output space is Cartesian three DOF space ($M = 3$). The manipulator kinematics then produce a relationship of the form

$$V = J\omega, \quad (10)$$

where

J = the manipulator Jacobian ($\in R^{3 \times 4}$)
 ω = joint angle rates ($\in R^4$)
 V = Cartesian fingertip velocity ($\in R^3$).

Salisbury (1982) suggested that it is possible to design manipulators such that the anticipated workspace is well conditioned. The criterion for workspace conditioning was based on the condition number of Eq. (10). If we perturb the system by disturbing slightly the input and output of the transformation defined by A , we may derive the following:

$$A(x + \delta x) = (b + \delta b),$$

so that

$$\delta x = A^{-1} \delta b \quad \text{and} \quad \|\delta x\| \leq \|A^{-1}\| \|\delta b\|.$$

Similarly,

$$\|b\| = \|Ax\| = \|A\| \|x\|.$$

Therefore,

$$\frac{\|\delta x\|}{\|x\|} \leq \kappa(A) \frac{\|\delta b\|}{\|b\|}$$

where

$$\kappa(A) = \|A\| \|A^{-1}\| = \text{the condition number of } A. \quad (11)$$

If the condition number is large, it effectively amplifies small relative perturbations in b , resulting in large relative perturbations in x . In light of the manipulability ellipsoid discussed below, small condition numbers ensure that the elliptical envelope is relatively large and uniform (isotropic). This feature of the manipulability ellipsoid can be used to define trajectories which are well conditioned.

The quantification of the manipulator in terms of its ability to control the application of forces and velocities to an object has been described in terms of the so-called *manipulability ellipsoid* (Yoshikawa 1984; 1985). The manipulability ellipsoid is defined by examining the singular value decomposition of the Jacobian. If $J \in R^{M \times N}$, then $\exists U \in R^{M \times M}$ and $V \in R^{N \times N}$

such that $J = U \Sigma V^T$, where

$$\Sigma = \begin{bmatrix} \sigma_1 & & & & & \\ & \sigma_2 & & & & \\ & & \sigma_3 & & & \\ & & & \ddots & & \\ & & & & \ddots & \\ & & & & & \sigma_m \\ & & & & & & 0 \end{bmatrix} \in R^{M \times N}$$

with $\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_m \geq 0$.

These σ_i represent the m singular values of J . If we constrain the joint space angular rates,

$$|\omega| = [\omega_1^2 + \omega_2^2 + \dots + \omega_n^2]^{1/2} \leq 1,$$

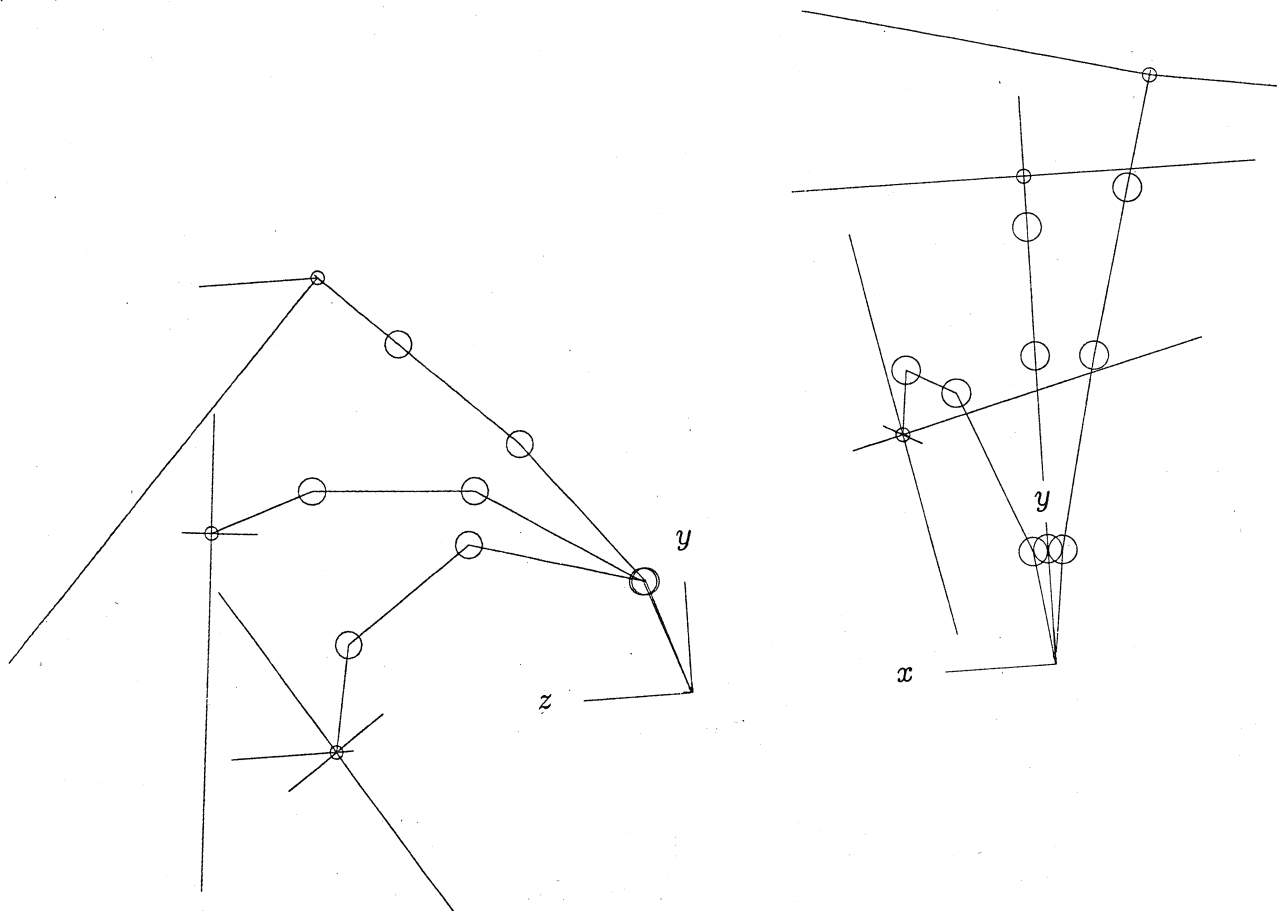
then we find that the resulting manipulability ellipse delimits the realizable Cartesian space velocities. The subspace ellipsoid is defined by the principally conditioned axes $[\sigma_1 u_1, \sigma_2 u_2, \dots, \sigma_m u_m]$, where the $u_i \in R^m$ are the m column vectors of the U matrix above. The manipulability ellipsoid describes the conditioning of the transformation from finger joint space to the fingertip Cartesian velocity space.

The volume of the manipulability ellipsoid may also be used to direct the manipulator into well-behaved configurations. Finger geometries that maximize this volume produce more anthropomorphic results than does the condition number approach. The volume can be approximated as the product of the singular values (Yoshikawa 1984).

The number of nonzero singular values define the rank r of the wrench system, and the first r column vectors of U define the Cartesian space spanned by the wrench system. These Cartesian directions, weighted by the corresponding singular values, define the *principally conditioned axes* (PCAs) of the manipulator. The PCAs represent the relative conditioning of the kinematic structure for forces or velocities along the corresponding direction.

While the manipulator geometry may be capable of generating instantaneous velocities, it may not be able to do so in the region immediately surrounding its current state. The PCAs are weighted to reflect the physical joint limits of the manipulator. This is accomplished by defining a weighting coefficient that pe-

Fig. 17. Weighted principally conditioned axes for the Utah/MIT finger [reprinted by permission from Grupen (1988) © IEEE].



nalizes a finger configuration which places joints at extreme positions. One such weighting coefficient is

$$W(\theta_i) = \cos \left\{ 2\pi \left(\frac{\theta_i - \theta_{i,\text{nom}}}{\theta_{i,\text{max}} - \theta_{i,\text{min}}} \right)^2 \right\}. \quad (12)$$

The contribution of the i th joint to the Cartesian direction defined by a particular PCA can thus be weighted to reflect the joint limits. The results of applying such a weighting coefficient to varying configurations of the Utah/MIT finger are presented in Fig. 17. A finger is viewed from the side and from behind to illustrate the weighted PCAs for a variety of finger geometries (Grupen and Henderson 1988). The product of the weighting coefficient over all joints in a finger may be used to modify the manipulability index presented earlier, resulting in a scalar conditioning metric which expresses joint limits.

The PCAs can be interpreted as a system of relative forces just as was derived from the contact friction

model earlier. As such, they may be used to define a wrench space vector defining the ability of the manipulator to address the task. Once again, an error with respect to the task may be defined by using Eq. (8). This error may be used to define migrations of the coordinate frame attached to the manipulator relative to the object frame, which improve the conditioning of a particular finger or the entire contact set.

5. Summary and Conclusions

General-purpose manipulation is a complicated problem involving many branches of knowledge and technology. We have discussed several aspects of the problem and presented research results which address these issues.

Since the human hand is the prototypical general-

purpose manipulator, we began our discussion here. The kinematic structure and the sensory apparatus of the human hand were presented.

Several methods of tactile sensing were discussed. Although a variety of technologies have been employed, the use of resistive, capacitive, piezoelectric, and optical effects seem to have dominated development efforts. Hysteresis, sensor fabrication, and reliability remain the primary obstacles to tactile sensing researchers. We also reviewed efforts to incorporate tactile sensing into machine perception systems. Multimodal sensing strategies and methods of sensory integration were presented.

The anthropomorphic design paradigm remains the dominant approach to general-purpose manipulators. A variety of multifingered dexterous hands were presented. Some of these manipulators are potentially suited to the role of general-purpose manipulator. The primary challenge to this technology is the control of the complex, multifingered hand. We briefly described the history of compliant manipulator control and the implications for control of general-purpose manipulators.

Finally, we discussed some of the properties of the object, the manipulator, and the task which constrain the selection of manipulation strategies. This area of research is still in its infancy, but is crucial to the development of a somewhat automatic general-purpose manipulator.

Unfortunately, much of the work surveyed pertains only to a small portion of the problem domain. Sensing, control and planning mechanisms are highly interdependent and should not, hereafter, be studied in isolation. There has been significant progress in the technology supporting robotic manipulation, enough to inspire optimism. Given the current state of research, we believe that general-purpose manipulation systems will appear on the scene in the not too distant future.

References

- Allen, P. K. 1984 (Atlanta, March). Surface descriptions from vision and touch. *Proc. IEEE Conf. on Robotics*. Washington: 394-397.
- Allen, P. K. 1986 (San Francisco, April). Sensing and describing 3-D structure. *Proc. IEEE Conf. on Robotics and Automation*. Washington: 126-131.
- Bardelli, R. et al. 1983. Piezo- and pyroelectric polymers skin-like tactile sensors for robots and prostheses. *Proc. Robotics International of SME*: 18-45 to 18-56.
- Barry Wright Corporation. 1984. *Sensoflex™* Tactile sensing system data sheet. Products for Flexible Automation Bulletin, Part No. TS 402-1, Watertown, MA: Barry Wright Corp.
- Begej, S. 1984. An optical tactile array sensor. *SPIE Intelligent Robots and Computer Vision*, 521:271-280.
- Begej Corporation. 1986. Product literature for the FTS-2 fingertip shaped tactile sensor. Technical Bulletin No. 2. Littleton, CO: Begej Corp.
- Bekey, G., and Tomovic, R. 1986 (San Francisco, April). Robot control by reflex actions. *Proc. IEEE Conf. on Robotics and Automation*. Washington: 240-247.
- Benton, A. 1982. Spatial thinking in neurological patients: historical aspects. *Spatial abilities: development and physiological foundations*. New York: Academic Press.
- Boie, R. A. 1984 (Atlanta, March). Capacitive impedance readout tactile image sensor. *Proc. IEEE Conf. on Robotics*. Washington: 370-378.
- Boissonnant, J. D. 1982. Stable matching between a hand structure and an object silhouette. *IEEE Trans. Pattern Analysis and Machine Intelligence*. PAMI-4(6):603-612.
- Bolle, R. M., and Cooper, D. B. 1986. On optimally combining pieces of information, with application to estimating 3-D complex object position from range data. *IEEE Trans. Pattern Analysis and Machine Intelligence* PAMI-8(5):619-638.
- Bolles, R. C., and Cain, R. A. 1982. Recognizing and locating partially visible objects: the local feature-focus method. *J. Robotics Research* 1(3):57-82.
- Bolles, R. C., and Houraud, P. 1986. 3DPO: A three-dimensional part orientation system. *J. Robotics Research* 5(3):3-26.
- Coiffet, P. 1981. *Robot technology*. Vol. 2, New York: Prentice-Hall.
- Cutkosky, M. R. 1985. *Robotic grasping and fine manipulation*. Boston: Kluwer Academic Publishers.
- Cutkosky, M. R., and Wright, P. K. 1986 (San Francisco, April). Modeling manufacturing grips and correlations with the design of robotic hands. *Proc. IEEE Conf. on Robotics and Automation*. Washington: 1533-1539.
- Dario, P. et al. 1984 (Atlanta, March). Ferroelectric polymer tactile sensors with anthropomorphic features. *Proc. IEEE Conf. on Robotics*. Washington: 332-340.
- Drake, S. H. 1977. Using compliance in lieu of sensory feedback for automatic assembly. D.Sc. thesis, Department

- of Mechanical Engineering, Massachusetts Institute of Technology.
- Ellis, R. E. 1984. Extraction of tactile features by passive and active sensing. *SPIE Intelligent Robots and Computer Vision* 521:289–295.
- Ellis, R. E. 1986 (San Francisco, April). A multiple-scale measure of static tactile texture. *Proc. IEEE Conf. on Robotics and Automation*. Washington: 1280–1285.
- Ernst, H. A. 1961. MH-1, A computer-operated mechanical hand. D.Sc. thesis, Massachusetts Institute of Technology.
- Fearing, R. S. 1983a. Simplified grasping and manipulation with dextrous robot hands. *J. Robotics Research* 2(4):188–195.
- Fearing, R. S. 1983b. Touch processing for determining a stable grasp. Master's thesis, Department of Electrical Engineering and Computer Science, Massachusetts Institute of Technology.
- Fearing, R. S. 1986 (San Francisco, April). Implementing a force strategy for object re-orientation. *Proc. IEEE Conf. on Robotics and Automation*. Washington: 96–102.
- Fearing R. S., and Hollerbach, J. M. 1984 (Atlanta, March). Basic solid mechanics for tactile sensing. *Proc. IEEE Conf. on Robotics*. Washington: 266–275.
- Geschke, C. G. 1983. A system for programming and controlling sensor-based robot manipulators. *IEEE Trans. Pattern Analysis and Machine Intelligence* PAMI-5(1):1–7.
- Golub, G. H., and Van Loan, C. F. 1983. *Matrix computations*. Baltimore: Johns Hopkins University Press.
- Gordon, G. 1978. *Active touch*. Elmsford, N.Y.: Pergamon Press.
- Grimson, W. E. L. 1986a (San Francisco, April). Disambiguating sensory interpretations using minimal sets of sensory data. *Proc. IEEE Conf. on Robotics and Automation*. Washington: 286–292.
- Grimson, W. E. L. 1986b. Sensing strategies for disambiguating among multiple objects in known poses. *J. Robotics Research* 2(4): 196–213.
- Grimson, W. E. L., and Lozano-Pérez, T. 1986. Model-based recognition and localization from tactile data. *J. Robotics Research* 3(3):3–35.
- Gruppen, R. A. et al. 1988. Task defined internal grasp wrenches. Technical Report UUCS-88-001, Salt Lake City: University of Utah Department of Computer Science.
- Gruppen, R. A., and Henderson, T. C. 1988 (Philadelphia, April 1988). A control paradigm for general purpose manipulation systems. *Proc. IEEE Conf. on Robotics and Automation*. Washington: to appear.
- Hanafusa, H., and Asada, H. 1977a (Tokyo). A robot hand with elastic fingers and its application to assembly process. *IFAC Symposium on Information and Control Problems in Manufacturing Technology*: 127–138.
- Hanafusa, H., and Asada, H. 1977b (Tokyo). Stable prehension by a robot hand with elastic fingers. *Proc. 7th Symp. on Industrial Robots*: 361–368.
- Hansen, C., and Henderson, T. 1987 (Miami Beach, November). CAGD-based computer vision. *Proc. IEEE Workshop on Computer Vision*: 100–106.
- Harmon, L. D. 1980. Touch-sensing technology: a review. Technical Report MSR80-03, Dearborn, Michigan: Society of Manufacturing Engineers.
- Harmon, L. D. 1982. Automated tactile sensing. Technical Report MSR82-02, Dearborn, Michigan: Society of Manufacturing Engineers.
- Harmon, L. D. 1984a (Atlanta, March). Automated touch sensing: a brief perspective and several new approaches. *Proc. IEEE Conf. on Robotics*. Washington: 326–331.
- Harmon, L. D. 1984b. Robotic taction for industrial assembly. *Communications of the J. Robotics Research* 3(1):72–76.
- Heimer, L. 1983. *The human brain and spinal cord*. New York: Springer-Verlag.
- Hemami, H., and Chen, B. 1984. Stability analysis and input design of a two-link planar biped. *J. Robotics Research* 3(2):93–100.
- Hillis, W. D. 1982. A high resolution image touch sensor. *J. Robotics Research* 1(2):33–44.
- Hogan, N. 1985. Impedance control: an approach to manipulation: I-theory, II-implementation, III-applications. *ASME J. Dynamic Systems, Measurement, and Control* 107:1–24.
- Hollerbach, J. M. 1987. Robot hands and tactile sensing. *AI in the 1980's and Beyond*, ed. W. E. L. Grimson and R. S. Patil, Cambridge: MIT Press, pp. 317–343.
- Jacobsen, S. C., et al. 1983 (Bretton Woods, NH, August). The Utah/MIT dextrous hand. *International Robotics Research Symposium*: 601–654.
- Jacobsen, S. C., et al. 1984. The Utah/MIT dextrous hand work in progress. *J. Robotics Research* 3(4):21–50.
- Jacobsen, S. C., et al. 1986 (San Francisco, April). Design of the Utah/MIT dextrous hand. *Proc. IEEE Conf. on Robotics and Automation*. Washington: 1520–1532.
- Jacobsen, S. C., et al. 1987 (Raleigh, April). Tactile sensing system design issues in machine manipulation. *Proc. IEEE Conf. on Robotics and Automation*. Washington: 2087–2096.
- Jameson, J. W., and Leifer, L. J. 1987. Automatic grasping: an optimization approach. *IEEE Trans. Systems, Man, and Cybernetics* SMC-17(5):806–814.
- Jones, F. W. 1920. *The principles of anatomy*. London: J. and A. Churchill.
- Kandel, E. R., and Schwartz, J. H. 1981. *Principles of neural science*. New York: Elsevier.

- Kapandji, I. 1970. *The physiology of the joints*. Vol. 1, New York: Churchill Livingstone.
- Kerr, J., and Roth, B. 1986. Analysis of multifingered hands. *J. Robotics Research* 4(4):3-17.
- Klein, C., and Blaho, B. 1987. Dexterity measures for the design and control of kinematically redundant manipulators. *J. Robotics Research* 6(2):72-83.
- Kobayashi, H. 1985. Control and geometrical considerations for an articulated robot hand. *J. Robotics Research* 4(1):3-12.
- Kobayashi, H. 1986 (San Francisco, April). Grasping and manipulation of objects by articulated hands. *Proc. IEEE Conf. on Robotics and Automation*. Washington: 1514-1519.
- Langley, L. L., Telford, I. R., and Christensen, J. B. 1974. *Dynamic anatomy and physiology*. New York: McGraw-Hill.
- Li, Z., and Sastry, S. 1987 (Raleigh, April). Task oriented optimal grasping by multifingered robot hands. *Proc. IEEE Conf. on Robotics and Automation*. Washington: 389-394.
- Lian, D., Peterson, S., and Donath, M. 1983 (Chicago, April). A three-fingered, articulated hand. *Proc. 13th Int. Symp. on Industrial Robots*, SME, pp. 18-91 to 18-101.
- Liapunov, A. M. 1966. *Stability of motion*. New York: Academic Press.
- Lord Corporation, 1984. Product literature for the Lord tactile sensor LTS series. Technical Report, Lord Corporate Development Center, Cary, NC.
- Lozano-Pérez, T. 1981. Automatic planning of manipulator transfer movements. *IEEE Trans. Systems, Man, and Cybernetics* 11(10):681-689.
- Luo, R. C., Wang, F., and Liu, Y. X. 1984. An imaging tactile sensor with magnetostrictive transduction. *SPIE Intelligent Robots and Computer Vision* 521:264-270.
- Lyons, D. 1986 (San Francisco, April). Tagged potential fields: an approach to specification of complex manipulator configurations. *Proc. IEEE Conf. on Robotics and Automation*. Washington: 1749-1754.
- Mason, M. T. 1981. Compliance and force control for computer controlled manipulators. *IEEE J. Systems, Man, and Cybernetics* 11(6):418-432.
- Mason, M. T., and Salisbury, J. K. 1985. *Robot hands and the mechanics of manipulation*. Cambridge, Mass.: MIT Press.
- McCammon, I. 1984. Design of a conformal tactile sensing array. *SPIE Intelligent Robots and Computer Vision* 521:296-301.
- Muthukrishnan, C., et al. 1987 (Raleigh, April). Edge detection in tactile images. *Proc. IEEE Conf. on Robotics and Automation*. Washington: 1500-1505.
- Nguyen, V. D. 1986 (San Francisco, April). The synthesis of stable grasps in the plane. *Proc. IEEE Conf. on Robotics and Automation*. Washington: 884-889.
- Okada, T. 1979. Object-handling system for manual industry. *IEEE J. of Systems, Man, and Cybernetics* 9(2):79-89.
- Okada, T. 1982. Computer control of multijointed finger system for precise object-handling. *IEEE J. Systems, Man, and Cybernetics* 12(3):289-299.
- Page, C. J., and Pugh, A. 1976 (Nottingham, UK, March). Novel techniques for tactile sensing in a three dimensional environment. *Proc. 3rd Conf. on Industrial Robot Technology*: C4-33 to C4-46.
- Paul, B. 1979. *Kinematics and dynamics of planar machinery*. Englewood Cliffs, N.J.: Prentice-Hall.
- Paul, R., and Shimano, B. 1976 (Purdue University, July). Compliance and control. *Proc. Joint Automatic Control Conf.*: 694-699.
- Pennywitt, K. E. 1986. (Jan.) Robotic tactile sensing. *Byte Magazine*: 177-200.
- Raibert, M. H. 1984 (Atlanta, March 1984). An all digital VLSI tactile array sensor. *Proc. IEEE Conf. on Robotics*. Washington: 314-319.
- Raibert, M. H., and Craig, J. J. 1981. Hybrid position/force control of manipulators. *J. Dynamic Systems, Measurements, and Control* 102:127-133.
- Rebman, J., and Trull, M. W. 1983 (Chicago). A robust tactile sensor for robotic applications. *Proc. Computers in Engineering*: 109-114.
- Salisbury, J. K. 1982. Kinematic and force analysis of articulated hands. Ph.D. thesis, Department of Mechanical Engineering, Stanford University.
- Salisbury, J. K. 1984 (Atlanta, March). Interpretation of contact geometries from force measurements. *Proc. IEEE Conf. on Robotics*. Washington: 240-247.
- Salisbury, J. K., and Craig, J. J. 1982. Articulated hands: force control and kinematic issues. *J. Robotics Research* 1(1):4-17.
- Schiff, W., and Foulke, E. 1982. *Tactual perception: a sourcebook*. New York: Cambridge University Press.
- Schneider, J. L. 1986 (San Francisco, April). An objective tactile sensing strategy for object recognition and localization. *Proc. IEEE Conf. on Robotics and Automation*. Washington: 1262-1267.
- Siegel, D. M., Garabieta, I., and Hollerbach, J. M. 1986 (San Francisco, April). An integrated tactile and thermal sensor. *Proc. IEEE Conf. on Robotics and Automation*. Washington: 1286-1291.
- Stansfield, S. A. 1986 (San Francisco, April). Primitives, features, and exploratory procedures: building a robot tac-

-
- tile perception system. *Proc. IEEE Conf. on Robotics and Automation*. Washington: 1274-1279.
- Tomovic, R., Bekey, G., and Karplus, W. 1987 (Raleigh, April). A strategy for grasp synthesis with multifingered robot hands. *Proc. IEEE Conf. on Robotics and Automation*. Washington: 83-89.
- Vranish, J. 1986 (San Francisco, April). Magnetoinductive skin for robots. *Proc. IEEE Conf. on Robotics and Automation*. Washington: 1292-1318.
- Whitney, D. E. 1977 (June). Force feedback control of manipulator fine motions. *ASME J. Dynamic Systems, Measurement, and Control*: 91-97.
- Whitney, D. E. 1987. Historical perspective and state of the art in robot force control. *J. Robotics Research* 6(1):3-14.
- Wright, P. 1983. *A manufacturing hand*. Annual Research Review, The Robotics Institute, Carnegie Mellon University, Pittsburg, PA.
- Yoshikawa, T. 1984. Analysis and control of robot manipulators with redundancy. *Robotics Research: First Int. Symp.*: 735-747.
- Yoshikawa, T. 1985. Manipulability of robotic mechanisms. *J. Robotics Research* 4(2):3-9.
- Zalucky, A. 1984. ITA Interim Technical Report No. 5. AFWAL/MLTC Wright-Patterson AFB, Adept Technology, Inc.