**Error Ellipses**

**Input:**
- one sigma uncertainty in the x-direction: $\sigma_x$
- one-sigma uncertainty in the y-direction: $\sigma_y$
- correlation coefficient $\rho_{xy}$
- center of the ellipse: $(x,y)$
- aspect ratio of the plot: $(\Delta y/\text{vertical plot length})/(\Delta x/\text{horizontal plot length})$

1) Calculate the covariance between $x$ and $y$:

$$\sigma_{xy} = \rho_{xy} \cdot \sigma_x \cdot \sigma_y$$

2) Construct the covariance matrix:

$$\text{covmat} = \begin{pmatrix} (\sigma_x)^2 & \sigma_{xy} \\ \sigma_{xy} & (\sigma_y)^2 \end{pmatrix}$$

3) Calculate the lengths of the ellipse axes, which are the square root of the eigenvalues of the covariance matrix:

$$\text{eigval} = \text{eigenvalues}(c)$$

4) Calculate the counter-clockwise rotation ($\theta$) of the ellipse:

$$\theta = \frac{1}{2} \cdot \tan^{-1}\left[\frac{1}{\text{aspect ratio}} \cdot \left(\frac{2 \cdot \sigma_{xy}}{(\sigma_x)^2 - (\sigma_y)^2}\right)\right]$$

5) To create a 95% confidence ellipse from the $1\sigma$ error ellipse, we must enlarge it by a factor of $\text{scalefactor} = 2.4477$.

6) Plot the ellipse:

7a) Plot an ellipse with semi-major and semi-minor axes parallel to the x- and y-axes of the graph, centered at $(x,y)$. These axis lengths are the square roots of the eigenvalues. The larger eigenvalue belongs to the axis with the larger uncertainty:

- **If** $\sigma_x > \sigma_y$
  - semi-major axis length parallel to $x = \sqrt{\text{maximum(eigenvalues)}} \cdot \text{scalefactor}$
  - semi-minor axis length parallel to $y = \sqrt{\text{minimum(eigenvalues)}} \cdot \text{scalefactor}$

- **If** $\sigma_y > \sigma_x$
  - semi-minor axis length parallel to $x = \sqrt{\text{minimum(eigenvalues)}} \cdot \text{scalefactor}$
  - semi-major axis length parallel to $y = \sqrt{\text{maximum(eigenvalues)}} \cdot \text{scalefactor}$

7b) Rotate the ellipse $\theta$ radians counter-clockwise from its original orientation.