# On the Equi-Area Partitioning Problem for Rectilinear Simple Polygons

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## ABSTRACT

We are given a rectilinear simple polygon $\mathcal{P}$ with $N$ vertices and the question is how to partition $\mathcal{P}$ into a set of $p$ sub-polygons of equal areas. This problem is motivated by an application in building design and evacuation planning on a rectilinear metric, where one seeks to locate $p$ stairwells in buildings so as to evacuate the occupants of the buildings in minimum clearance time.

The problem of finding the equi-area partitioning of $\mathcal{P}$ is a classical isoperimetric optimization problem and is $\mathcal{NP}$-hard in the number of sub-polygons to within any factor independent of the shape of the polygon, Bast and Hert [1].

There aren't many algorithms that directly address this problem, among some that are related there is CGAL*; CGAL 3.1 provides a library which can be used to decompose a polygon into a set of smaller convex/non-convex polygons, but the CGAL library cannot be used to partition $\mathcal{P}$ according to the area of each piece. Bast and Hert [1] stated that their area partitioning algorithm can produce non-optimal, but often quite reasonable, area partitionings for arbitrary $\mathcal{P}$. However, their implementation does not yield often reasonable results as stated in their paper, since the resulting sub-polygons are often not rectilinear or are skewed. So, in our paper, we show that we can use a sweep line heuristic to carry out the partitioning of the rectilinear polygon. Our heuristic is based on the straight skeleton of a polygon introduced by

Aichholzer [2] and Oliver [3]. Felkel and Obdrzalek [4] have implemented the straight skeleton problem following their own ideas [5] in C++.

Using the straight skeleton generated by Felkel and Obdrzalek's algorithm, we prune the branches of the skeleton only to keep the inner backbone and then extend the backbone to intersect the boundary of the polygon. From one of the intersection points (without loss of generality, we choose the left most intersection point), and recursively traverse the extended backbone, and construct the skeleton lines tree data structure. This tree data structure contains all the information that helps us to determine the sweep line movements. Initially, the sweep line starts from the left most intersection point, moves along the direction that's determined by the root skeleton line of the tree structure of the extended backbone. After the sweep line has swept the full length of a skeleton line, one of its children is picked and its direction becomes the current sweep line movement direction. Since $\mathcal{P}$ has an arbitrary shape of a rectilinear boundary, the sweep line sometimes will meet corners, which can result in convex or reflex angles. In the heuristic, convex angles don't change the sweep line movement, but when the sweep line comes to a reflex angle, (it depends on the shape of the polygon), the sweep line might keep moving (along the current skeleton line that is in use) or it might anchor at the corner point and start to rotate. In each $\epsilon-$movement step (either movement along the skeleton line, or rotation anchored at a corner), the heuristic automatically calculates the swept area portion of the polygon. The heuristic marks the cut when it swept through a portion of polygon with a specified area and keeps moving until it finishes sweeping the whole polygon.

The heuristic runs in a $O(S^2 + SN + S\log S)$ time, and the space complexity of the heuristic is $O(N + S)$, where $N$ is the number of vertices of the polygon and $S$ is the number of the skeleton line segments in the extended skeleton backbone of $\mathcal{P}$.

* Computational Geometry Algorithm Library (CGAL)

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