On Guarding and Partitioning Polygons

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Abstract

We do an experimental study of heuristics for guarding polygons in the plane. Our heuristics include simple methods based on greedy set cover for vertex guarding, as well as partitioning polygons into (approximately) the minimum number of star-shaped pieces. We test also heuristics for computing lower bounds on the guard number, based on placing a large set of "independent" witness points. These upper and lower bounds allow us to investigate the approximation ratio of our heuristics in practice, on a data base of "randomly" generated polygons.

1 Overview

The art gallery problem deals with covering polygons by guards who cannot see through walls and may have other constraints on visibility. Many variants to the problem have appeared in the literature since the initial exploration started in the early 1970's. Variants include domains with holes, constraints on the shape of the polygon (such as orthogonality), altering the notion of guarding (edge guards, diagonal guards), limits on the visibility of the guards, etc. A good source of information on the problem is [3, 4] . A major algorithmic challenge is to obtain a small set of guards that are sufficient to cover the polygon. The original art gallery theorem shows that for a polygon with n vertices, $\lfloor n/3 \rfloor$ guards are always sufficient and sometimes necessary. However, this result can be far from the optimal in many cases. Unfortunately, finding the smallest set of guards is known to be NP-hard for many instances of this problem; thus, attention has been focused recently on approximation techniques for obtaining small covering sets.

Since visibility is a key point in the art gallery problem, visibility graphs are important in our study of the problem, particularly since we focus some of our attention on computing *vertex guards* (guards that must be placed at vertices, or at pre-specified discrete points within the region).

One useful direction for approximation is to decompose the polygon into simple subpolygons (such as convex, star-shaped, monotone, etc.), each of which can be more easily analyzed for guarding. For instance, after decomposing into convex polygons, we can simply position one guard in each subpolygon and cover the entire polygon. A good candidate shape for such decomposition is the star-shaped polygon. An $O(n^5 r^2 \log n)$ dynamic programming algorithm is known (Keil [2]) for computing a decomposition of a simple polygon into the minimum number of star-shaped pieces. We explore approximation methods (based on the Hertel-Mehlhorn [1] method of approximating convex decompositions) and apply these experimentally to the guard *coverage* problem.

Since we also apply heuristics to determine a "large" set of "independent" witness points (no two of which are seen by a common point/vertex), we are able to give lower bounds on the guard number and thereby give approximation bounds on the coverages we compute.

2 Our Work

Although it is hard to compute a minimum guard cover for simple polygons or polygons with holes, lower and upper bounds can be established much more easily. Any decomposition into k convex or star-shaped polygons yields an upper bound of k on the coverage number; this bound can, in theory, be very poor (examples exist with coverage number $O(1)$ and partition number $\Omega(n)$). Part of our goal is to explore experimentally how close to optimal heuristics are that are based on simple methods of partitioning P or of greedily placing guards in P.

We determine a lower bound on the coverage number for P by computing a large set (hopefully close to maximum) of I "independent" witness points in P, no two of which are seen by a common point (or a common vertex, in the case of vertex guarding). Then, I is

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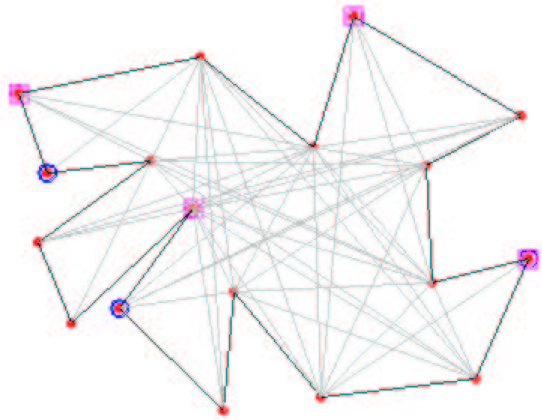

Figure 1: A polygon with 18 vertices: Witnesses are marked by blue circles and guards by magenta squares

a lower bound on the optimal coverage number.

3 Experimental Results

In Table 1 we show some experimental results obtained with our software for randomly generated simple polygons of various sizes (for which we used the RPG package of Auer and Held). These results are based on a greedy placement of guards, and a greedy placement of independent witness points. In all cases, we see that the approximation ratio is between 1 and 2.

For clarity, in Figure 1 we present the results of computing lower and upper bounds for a polygon with only 18 vertices

# vertices	# withnesses (W)	# guards (G)	G / W
10	1	1	1
20	2	2	1
10	1	1	1
20	1	1	1
10	2	2	1
20	2	2	1
50	8	9	1.12
100	8	12	1.5
50	8	10	1.25
100	8	10	1.25
50	8	10	1.25
100	8	13	1.62
100	13	15	1.15
200	12	18	1.5
100	13	20	1.54
200	14	21	1.5
100	13	18	1.38
200	17	22	1.29
200	32	37	1.19
400	35	50	1.42
200	23	31	1.34
400	25	38	1.52
200	25	35	1.4
400	28	39	1.32
500	64	85	1.32
1000	63	115	1.85
500	64	91	1.42
1000	71	110	1.55
500	61	87	1.42
1000	64	103	1.60

Table 1: Lower and upper bounds

References

[1] S. Hertel and K. Mehlhorn. Fast triangulation of the plane with respect to simple polygons. *Inform. Control*, 64:52–76, 1985.

[2] J. M. Keil. Decomposing a polygon into simpler components. *SIAM J. Comput.*, 14:799–817, 1985.

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