The Fold-and-Cut Problem

Problem Statement:

Take a piece of paper, fold it into any flat origami, and make one complete straight cut. After unfolding the pieces, what kind of shapes can you get from it?

Theorem:

Every pattern (plane graph) can be made by folding and one complete straight cut. This includes single (even non-convex) polygons, multiple disjoint polygons, nested polygons, adjoining polygons, and even floating line segments and points.

Straight-skeleton method:

The straight skeleton of a simple polygon is defined by shrinking the polygon by translating each of its edges at a fixed rate, keeping sharp corners at the reflex vertices, and watching where the vertices go. The straight skeleton consists of the majority of the creases and achieve the desired lining up of cuts.

Algorithm Outline for Straight-Skeleton method:

- For each face of the desired cut pattern (region between cuts), shrink the face so that the edges stay parallel and move a constant speed in a perpendicular direction.
- Stop whenever the boundary becomes non-simple (intersects itself), and continue shrinking each piece.
- The straight skeleton is the trajectories of the vertices of the desired cut pattern during this shrinking process.

Implementation:

We try to implement the straight skeleton method using CGAL which provides provide efficient geometric algorithms.