## Lecture 14: Interconnection Networks

- Topics: dimension vs. arity, deadlock


## Interconnection Networks

- Recall: fully connected network, arrays/rings, meshes/tori, trees, butterflies, hypercubes
- Consider a k-ary d-cube: a d-dimension array with k elements in each dimension, there are links between elements that differ in one dimension by $1(\bmod k)$
- Number of nodes $N=k^{d}$

| Number of switches: | Avg. routing distance: |
| :--- | :--- |
| Switch degree $:$ | Diameter |
| Number of links | $:$ |
| Pins per node | $:$ |$\quad$ Bisection bandwidth :

Should we minimize or maximize dimension?

## Interconnection Networks

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- Consider a k-ary d-cube: a d-dimension array with k elements in each dimension, there are links between elements that differ in one dimension by $1(\bmod k)$
- Number of nodes $\mathrm{N}=\mathrm{k}^{\mathrm{d}}$

| Number of switches : | N |
| :--- | :--- |
| Switch degree | $: 2 \mathrm{~d}+1$ |
| Number of links | $: \mathrm{Nd}$ |
| Pins per node | $: 2 \mathrm{wd}$ |

Avg. routing distance: Diameter Bisection bandwidth Switch complexity : $(2 d+1)^{2}$

Should we minimize or maximize dimension?

## Bisection Bandwidth

Break the $k^{d}$ nodes into two groups such that all elements in group-1 are of the form: [0-k/2-1] [*][*]...[*] in group-2 are of the form: [ $\left.\mathrm{k} / 2-\mathrm{k}]\left[{ }^{*}\right]\left[{ }^{*}\right] \ldots{ }^{*}{ }^{*}\right]$

- Each node has an edge to other nodes that differ in only one dimension by one
- Any node in group-1 differs from any node in group-2 in at least the first dimension - hence, any edge from group- 1 to group-2 is an edge that connects nodes that are identical in $\mathrm{d}-1$ dimensions and differ in the first dimension by 1
- If we fix the co-ordinates of the d - 1 dimensions, we can identify two edges: $\left[0, \mathrm{i}_{1}, \ldots, \mathrm{i}_{-1}\right]-\left[k-1, \mathrm{i}_{1}, \ldots, \mathrm{i}_{\mathrm{d}-1}\right]$ and $\left[k / 2-1, i_{1}, \ldots, \mathrm{i}_{\mathrm{d}-1}\right]-\left[k / 2, \mathrm{i}_{1}, \ldots, \mathrm{i}_{\mathrm{d}-1}\right]$ : there are totally $2 \mathrm{k}^{\mathrm{d}-1}$ edges


## Dimension

- For a fixed machine size N, low-dimension networks have significantly higher latencies for a packet - scalable machines should employ high dimensionality (high cost!)
- For a fixed number of pins, message latency decreases at first, then increases (as we increase dimensionality)
-What if we keep constant bisection bandwidth?

| Number of switches : | N |
| :--- | :--- |
| Switch degree | $: 2 \mathrm{~d}+1$ |
| Number of links | $: \mathrm{Nd}$ |
| Pins per node | $: 2 \mathrm{wd}$ |

Avg. routing distance: $d(k-1) / 2$
Diameter : d(k-1)
Bisection bandwidth : $2 w^{d-1}$
Switch complexity : $(2 d+1)^{2} 5$
$N=k^{d}$

## Butterfly Network



## Routing

- Deterministic routing: given the source and destination, there exists a unique route
- Adaptive routing: a switch may alter the route in order to deal with unexpected events (faults, congestion) - more complexity in the router vs. potentially better performance
- Example of deterministic routing: dimension order routing: send packet along first dimension until destination co-ord (in that dimension) is reached, then next dimension, etc.


## Deadlock

- Deadlock happens when there is a cycle of resource dependencies - a process holds on to a resource (A) and attempts to acquire another resource (B) - $A$ is not relinquished until $B$ is acquired


## Deadlock Example


$\square$ Packets of message 1
$\square$ Packets of message 2
$\square$ Packets of message 3Packets of message 4

Each message is attempting to make a left turn - it must acquire an output port, while still holding on to a series of input and output ports

## Deadlock-Free Proofs

- Number edges and show that all routes will traverse edges in increasing (or decreasing) order - therefore, it will be impossible to have cyclic dependencies
- Example: k-ary 2-d array with dimension routing: first route along x-dimension, then along y



## Breaking Deadlock I

- The earlier proof does not apply to tori because of wraparound edges
- Partition resources across multiple virtual channels
- If a wraparound edge must be used in a torus, travel on virtual channel 1, else travel on virtual channel 0


## Breaking Deadlock II

- Consider the eight possible turns in a 2-d array (note that turns lead to cycles)
- By preventing just two turns, cycles can be eliminated
- Dimension-order routing disallows four turns
- Helps avoid deadlock even in adaptive routing


West-First


North-Last


Negative-First


Can allow deadlocks

## Title

- Bullet

