A Belief-Based Account of Decision under Uncertainty

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Outline

• Problem Definition
• Decision under Uncertainty (classical Theory)
• Two-Stage Model
• Probability Judgment and Support Theory
• The Case Studies and Discussion
Decision under Uncertainty

• Judgment of probability

• Decision under Risk
Two studies

• 1995 professional basketball playoffs

• Movement of economic indicators in a simulated economy

• Results ....
  – Consistent with the belief-based account
  – Violated the partition inequality (implied by classical theory of decision under uncertainty)
Decision Making

• *Decision*: Depends on the strength of people’s *belief* an event happens.

• Question: How to measure these beliefs ?!
Decision under Uncertainty (classical Theory)

• “... derives beliefs about the likelihood of uncertain events from people’s **choices** between prospects whose consequences are contingent on these events.”

• Simultaneous measurement of utility and subjective probability
Challenges

• From psychological perspective:
  1) Belief precedes preference
  2) Probability Judgment
  3) The assumption of the derivation of belief from preference

• Belief based approach uses probability judgment to predict decisions under uncertainty
Background

• Risky prospects – known probabilities
  – Decision under risk
  – Non-linear weighting function
Background

• Real world decisions – uncertain prospects

• Extension to the domain of uncertainty
Cumulative Prospect Theory

- Assumes that an event has more impact on choice when:
  - *Possibility Effect*: It turns an impossibility into a possibility
  - *Certainty Effect*: It turns a possibility into a certainty than when it merely makes a possibility more or less likely.

*Bounded Subadditivity*
Bounded Subadditivity

• Tested on both risky and uncertain prospects.

• Data satisfied bounded subadditivity for both risk and uncertainty.

• Departure from expected utility theory
Two-Stage Model

- Decision makers:
  1) Assess the probability (P) of an uncertain event (A)
  2) Then, transform this value using the risky weighting function (w)
Two-Stage Model Terminology

• Simple prospect:
  – \((x, A)\): Pay $x$, if the target event \((A)\) obtains and nothing otherwise.

• Overall value of a prospect \((V)\):
  – \(V(x, A) = v(x)W(A) = v(x)w[P(A)]\)
    - \(P(A)\): judged probability of \(A\)
    - \(v\): value function for monetary gains
    - \(w\): risky weighting function
Probability Judgment

- People’s intuitive probability judgments are often inconsistent with the laws of chance.

- Support Theory: Probabilities are attached to *description of events* (called hypothesis) rather than the *events*. 
Support Theory

• Hypothesis, A, has a nonnegative support value, s(A).

• Judged probability P(A, B):
  – Hypothesis A rather than B holds.
  – Interpreted as the support of the focal hypothesis, A, relative to the alternative hypothesis, B.

\[
P(A, B) = \frac{s(A)}{s(A) + s(B)}
\]
Support Theory

- The judged probability of the union of disjoint events is generally smaller than the sum of judged probabilities of these events.

\[ s(A) \leq s(A_1 \lor A_2) \leq s(A_1) + s(A_2) \]

*Unpacking principle*
Support Theory

• Binary Complementarity:
  – $P(A, B) + P(B, A) = 1$

• Subaditivity:
  – For finer partitioning (i.e., more than 2 events), the judged probability is \textit{less than or equal to} the sum of judged probabilities of its disjoint components.

\[
s(A) \leq s(A_1) + s(A_2)
\]
Implications

- Contrast between two-stage modern and expected utility theory with risk aversion:
  - The effect of partitioning
Implications

• Contrast between two-stage modern and expected utility theory with risk aversion:
  – The effect of partitioning
  – The classical model follows *partition inequality*:

\[ C(x, A) : \sum_{i=1}^{n} C(x, A_i) \leq C(x, A), \]

• \( C(x, A) \): pays $x if A occurs, and nothing otherwise.
• Doesn’t necessary hold considering the two-stage model.
Two-Stage Model

• Predict the certainty equivalent of an uncertain prospect, $C(x, A)$ from two independent responses:
  – The judged probability of the target event, $P(A)$
  – The certainty equivalent of the risky prospect, $C(x, P(A))$
Study 1

• Four tasks:
  1) Estimating subjects’ certainty equivalents (C) for risky prospects.
     Random draw of a single poker ship from an urn
  2) Estimating subjects’ certainty equivalents (C) for uncertain prospect.
     offering reward if a particular team, division, or conference would win the 1995 NBA.
  3) An independent test of risk aversion
  4) Estimate the probability of target events.
Result of Study 1

• Fit of the models to the data

• Unpacking principle VS. monotonicity
Study 2

• More simulated environment:
  – Subjects have identical information
  – Compare the judged probabilities vs. actual probabilities
Result of Study 2

• Binary partitioning:
  – Judged probabilities (nearly) satisfy binary complementarity
  – Certainty equivalents satisfy the partition inequality

• Finer partitioning:
  – Subadditivity of judged probabilities
  – Reversal of the partition inequality for certainty equivalents
Discussion

• The event spaces under study have some structure (hierarchical, product, ...)
• Subadditivity of judged probability is a major cause of violations of the partition inequality
• Generalization of the two-stage model for source preference.
• Particular description of events on which outcomes depend may affect a person’s willingness to act (unpacking principle)