L20: Introduction to "Irregular" Algorithms

November 9, 2010

Administrative

- Guest Lecture, November 18, Matt Might
- Project proposals
  - Reviewed all of them, will mail comments to CADE account during office hours
- CUDA Projects status
  - Available on CADE Linux machines (lab1 and lab3) and Windows machines (lab5 and lab6)
  - Windows instructions hopefully today
  - Still do not have access to "cutil" library, where timing code resides, but expect that later today

Programming Assignment #3: Simple CUDA Due Monday, November 18, 11:59 PM

Today we will cover Successive Over Relaxation. Here is the sequential code for the core computation, which we parallelize using CUDA:

```
for(i=1;i<N-1;i++) {
  for(j=1;j<N-1;j++) {
    B[i][j] = (A[i-1][j]+A[i+1][j]+A[i][j-1]+A[i][j+1])/4;
  }
}
```

You are provided with a CUDA template (sor.cu) that (1) provides the sequential implementation; (2) times the computation; and (3) verifies that its output matches the sequential code.

Programming Assignment #3, cont.

- Your mission:
  - Write parallel CUDA code, including data allocation and copying to/from CPU
  - Measure speedup and report
  - 45 points for correct implementation
  - 5 points for performance
  - Extra credit (10 points): use shared memory and compare performance

Programming Assignment #3, cont.

- You can install CUDA on your own computer
  - http://www.nvidia.com/cudazone/
- How to compile under Linux and MacOS
  - Version 2.x:

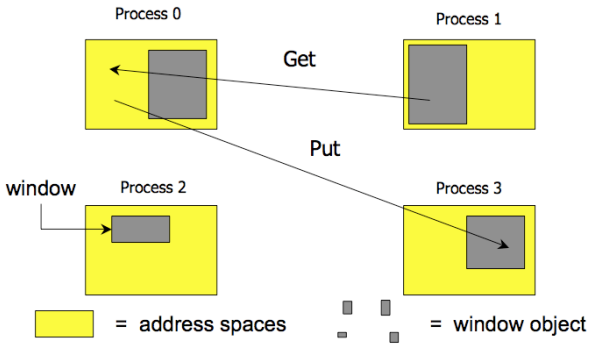
    nvcc -I/Developer/CUDA/common/inc \
    -L/Developer/CUDA/lib sor.cu -lcutil
  - Version 3.x:

    nvcc -I/Developer/GPU\ Computing/C/common/inc -L/ Developer/GPU\ Computing/C/lib sor.cu -lcutil
- Turn in
  - Handin cs4961 p03 <file> (includes source file and explanation of results)

Outline

- Finish MPI discussion
  - Review blocking and non-blocking communication
  - One-sided communication
- Irregular parallel computation
  - Sparse matrix operations and graph algorithms
- Sources for this lecture:
  - http://mpi.deino.net/mpi_functions/
  - Kathy Yelick/Jim Demmel (UC Berkeley): CS 267, Spr 07 • http://www.eecs.berkeley.edu/~yelick/cs267_sp07/lectures
  - "Implementing Sparse Matrix-Vector Multiplication on Throughput Oriented Processors," Bell and Garland (Nvidia), SC09, Nov. 2009.
  - Slides accompanying textbook "Introduction to Parallel Computing" by Grama, Gupta, Karypis and Kumar

One-Sided Communication



MPI One-Sided Communication or Remote Memory Access (RMA)

- Goals of MPI-2 RMA Design
  - Balancing efficiency and portability across a wide class of architectures
  - shared-memory multiprocessors
  - NUMA architectures
  - distributed-memory MPP's, clusters
  - Workstation networks
- Retaining "look and feel" of MPI-1
- Dealing with subtle memory behavior issues: cache coherence, sequential consistency

MPI Constructs supporting One-Sided Communication (RMA)

- MPI_Win_create exposes local memory to RMA operation by other processes in a communicator
  - Collective operation
  - Creates window object
- MPI_Win_free deallocates window object
- MPI_Put moves data from local memory to remote memory
- MPI_Get retrieves data from remote memory into local memory
- MPI_Accumulate updates remote memory using local values

Simple Get/Put Example

```
i = MPI_Alloc_mem(200 * sizeof(int), MPI_INFO_NULL, &A);
i = MPI_Alloc_mem(200 * sizeof(int), MPI_INFO_NULL, &B);
if (rank == 0) {
  for (i=0; i<200; i++) A[i] = B[i] = i;
  MPI_Win_create(NULL, 0, 1, MPI_INFO_NULL, MPI_COMM_WORLD, &win);
  MPI_Win_start(group, 0, win);
  for (i=0; i<100; i++) MPI_Put(A+i, 1, MPI_INT, 1, i, 1, MPI_INT, win);
  for (i=0; i<100 MPI_Get(B+i, 1, MPI_INT, 1, 100+i, 1, MPI_INT, win);
  MPI_Win_complete(win);
  for (i=0; i<100; i++)
    if (B[i] != (-4)*(i+100)) {
      printf("Get Error: B[i] is %d, should be %d\n", B[i], (-4)*(i+100));
      fflush(stdout);
      errs++;
    }
}
```


Simple Put/Get Example, cont.

```
else { /* rank=1 */

    for (i=0; i<200; i++) B[i] = (-4)*i;
      MPI_Win_create(B, 200*sizeof(int), sizeof(int), MPI_INFO_NULL,
MPI_COMM_WORLD, &win);
      destrank = 0;
      MPI_Group_incl(comm_group, 1, &destrank, &group);
      MPI_Win_post(group, 0, win);
      MPI_Win_wait(win);

      for (i=0; i<100; i++) {
        if (B[i] != i) {
          printf("Put Error: B[i] is %d, should be %d\n", B[i], i);fflush(stdout);
          errs++;
        }
      }
  }
```


MPI Critique (Snyder)

- Message passing is a very simple model
- Extremely low level; heavy weight
  - Expense comes from λ and lots of local code
  - Communication code is often more than half
  - Tough to make adaptable and flexible
  - Tough to get right and know it
  - Tough to make perform in some (Snyder says most) cases
- Programming model of choice for scalability
- Widespread adoption due to portability, although not completely true in practice

Motivation: Dense Array-Based Computation

- Dense arrays and loop-based data-parallel computation has been the focus of this class so far
- Review: what have you learned about parallelizing such computations?
  - Good source of data parallelism and balanced load
  - Top500 measured with dense linear algebra
    - How fast is your computer?" = "How fast can you solve dense Ax=b?"
  - Many domains of applicability, not just scientific computing
    - Graphics and games, knowledge discovery, social networks, biomedical imaging, signal processing
- What about "irregular" computations?
  - On sparse matrices? (i.e., many elements are zero)
  - On graphs?
  - Start with representations and some key concepts

Sparse Matrix or Graph Applications

- Telephone network design
  - Original application, algorithm due to Kernighan
- Load Balancing while Minimizing Communication
- Sparse Matrix times Vector Multiplication
  - Solving PDEs • N = {1,…,n}, (j,k) in E if A(j,k) nonzero, •
  - WN(j) = #nonzeros in row j, WE(j,k) = 1
- VLSI Layout
  - N = {units on chip}, E = {wires}, WE(j,k) = wire length
- Data mining and clustering
- Analysis of social networks
- Physical Mapping of DNA

Dense Linear Algebra vs. Sparse Linear Algebra

Matrix vector multiply:

```
for (i=0; i<n; i++)
    for (j=0; j<n; j++)
        a[i] = c[i][j]*b[j];
```

- What if n is very large, and some large percentage (say 90%) of c is zeros?
- Should you represent all those zeros? If not, how to represent "c"?

Sparse Linear Algebra

- Suppose you are applying matrix-vector multiply and the matrix has lots of zero elements
  - Computation cost? Space requirements?
- General sparse matrix representation concepts
  - Primarily only represent the nonzero data values
  - Auxiliary data structures describe placement of nonzeros in "dense matrix"

Some common representations

$$A = \begin{bmatrix} 1 & 7 & 0 & 0 \\ 0 & 2 & 8 & 0 \\ 5 & 0 & 3 & 9 \\ 0 & 6 & 0 & 4 \end{bmatrix}$$

$$\text{data} = \begin{bmatrix} * & 1 & 7 \\ * & 2 & 8 \\ 5 & 3 & 9 \\ 6 & 4 & * \end{bmatrix} \quad \text{offsets} = [-2\ 0\ 1]$$

DIA: Store elements along a set of diagonals.

ptr = [0 2 4 7 9]
indices = [0 1 1 2 0 2 3 1 3]
data = [1 7 2 8 5 3 9 6 4]

Compressed Sparse Row (CSR): Store only nonzero elements, with "ptr" to beginning of each row and "indices" representing column.

$$\text{data} = \begin{bmatrix} 1 & 7 & * \\ 2 & 8 & * \\ 5 & 3 & 9 \\ 6 & 4 & * \end{bmatrix} \quad \text{indices} = \begin{bmatrix} 0 & 1 & * \\ 1 & 2 & * \\ 0 & 2 & 3 \\ 1 & 3 & * \end{bmatrix}$$

ELL: Store a set of K elements per row and pad as needed. Best suited when number non-zeros roughly consistent across rows.

row = [0 0 1 1 2 2 2 3 3]
indices = [0 1 1 2 0 2 3 1 3]
data = [1 7 2 8 5 3 9 6 4]

COO: Store nonzero elements and their corresponding "coordinates".

CSR Example (UPDATE)

```
for (j=0; j<nr; j++) {
  for (k = ptr[j]; k<ptr[j+1]-1; k++)
    t[j] = t[j] + data[k] * x[indices[k]];
```

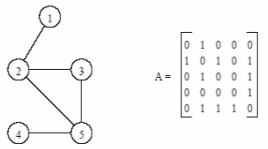
Other Representation Examples

- Blocked CSR
  - Represent non-zeros as a set of blocks, usually of fixed size
  - Within each block, treat as dense and pad block with zeros
  - Block looks like standard matvec
  - So performs well for blocks of decent size
- Hybrid ELL and COO
  - Find a "K" value that works for most of matrix
  - Use COO for rows with more nonzeros (or even significantly fewer)

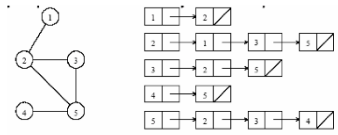
Basic Definitions: A Quiz

- Graph
  - undirected, directed
- Path, simple path
- Forest
- Connected component

Representing Graphs



An undirected graph and its adjacency matrix representation.



An undirected graph and its adjacency list representation.

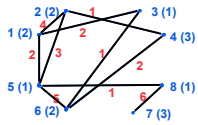
Common Challenges in Graph Algorithms

- Localizing portions of the computation
  - How to partition the workload so that nearby nodes in the graph are mapped to the same processor?
  - How to partition the workload so that edges that represent significant communication are co-located on the same processor?
- Balancing the load
  - How to give each processor a comparable amount of work?
  - How much knowledge of the graph do we need to do this since complete traversal may not be realistic?

All of these issues must be addressed by a **graph partitioning algorithm** that maps individual subgraphs to individual processors.

Definition of Graph Partitioning

- Given a graph $G = (N, E, W_N, W_E)$
  - N = nodes (or vertices),
  - W_N = node weights
  - E = edges
  - W_E = edge weights



- Ex: N = {tasks}, W_N = {task costs}, edge (j,k) in E means task j sends $W_E(j,k)$ words to task k
- Choose a partition $N = N_1 \cup N_2 \cup \ldots \cup N_P$ such that
  - The sum of the node weights in each N_j is "about the same"
  - The sum of all edge weights of edges connecting all different pairs N_j and N_k is minimized
- Ex: balance the work load, while minimizing communication
- Special case of $N = N_1 \cup N_2$: Graph Bisection

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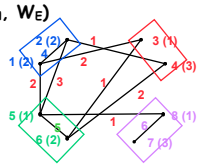

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  - The sum of the node weights in each N_j is "about the same"
  - The sum of all edge weights of edges connecting all different pairs N_j and N_k is minimized (shown in black)
- Ex: balance the work load, while minimizing communication
- Special case of $N = N_1 \cup N_2$: Graph Bisection

Summary of Lecture

- Summary
  - Regular computations are easier to schedule, more amenable to data parallel programming models, easier to program, etc.
  - Performance of irregular computations is heavily dependent on representation of data
  - Choosing this representation may depend on knowledge of the problem, which may only be available at run time
- Next Time
  - Introduction to parallel graph algorithms
    - Minimizing bottlenecks