

# The Arbitrariness of Identifiers

Are the following two programs equivalent?

```
(define (f x) (+ x 1))  
(f 10)
```

```
(define (f y) (+ y 1))  
(f 10)
```

yes

argument is consistently renamed

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**no**

not a use of the argument anymore

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(f 10)
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**no**

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## The Arbitrariness of Identifiers

Are the following two programs equivalent?

```
(define (f x) (+ y 1))  
(f 10)
```

```
(define (f z) (+ y 1))  
(f 10)
```

yes

argument never used, so almost any name is ok

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Are the following two programs equivalent?

```
(define (f x) (+ y 1))  
(f 10)
```

```
(define (f y) (+ y 1))  
(f 10)
```

**no**

now a use of the argument

# The Arbitrariness of Identifiers

Are the following two programs equivalent?

```
(define (f x) (+ y 1))  
(f 10)
```

```
(define (f x) (+ z 1))  
(f 10)
```

**no**

still an undefined identifier, but a different one

## The Arbitrariness of Identifiers

Are the following two programs equivalent?

```
(define (f x)
  (local [(define y 10)]
    (+ x y)))
(f 0)
```

```
(define (f z)
  (local [(define y 10)]
    (+ z y)))
(f 0)
```

yes

argument is consistently renamed

## The Arbitrariness of Identifiers

Are the following two programs equivalent?

```
(define (f x)
  (local [(define y 10)]
    (+ x y)))
(f 0)
```

```
(define (f x)
  (local [(define z 10)]
    (+ x z)))
(f 0)
```

yes

local identifier is consistently renamed



# The Arbitrariness of Identifiers

Are the following two programs equivalent?

```
(define (f x)
  (local [(define y 10)]
    (+ x y)))
(f 0)
```

```
(define (f x)
  (local [(define x 10)]
    (+ x x)))
(f 0)
```

no

local identifier now hides the argument

# The Arbitrariness of Identifiers

Are the following two programs equivalent?

```
(define (f x)
  (local [(define y 10)]
    (+ x y)))
(f 0)

(define (f y)
  (local [(define y 10)]
    (+ y y)))
(f 0)
```

no

local identifier now hides the argument

## Free and Bound Identifiers

An identifier for the argument of a function or the name of a local identifier is a *binding occurrence*

```
(define (f x y) (+ x y z))
```

```
(local [(define a 3)
        (define c 4)]
  (+ a b c))
```

## Free and Bound Identifiers

A use of a function argument or a local identifier is a ***bound occurrence***

```
(define (f x y) (+ x y z))
```

```
(local [(define a 3)
        (define c 4)]
  (+ a b c))
```

# Free and Bound Identifiers

A use of an identifier that is not function argument or a local identifier is a *free identifier*

```
(define (f x y) (+ x y z))
```

```
(local [(define a 3)
        (define c 4)]
  (+ a b c))
```

# Arithmetic Language

$$\begin{aligned} \langle \text{AE} \rangle & ::= \langle \text{num} \rangle \\ & \quad | \{ + \langle \text{AE} \rangle \langle \text{AE} \rangle \} \\ & \quad | \{ - \langle \text{AE} \rangle \langle \text{AE} \rangle \} \end{aligned}$$

```
(define-type AE
  [num (n number?)]
  [add (lhs AE?)
       (rhs AE?)]
  [sub (lhs AE?)
       (rhs AE?)])
```



# Arithmetic Language

```
<AE> ::= <num>
      | { + <AE> <AE> }
      | { - <AE> <AE> }
```

```
(define (interp an-ae)
  (type-case AE an-ae
    [num (n) n]
    [add (l r) (+ (interp l) (interp r))]
    [sub (l r) (- (interp l) (interp r))]))
```

*No identifiers to help us study binding...*



# With Arithmetic Language

$\langle \text{WAE} \rangle ::= \langle \text{num} \rangle$   
|  $\{ + \langle \text{WAE} \rangle \langle \text{WAE} \rangle \}$   
|  $\{ - \langle \text{WAE} \rangle \langle \text{WAE} \rangle \}$   
|  $\{ \text{with} \{ \langle \text{id} \rangle \langle \text{WAE} \rangle \} \langle \text{WAE} \rangle \}$    
|  $\langle \text{id} \rangle$  

$\{ \text{with} \{ x \{ + 1 2 \} \}$   
 $\{ + x x \} \} \Rightarrow 6$





# With Arithmetic Language

**<WAE>** ::= **<num>**  
| **{ + <WAE> <WAE> }**  
| **{ - <WAE> <WAE> }**  
| **{ with { <id> <WAE> } <WAE> }**   
| **<id>** 

**x** ⇒ *error: free identifier*



# With Arithmetic Language

```
<WAE> ::= <num>
        | { + <WAE> <WAE> }
        | { - <WAE> <WAE> }
        | { with { <id> <WAE> } <WAE> }
        | <id>
```



```
{ + { with { x { + 1 2 } }
      { + x x } }
  { with { x { - 4 3 } }
    { + x x } } } ⇒ 8
```

# With Arithmetic Language

$\langle \text{WAE} \rangle ::= \langle \text{num} \rangle$   
|  $\{ + \langle \text{WAE} \rangle \langle \text{WAE} \rangle \}$   
|  $\{ - \langle \text{WAE} \rangle \langle \text{WAE} \rangle \}$   
|  $\{ \text{with} \{ \langle \text{id} \rangle \langle \text{WAE} \rangle \} \langle \text{WAE} \rangle \}$    
|  $\langle \text{id} \rangle$  

$\{ + \{ \text{with} \{ x \{ + 1 2 \} \}$   
   $\{ + x x \} \}$   
   $\{ \text{with} \{ y \{ - 4 3 \} \}$   
     $\{ + y y \} \} \}$   $\Rightarrow$  8

# With Arithmetic Language

$\langle \text{WAE} \rangle ::= \langle \text{num} \rangle$   
|  $\{ + \langle \text{WAE} \rangle \langle \text{WAE} \rangle \}$   
|  $\{ - \langle \text{WAE} \rangle \langle \text{WAE} \rangle \}$   
|  $\{ \text{with} \{ \langle \text{id} \rangle \langle \text{WAE} \rangle \} \langle \text{WAE} \rangle \}$    
|  $\langle \text{id} \rangle$  

$\{ \text{with} \{ x \{ + 1 2 \} \}$   
   $\{ \text{with} \{ x \{ - 4 3 \} \}$   
     $\{ + x x \} \} \}$   $\Rightarrow$  2

# With Arithmetic Language

```
<WAE> ::= <num>
| { + <WAE> <WAE> }
| { - <WAE> <WAE> }
| { with { <id> <WAE> } <WAE> }
| <id>
```


NEW

NEW

```
{ with { x { + 1 2 } }
  { with { y { - 4 3 } }
    { + x x } } } ⇒ 6
```

# With Arithmetic Language



```
<WAE> ::= <num>
        | { + <WAE> <WAE> }
        | { - <WAE> <WAE> }
        | { with { <id> <WAE> } <WAE> }
        | <id>
```



```
(define-type WAE
  [num (n number?)]
  [add (lhs WAE?)
       (rhs WAE?)]
  [sub (lhs WAE?)
       (rhs WAE?)]
  [with (name symbol?)
        (named-expr WAE?)
        (body WAE?)]
  [id (name symbol?)])
```

## With Arithmetic Language


```
<WAE> ::= <num>
        | { + <WAE> <WAE> }
        | { - <WAE> <WAE> }
        | { with { <id> <WAE> } <WAE> }
        | <id>
```



```
(define (interp a-wae)
  (type-case WAE a-wae
    [num (n) n]
    [add (l r) (+ (interp l) (interp r))]
    [sub (l r) (- (interp l) (interp r))]
    [with (bound-id named-expr body-expr)
      ...]
    [id (name) ...])))
```

## With Arithmetic Language

```
<WAE> ::= <num>
        | { + <WAE> <WAE> }
        | { - <WAE> <WAE> }
        | { with { <id> <WAE> } <WAE> }
        | <id>
```




```
(define (interp a-wae)
  (type-case WAE a-wae
    [num (n) n]
    [add (l r) (+ (interp l) (interp r))]
    [sub (l r) (- (interp l) (interp r))]
    [with (bound-id named-expr body-expr)
      ...]
    [id (name) (error 'interp "free variable")]))
```



## With Arithmetic Language


```
<WAE> ::= <num>
        | { + <WAE> <WAE> }
        | { - <WAE> <WAE> }
        | { with { <id> <WAE> } <WAE> }
        | <id>
```



```
(define (interp a-wae)
  (type-case WAE a-wae
    [num (n) n]
    [add (l r) (+ (interp l) (interp r))]
    [sub (l r) (- (interp l) (interp r))]
    [with (bound-id named-expr body-expr)
      ... (interp named-expr) ...]
    [id (name) (error 'interp "free variable")]))
```

## With Arithmetic Language

```
<WAE> ::= <num>
        | { + <WAE> <WAE> }
        | { - <WAE> <WAE> }
        | { with { <id> <WAE> } <WAE> }
        | <id>
```



```
(define (interp a-wae)
  (type-case WAE a-wae
    [num (n) n]
    [add (l r) (+ (interp l) (interp r))]
    [sub (l r) (- (interp l) (interp r))]
    [with (bound-id named-expr body-expr)
     ... (interp named-expr)
     ... (interp body-expr) ...]
    [id (name) (error 'interp "free variable")]))
```

## With Arithmetic Language

```
<WAE> ::= <num>
        | { + <WAE> <WAE> }
        | { - <WAE> <WAE> }
        | { with { <id> <WAE> } <WAE> }
        | <id>
```

NEW

NEW

```
(define (interp a-wae)
  (type-case WAE a-wae
    [num (n) n]
    [add (l r) (+ (interp l) (interp r))]
    [sub (l r) (- (interp l) (interp r))]
    [with (bound-id named-expr body-expr)
      (interp (subst body-expr
                     bound-id
                     (interp named-expr)))]
    [id (name) (error 'interp "free variable")]))
```

# Substitution

```
; subst : WAE symbol num -> WAE
(define (subst a-wae sub-id val)
  (type-case WAE a-wae
    [num (n) ...]
    [add (l r) ...]
    [sub (l r) ...]
    [with (bound-id named-expr body-expr)
     ...]
    [id (name) ...])))
```

*Let's make examples/tests first...*

## Example Substitutions

`; 10 for x in {+ 1 x} ⇒ {+ 1 10}`

```
(test (subst (add (num 1) (id 'x)) 'x 10)
      (add (num 1) (num 10)))
```

`; 10 for x in x ⇒ 10`

```
(test (subst (id 'x) 'x 10)
      (num 10))
```

`; 10 for x in y ⇒ y`

```
(test (subst (id 'y) 'x 10)
      (id 'y))
```

`; 10 for y in {- x 1} ⇒ {- x 1}`

```
(test (subst (sub (id 'x) (num 1)) 'y 10)
      (sub (id 'x) (num 1)))
```

# Substitution

```
; subst : WAE symbol num -> WAE
(define (subst a-wae sub-id val)
  (type-case WAE a-wae
    [num (n) a-wae]
    [add (l r) (add (subst l sub-id val)
                    (subst r sub-id val))]
    [sub (l r) (sub (subst l sub-id val)
                   (subst r sub-id val))]
    [with (bound-id named-expr body-expr)
          ...]
    [id (name) (if (symbol=? name sub-id)
                  (num val)
                  a-wae))]))
```

## Example Substitutions

```
; 10 for x in {with {y 17} x} ⇒ {with {y 17} 10}
(test (subst (with 'y (num 17) (id 'x)) 'x 10)
      (with 'y (num 17) (num 10)))
```

```
; 10 for x in {with {y x} y} ⇒ {with {y 10} y}
(test (subst (with 'y (id 'x) (id 'y)) 'x 10)
      (with 'y (num 10) (id 'y)))
```

```
; 10 for x in {with {x y} x} ⇒ {with {x y} x}
(test (subst (with 'x (id 'y) (id 'x)) 'x 10)
      (with 'x (id 'y) (id 'x)))
```

Substitution replaces

- free identifiers with the same name
- no binding identifiers
- no bound identifiers

# Substitution

An identifier is bound when it appears in the body of a **with** binding the same name

Conversely, a free variable of a name appears in a **with** only if the **with** doesn't bind the name

```
; subst : WAE symbol num -> WAE
(define (subst a-wae sub-id val)
  (type-case WAE a-wae
    ...
    [with (bound-id named-expr body-expr)
      (with bound-id
        (subst named-expr sub-id val)
        (if (symbol=? bound-id sub-id)
            body-expr
            (subst body-expr sub-id val))))])
    ...))
```



## HW 2

Implement

```
; free-ids : WAE -> list-of-sym
```

```
; binding-ids : WAE -> list-of-sym
```

```
; bound-ids : WAE -> list-of-sym
```