CS6931 Database Seminar

Lecture 2: External Memory Indexing Structures

External Memory Data Structures

- Names:
  - I/O-efficient data structures
  - Disk-based data structures (index structures) used in DB
  - Disk-resilient data structures (index structures) used in DB
  - Secondary indexes used in DB
- Other Data structures

Mainly used in algorithms

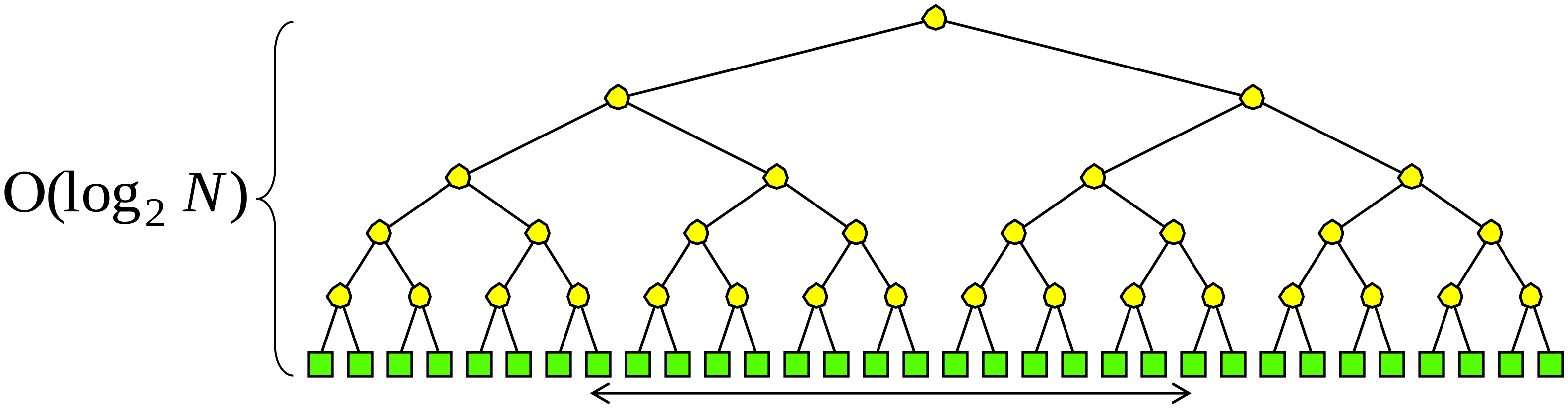
  - Queue, stack
    - $O(N/B)$ space, $O(1/B)$ push, $O(1/B)$ pop
  - Priority queue
    - $O(N/B)$ space, $O(1/B \cdot \log_{M/B} N/B)$ insert, delete-max

External Memory Data Structures

- General-purpose data structures
  - Space: linear or near-linear (very important)
  - Query: logarithmic in B or 2 for any query (very important)
  - Update: logarithmic in B or 2 (important)
- In some sense, more useful than I/O-algorithms
  - Structure stored in disk most of the time
  - DB typically maintains many data structures for many different data sets: can’t load all of them to memory
  - Nearly all index structures in large DB are disk based

External Search Trees

- *Binary search tree:*
  - *Standard method for search among N elements*
  - *We assume elements in leaves*

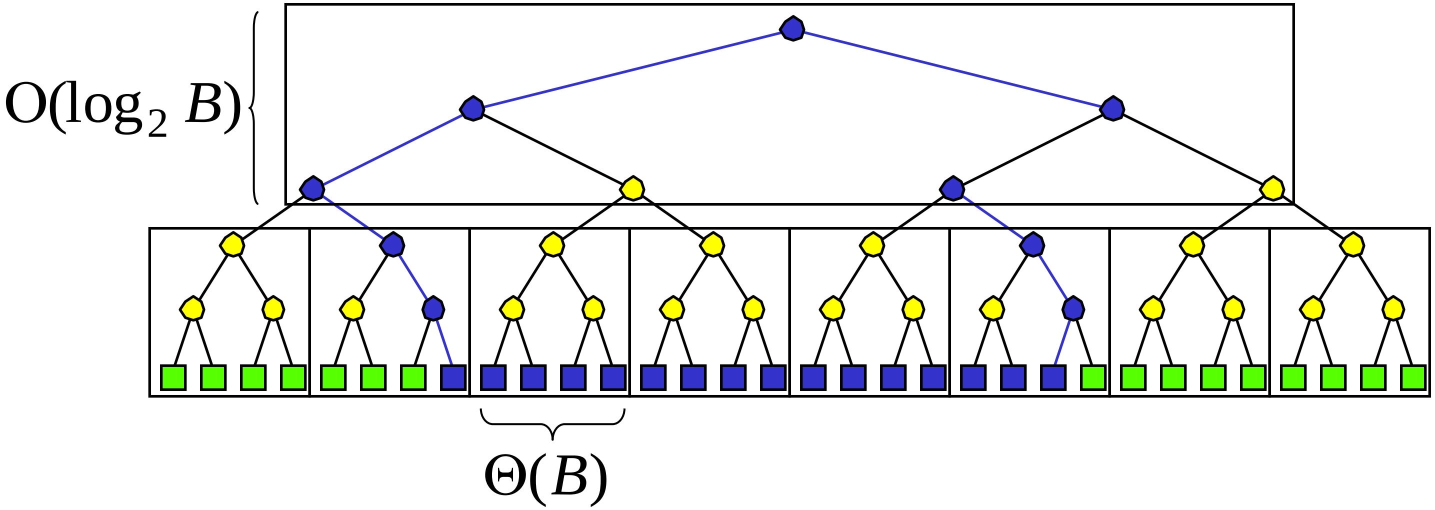


  - *Search traces at least one root-leaf path*
  - *If nodes stored arbitrarily on disk*

    $\Rightarrow$ *Search in* $O(\log_2 N)$ *I/Os*

    $\Rightarrow$ *Rangesearch in* $O(\log_2 N + T)$ *I/Os*

External Search Trees



- *Bottom-up BFS blocking:*
  - *Block height* $O(\log_2 N) / O(\log_2 B) = O(\log_B N)$
  - *Output elements blocked*

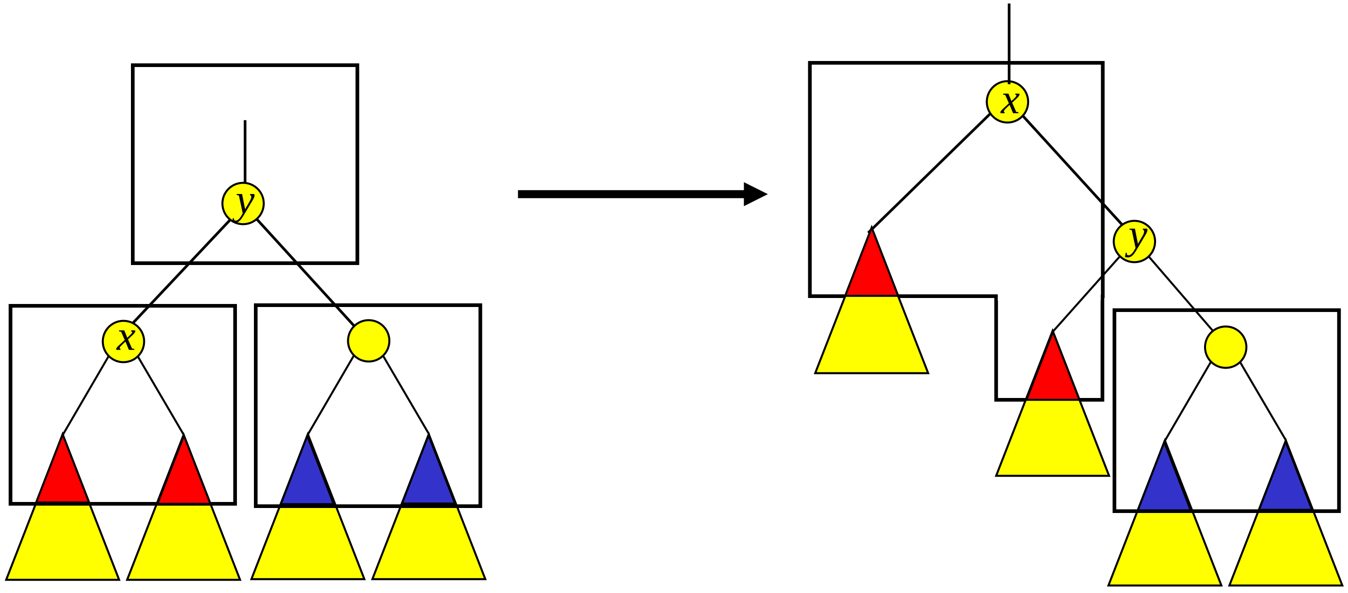
$\Downarrow$

Range query in $O(\log_B N + T/B)$ *I/Os*

- *Optimal: O(N/B) space and* $O(\log_B N + T/B)$ *query*

External Search Trees

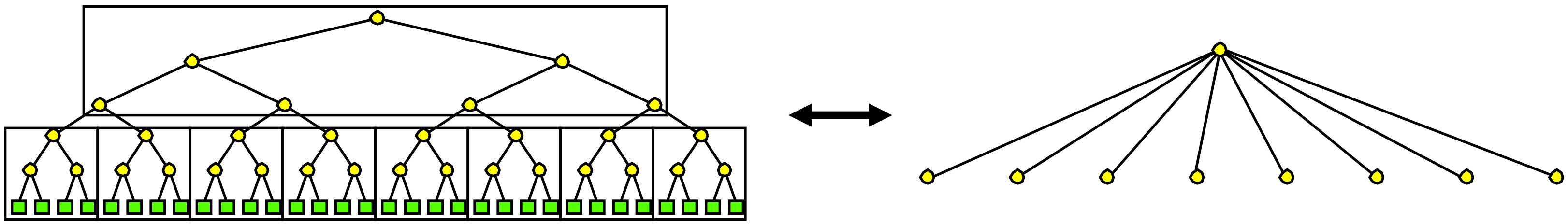
- Maintaining BFS blocking during updates?
  - Balance normally maintained in search trees using rotations



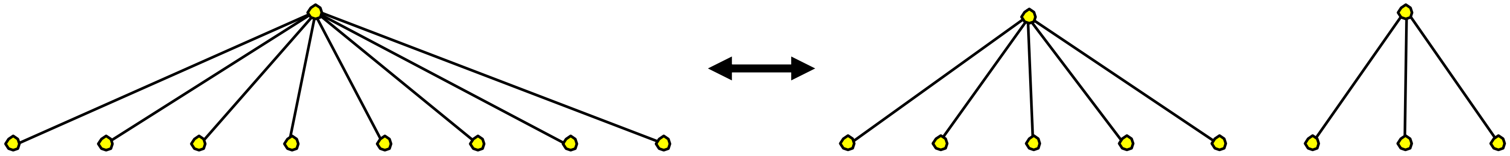
- Seems very difficult to maintain BFS blocking during rotation
  - Also need to make sure output (leaves) is blocked!

B-trees

- BFS-blocking naturally corresponds to tree with fan-out $\Theta(B)$

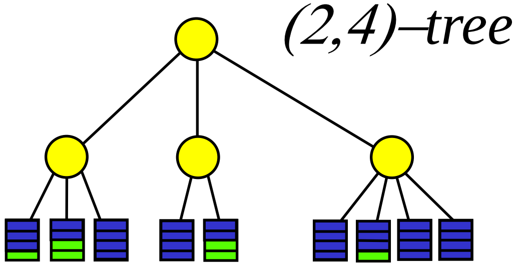


- B-trees balanced by allowing node degree to vary
  - Rebalancing performed by splitting and merging nodes



(a,b)-tree

- T is an (a,b)-tree ($a \geq 2$ and $b \geq 2a-1$)
  - All leaves on the same level (contain between a and b elements)
  - Except for the root, all nodes have degree between a and b
  - Root has degree between 2 and b



- *(a,b)-tree uses linear space and has height* $O(\log_a N)$

$\Downarrow$

Choosing a,b = $\Theta(B)$ *each node/leaf stored in one disk block*

$\Downarrow$

O(N/B) space and $O(\log_B N + T/B)$ *query*

(*a*,*b*)-Tree Insert

- Insert:

Search and insert element in leaf *v*

DO *v* has *b+1* elements/children

Split *v*:

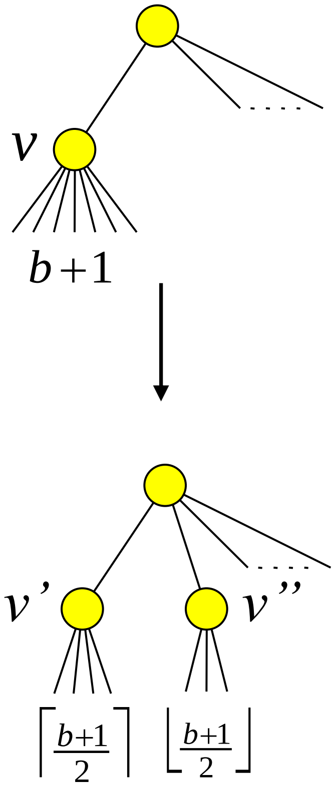
make nodes *v'* and *v''* with

$\left\lceil \frac{b+1}{2} \right\rceil \leq b$ and $\left\lfloor \frac{b+1}{2} \right\rfloor \geq a$ elements

insert element (ref) in *parent(v)*

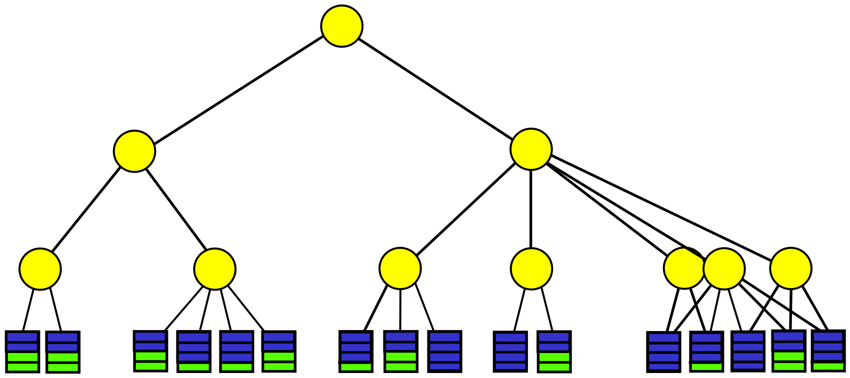
(make new root if necessary)

v=parent(v)



- Insert touch $O(\log_a N)$ nodes

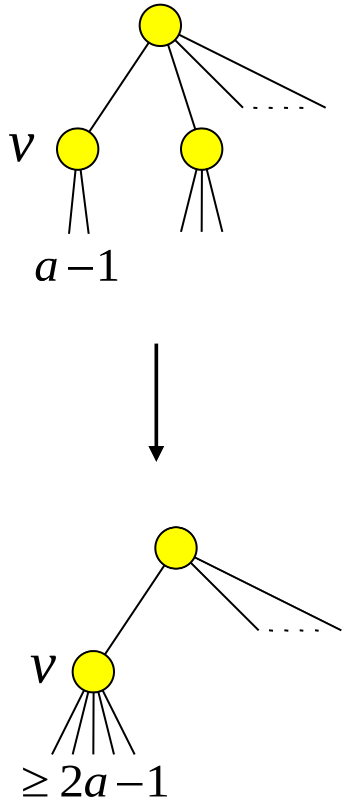
(a,b)-Tree Insert



(a,b)-Tree Delete

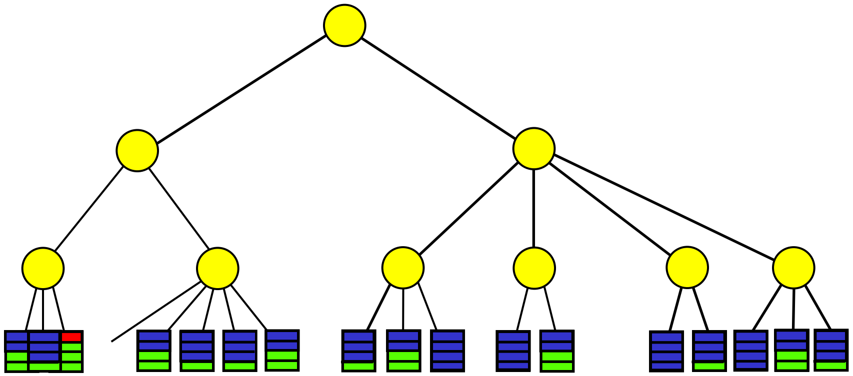
- Delete:

Search and delete element from leaf v
DO v has a-1 elements/children
  Fuse v with sibling v':
    move children of v' to v
    delete element (ref) from $parent(v)$
    (delete root if necessary)
  If v has $>b$ (and $\leq a+b-1<2b$) children split v
$v=parent(v)$

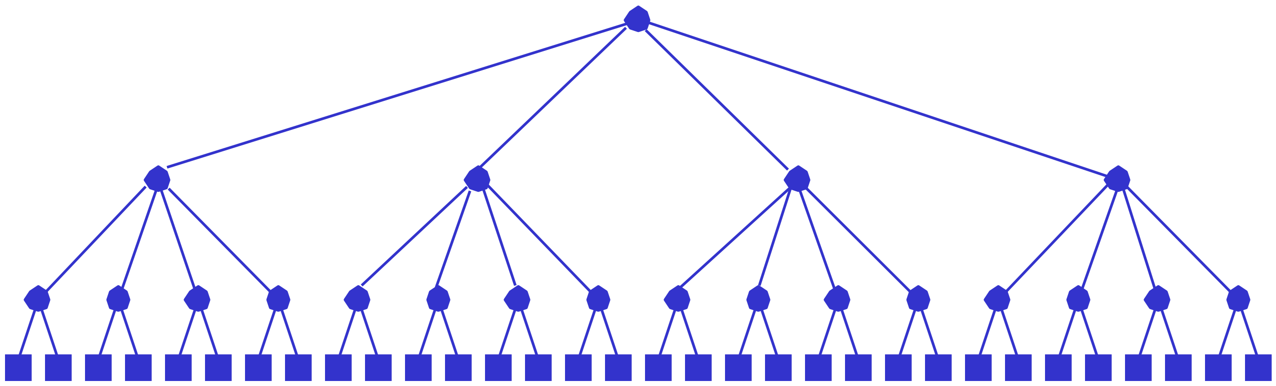


- Delete touch $O(\log_a N)$ nodes

(*a*,*b*)-Tree Delete



External Searching: B-Tree



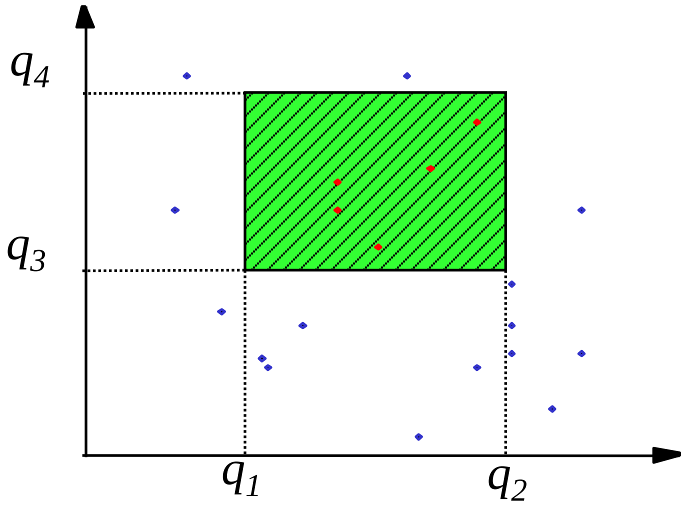
- Each node (except root) has fan-out between $B/2$ and B
- Size: $O(N/B)$ blocks on disk
- Search: $O(\log_B N)$ I/Os following a root-to-leaf path
- Insertion and deletion: $O(\log_B N)$ I/Os

Summary/Conclusion: B-tree

- B-trees: (a,b)-trees with $a,b = \Theta(B)$
  - $O(N/B)$ space
  - $O(\log_B N + T/B)$ query
  - $O(\log_B N)$ update

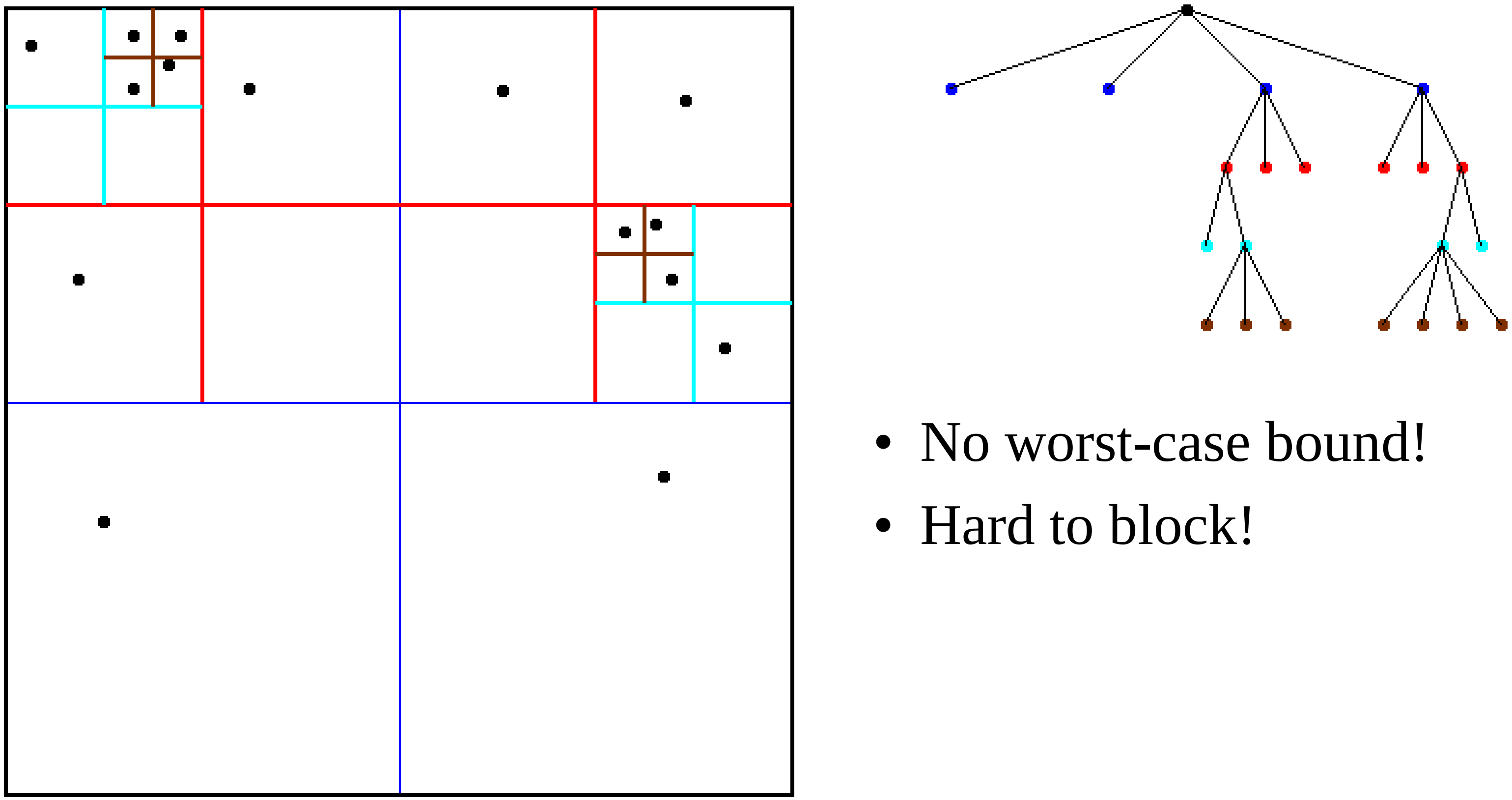
- B-trees with elements in the leaves sometimes called B^+-tree
  - Now B-tree and B^+tree are synonyms
- Construction in $O(\frac{N}{B} \log_{M/B} \frac{N}{B})$ I/Os
  - Sort elements and construct leaves
  - Build tree level-by-level bottom-up

2D Range Searching



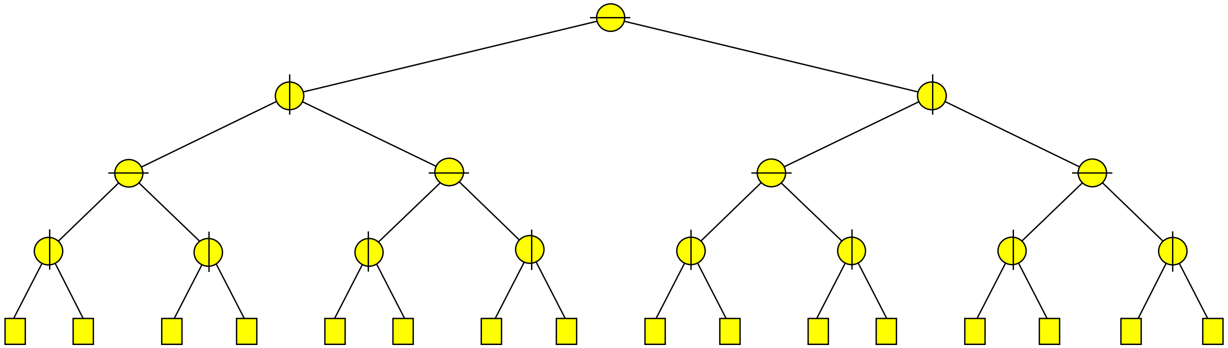
Quadtree

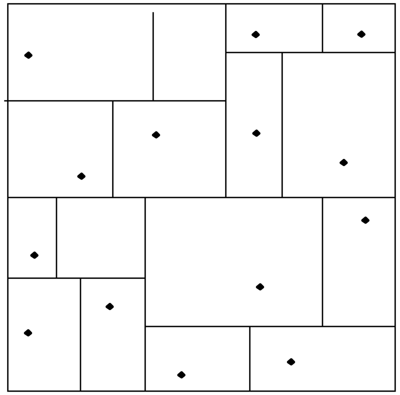
Adaptive quadtree where no square contains more than 1 particle



- No worst-case bound!
- Hard to block!

kd-tree





- kd-tree:
  - Recursive subdivision of point-set into two half using vertical/horizontal line
  - Horizontal line on even levels, vertical on uneven levels
  - One point in each leaf

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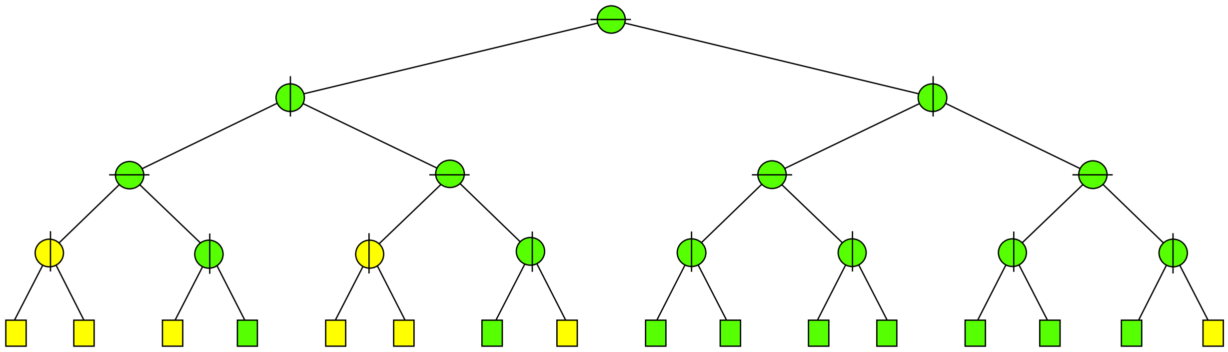
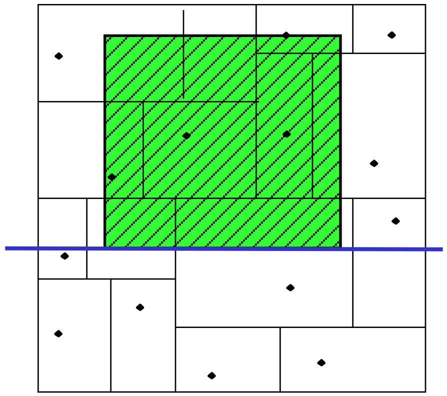
Linear space and logarithmic height

kd-Tree: Query

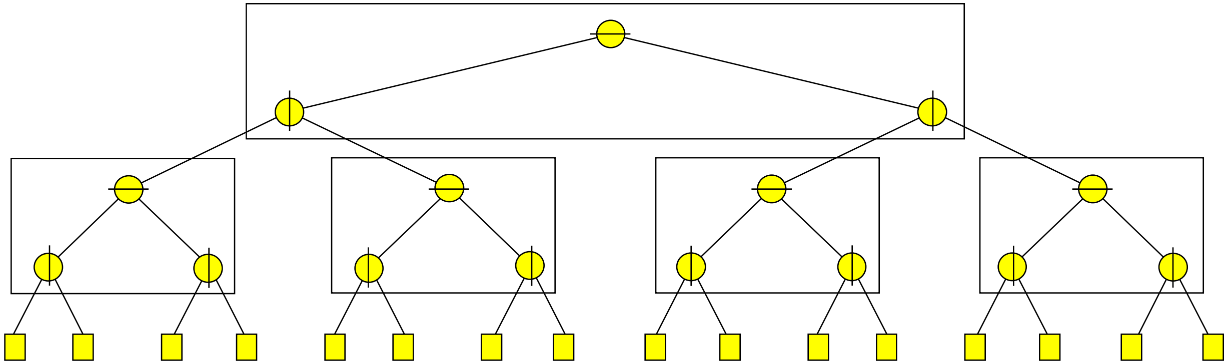


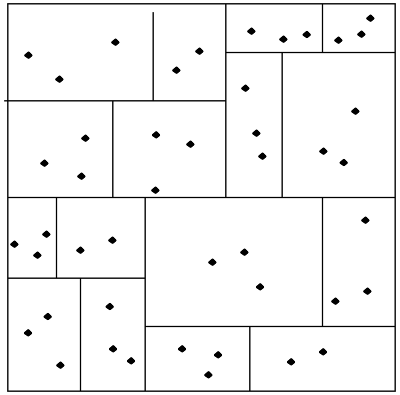


- Query
  - Recursively visit nodes corresponding to regions intersecting query
  - Report point in trees/nodes completely contained in query
- Query analysis
  - Horizontal line intersect $Q(N) = 2+2Q(N/4) = O(\sqrt{N})$ regions
  - Query covers T regions

$\Rightarrow O(\sqrt{N}+T)$ I/Os worst-case

kdB-tree





- kdB-tree:
  - Bottom-up BFS blocking
  - Same as B-tree
- Query as before
  - Analysis as before but each region now contains $\Theta(B)$ points

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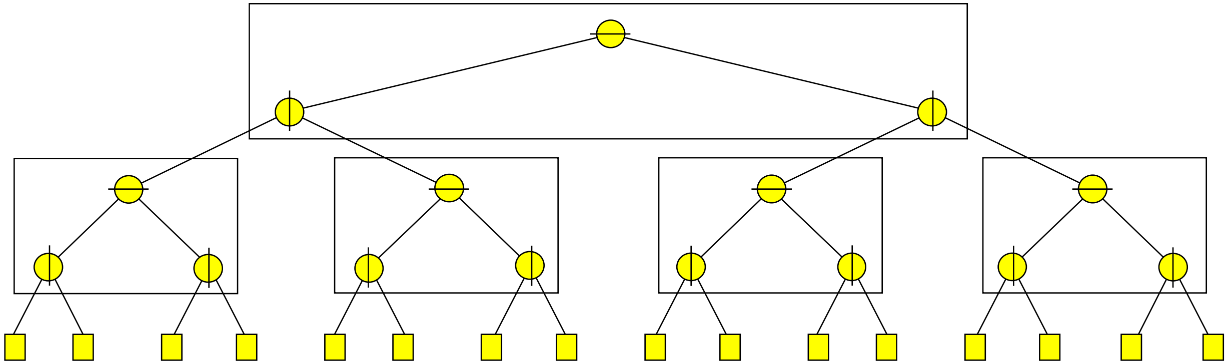
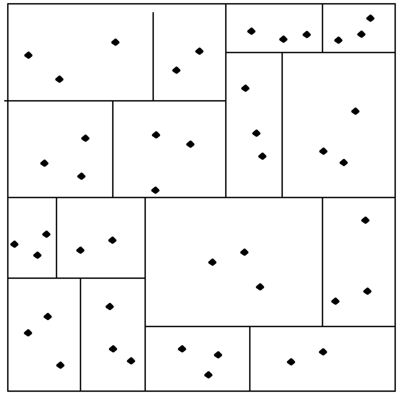
$O(\sqrt{N/B} + T/B)$ I/O query

Construction of kdB-tree

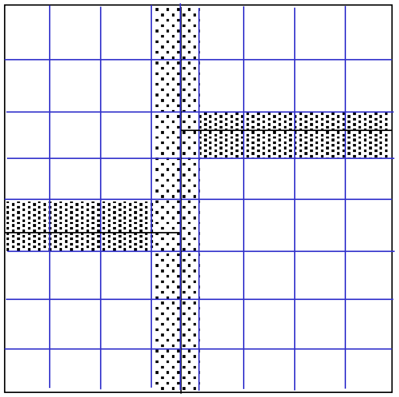




- Simple $O(\frac{N}{B}\log_2\frac{N}{B})$ algorithm
  - Find median of *y*-coordinates (construct root)
  - Distribute point based on median
  - Recursively build subtrees
  - Construct BFS-blocking top-down (can compute the height in advance)
- Idea in improved $O(\frac{N}{B}\log_{M/B}\frac{N}{B})$ algorithm
  - Construct $\log\sqrt{M/B}$ levels at a time using $O(N/B)$ I/Os

Construction of kdB-tree

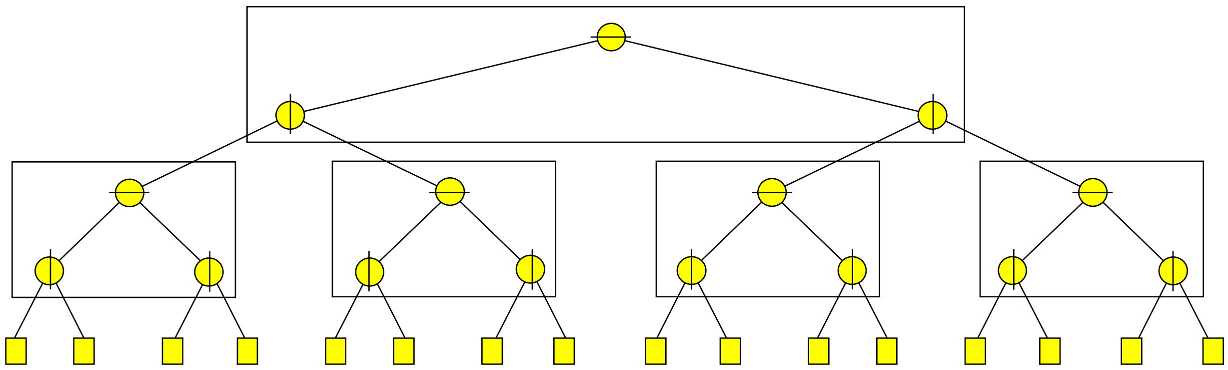
- Sort N points by x- and by y-coordinates using $O(\frac{N}{B}\log_{M/B}\frac{N}{B})$ I/Os
- Building $\log\sqrt{M/B}$ levels ($\sqrt{M/B}$ nodes) in $O(N/B)$ I/Os:
  1. Construct $\sqrt{M/B}$ by $\sqrt{M/B}$ grid with $N/\sqrt{M/B}$ points in each slab
  2. Count number of points in each grid cell and store in memory
  3. Find slab s with median x-coordinate
  4. Scan slab s to find median x-coordinate and construct node
  5. Split slab containing median x-coordinate and update counts
  6. Recurse on each side of median x-coordinate using grid (step 3)

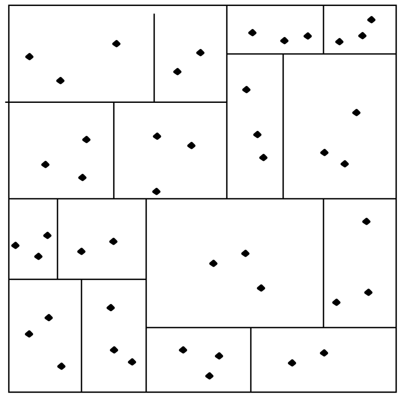


$\Rightarrow$ Grid grows to $M/B + \sqrt{M/B}\cdot\sqrt{M/B} = \Theta(M/B)$ during algorithm

$\Rightarrow$ Each node constructed in $O(N/(\sqrt{M/B}\cdot B))$ I/Os

kdB-tree





- kdB-tree:
  - Linear space
  - Query in $O(\sqrt{N/B} + T/B)$ I/Os
  - Construction in O(sort(N)) I/Os
  - Height $O(\log_B N)$
- Dynamic?
  - Difficult to do splits/merges or rotations …