Problem 1. [64pts]
Consider the following relational database that stores information about the business of a set of bars:

Drinker (name, address)
Bar (name, address)
Beer (name, brewer)
Frequents (drinker, bar, times) /
Likes (drinker, beer)
Serves (bar, beer, price)

Write the following queries in relational algebra:
(a) Find all drinkers who frequent James Joyce Pub.
(b) Find all bars that serve both Amstel and Corona.
(c) Find all bars that serve at least one of the beers Amy likes for no more than $2.50.
(d) For each bar, find all beers served at this bar that are liked by none of the drinkers who frequent that bar.
(e) Find all drinkers who frequent only those bars that serve some beers they like.
(f) Find all drinkers who frequent every bar that serves some beers they like.
(g) Find those drinkers who enjoy exactly the same set of beers as Amy.
(h) For each beer, find the bars that serve it at the lowest price.

Problem 2. [20pts]
As discussed in class, the core operators in relational algebra are selection ($\sigma$), projection ($\pi$), cross product ($\times$), union ($\bigcup$), and difference ($-$). Show that the selection operator is necessary; that is, some queries that use the selection operator cannot be expressed using any combination of the other core operators.

Problem 3. [16pts]
Consider the following two tables, T1 and T2:

<table>
<thead>
<tr>
<th>T1</th>
<th>T2</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>A</td>
</tr>
<tr>
<td>Q</td>
<td>B</td>
</tr>
<tr>
<td>R</td>
<td>C</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>15</td>
<td>25</td>
</tr>
<tr>
<td>25</td>
<td>10</td>
</tr>
</tbody>
</table>

Show the results of the following relational algebra queries:
1. $T_1 \bowtie_{T_1.P=T_2.A} T_2$
2. $T_1 \bowtie_{T_1.Q=T_2.B} T_2$
3. $T_1 \bowtie T_2$ (assume the natural join happens for columns P and A)
4. $T_1 \bowtie_{T_1.P=T_2.A \text{ AND } T_1.R=T_2.C} T_2$