Abstract
The intergovernmental panel on climate change (IPCC) has estimated between 1.3 and 5.8 m sea level rise over the next hundred years. Of this, only 0.1 to 0.4 m is attributed to the largest ice masses on the planet, the Antarctic and Greenland ice sheets. Over the last decade, dramatic activity in the outlet glaciers of Greenland and the Antarctic Peninsula raise the possibility that these large ice sheets will have a much greater contribution to sea level rise over the next century than was predicted by the IPCC. Recent studies have shown these areas are exhibiting significant scale changes in response to climate forcings, whereas IPCC models show that ice is not responsive to climate change over such short periods of time. Many believe the IPCC type models fail to show short term climate responses due to the simplifications they make to ice sheet mechanics. Here, we develop a higher order model — a new ice sheet model which contains all relevant flow physics. In addition, we use a rescaled coordinate system to simplify domain geometry, adding simplicity, flexibility, and robustness to the model. Our hope is that this scheme allows us to recreate sea level change and reevaluate the estimations made by the IPCC. Our immediate focus is validation and verification of our model around a structured set of experiments.

Proper resolution of the decadal responses of continental ice sheets to climate forcings requires a model incorporating both fluid Stokes flow physics and unstructured meshes. Such a model can be produced with finite-element methods and a rescaled coordinate system.

1. Introduction

Background
In recent years, attention has been drawn to the Earth’s changing climate and the potential impact such a change would have on its inhabitants. Of noteworthy concern is the possibility of sea level rise, where a few additional meters of water would displace millions of people worldwide. One possible contributor to sea level rise is melts from ice in mountain glaciers and the large ice masses of Greenland and Antarctica. Reacting to global concerns, the Intergovernmental Panel on Climate Change (IPCC) related a climate change event which included an estimation of sea level rise over the next hundred years.

However, recent dramatic activity in the outlet glaciers of Greenland and the Antarctic Peninsula raises the possibility that these large ice sheets will have a much greater contribution to sea level rise over the next century than was predicted by the IPCC. Studies have shown that these areas exhibit changes over much smaller time scales than previously assumed, responding to climate forcings on a decadal scale. The models used by IPCC are generally believed to not account for this recently discovered phenomena due to their simplification of ice sheet mechanics.

![Figure 1: IPCC estimation for sea level rise over the next century.](image)

In response to the shortcomings of existing solutions, we developed a higher order ice sheet model, one that contains all the relevant flow physics. In addition, our model uses a rescaled coordinate system, which provides several advantages discussed in section 5. Ultimately, our goal is to revisit the IPCC’s findings and reevaluate the contribution of glaciers to sea level rise.

2. Hypothesis

Proper resolution of the decadal responses of continental ice sheets to climate forcings requires a model incorporating both fluid Stokes flow physics and unstructured meshes. Such a model can be produced with finite-element methods and a rescaled coordinate system.

![Figure 2: An example of the unstructured grid used in our model. Notice the simplicity and clarity present in the mesh from the rescaled coordinate system.](image)

3. Methods

Numerics
A higher-order ice sheet model is one that incorporates all relevant stresses, including longitudinal stress gradients. By treating ice as an incompressible fluid with constant density, we can write the equations for conservation of mass and momentum as:

\[ \nabla \cdot \mathbf{u} = 0, \]  
\[ \rho \mathbf{u} = -\nabla p + \mathbf{\tau}, \]  

where \( \rho \) is the ice density, \( g \) gravitational acceleration, \( \mathbf{u} \) the velocity vector, and \( \mathbf{\tau} \) the stress tensor.

Furthermore, substituting the deviatoric stress tensor \( \mathbf{\tau} \) in our model results in:

\[ \nabla \cdot \mathbf{u} = 0, \]  
\[ \rho \mathbf{u} = -\nabla p + \mathbf{\tau}, \]  

taking into account all relevant terms. Lower order ice sheet models use only the terms above shown in blue. Multi-scale higher order models only include some of the additional numerics, whereas our model includes every term.

4. Results

Coordinate Transformation
For numerical convenience and to simplify the underlying mesh used by the finite element solver, this model works on a transformed domain. Rather than explicitly defining the domain, a 1x1 square (in the 2D case) is used and a coordinate transformation captures the true geometry. The transformation, shown here in two dimensions, but easily extended to three, maps \( (x, y) \) to \( (x', y') \) where:

\[ x' = 1 - (1 - 1000 + 500 \frac{x}{L}), \]  
\[ y' = 1 - \frac{y}{L}, \]  

where the vertical coordinate \( y' \) is defined such that \( y' = 0 \) at the surface and \( y' = 0 \) at the bed.\( [4] \)

![Figure 3: The rescaling reduces the geometry to a uniform shape and size. In this three dimensional example, the test domain on the left is reduced to a 1x1x1 cube on the right.](image)

In addition, the function derivatives must also be transformed to reflect the change in coordinates. Using the chain rule, the derivative for \( x \) (2D) can be found similarly to:

\[ \frac{\partial f}{\partial x} = \nabla f \cdot \mathbf{\tau} = \left( \frac{\partial f}{\partial x'} \frac{\partial x'}{\partial x} \right)^2 + \left( \frac{\partial f}{\partial y'} \frac{\partial y'}{\partial x} \right)^2 \]  

Taking this into account, we can write:

\[ \frac{\partial f}{\partial x} = \frac{\partial f}{\partial x'} \frac{\partial x'}{\partial x} + \frac{\partial f}{\partial y'} \frac{\partial y'}{\partial x} \]  

Next, we introduce a convenience variable \( \alpha \). After deriving \( \partial f/\partial x \) and reordering terms using algebraic manipulation, we have:

\[ \alpha_x = -\frac{1}{1 + \frac{\partial y}{\partial x}} \]  

The complete form of the first order derivative of \( x \) can be found using substitution into equation 7.

Using a rescaled coordinate system provides simplicity and uniformity to all experiments. With this scheme, there is no need to recreate the true geometry in order to run a simulation. Instead, every simulation uses the same geometry — a unit square or unit cube, in two and three dimensions, respectfully. Further, this domain consistency across models eliminates the need to treat the model’s geometry for each experiment you are interested in.

![Figure 4: Results from ISMIP-HOM experiment B.](image)

We are confident that the core of our model is working properly, though we need more time to fully understand the impact the “higher-order” terms have on the results. Experiment B involves ice flow over a simple topography where the surface and bed are linear and parallel. Again, periodic boundary conditions exist at the edges and the surface is an open boundary. What is unique about this experiment is that the ice is allowed to slide over the bed. The basal friction field is defined as:

\[ f_b = 100 \frac{1}{1 + 1000 \frac{x}{L}}, \]  

where a high value for \( f_b \) corresponds to a large amount of friction between the ice and bedrock.

![Figure 5: Results from ISMIP-HOM experiment D.](image)

5. Conclusions and Future Work

Our results demonstrate that our model is producing accurate results over a range of different scenarios. We are confident that the core of our model is working properly, though we have a few anomalies that need to be reexamined. The remainder of our work will focus on improving sliding and free surface evolution.

While our initial results are promising, there is still a great deal of work to be done. Currently, we have only implemented steady state experiments, where the ice doesn’t evolve over time. In order to do that, our model must be thermodynamically coupled, work we are currently pursuing.

References