Introduction to Parallel Algorithm Analysis

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C. A. Petri [1962] introduced analysis model for concurrent systems.

- Flow chart
- Described data flow and dependencies.
- Very low level (we want something more high-level)
- Reachability EXP-SPACE-HARD, Decidable
Critical Regions Problem

Edsger Dijkstra [1965]

- Mutex: “Mutual exclusion” of variable
- Semaphores: Locks/Unlocks access to (multiple) data.
- Semaphore more general - keeps a count. Mutex binary.
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Important, but lower level details.
Amdahl’s and Gustafson’s Laws

**Amdahl’s Law**: Gene Amdahl [1967]
- Small portion (fraction $\alpha$) non-parallelizable
- Limits max speed-up $S = 1/\alpha$.

**Gustafson’s Law**: Gustafson+Barsis [1988]
- Small portion (fraction $\alpha$) non-parallelizable
- $P$ processors
- Limits max speed-up $S(P) = P - \alpha(P - 1)$. 

\[
S = \frac{T_{\text{seq}}}{T_{\text{par}}}
\]

\[
S(P) = \frac{T_{\text{seq}}}{T_{\text{par}}(P)}
\]
Leslie Lamport [1978]

- Posed parallel problems as finite state machine
- Preserved (only) partial order: “happens before” mutex
Logical Clocks

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Highlights nuances and difficulties in clock synchronization.
DAG Model

Directed Acyclic Graph:

- Each node represents a chunk of computation that is to be done on a single processor.
- Directed edges indicate that the from node must be completed before the to node.
- The longest path in the DAG represents the total amount of parallel time of the algorithm.
- The width of the DAG indicates the number of processors that can be used at once.
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- $P$ processors which operate on a shared data
- For each processor read, write, op (e.g. $+$, $-$, $\times$) constant time.
PRAM Model

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Message Passing Model

Emphasizes Locality

- **send**\((X, i)\) : sends \(X\) to \(P_i\)
- **receive**\((Y, j)\) : receives \(Y\) from \(P_j\)
- Fixed topology, can only **send/receive** from neighbor
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Common Topologies:

- Array/Ring Topology
  - ●
- Mesh Topology
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- Hypercube Topology
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- Hypercube Topology
  - \( (\Omega(\log p)) \) rounds
Programming in MPI

Open MPI:

- (Open Source High Performance Computing).
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When to use MPI?

- Critical to exploit locality (i.e. scientific simulations)
- Complication in only talking to neighbor
Les Valiant [1989] BSP
Creates “barriers” in parallel algorithm.

1. Each processor computes in data
2. Processors send/receive data
3. Barrier: All processors wait for communication to end globally

Allows for easy synchronization. Easier to analysis since handles many messy synchronization details if this is emulated.
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Many parameters:

- $P$: number of processors
- $M$: Memory/Cache Size
- $B$: Block Size/Cost
- $L$: Synchronization Costs

Argues: any portable and efficient parallel algorithm, must take into account all of these parameters.
Bridging Model for Multi-Core Computing


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- Analyzes all levels of architecture together
- Like Cache-Oblivious, but not oblivious
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At depth $d$ uses parameters:
$\bigcup_i (p_i, g_i, L_i, m_i)$
- $p_i$: number of subcomponents (processors at leaf)
- $g_i$: communication bandwidth (e.g. I/O cost)
- $L_i$: synchronization cost
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Matrix Multiplication, Fast Fourier Transform, Sorting
Two types of programmers

1. Wants to optimize the heck out of everything, tune all parameters
2. Wants to get something working, not willing to work too hard
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MapReduce

Each Processor has full hard drive, data items \( \langle \text{KEY}, \text{VALUE} \rangle \). Parallelism Proceeds in Rounds:

- Map: assigns items to processor by KEY.
- Reduce: processes all items using VALUE. Usually combines many items with same KEY.

Repeat M+R a constant number of times, often only one round.

- Optional post-processing step.

Pro: Robust (duplication) and simple. Can harness Locality

Con: Somewhat restrictive model
General Purpose GPU

Massive parallelism on your desktop. Uses Graphics Processing Unit. Designed for efficient video rasterizing. Each processor corresponds to pixel $p$

- depth buffer:
  $$D(p) = \min_i \|x - w_i\|$$
- color buffer: $C(p) = \sum_i \alpha_i \chi_i$
- ...

Pro: Fine grain, massive parallelism. Cheap.
Con: Somewhat restrictive model. Small memory.
Google Sawzall?

▶ Compute statistics on massive distributed data.
▶ Separates local computation from aggregation.
... and Beyond

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Processing in memory?
▶ GPU has large cost in transferring data
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Massive, Unorganized, Distributed Computing
- Bit-Torrent (distributed hash tables)
- SETI @ Home
- sensor networks