CS7960 L13 : Parallel | (Prefix) Sum

PRAM

1 disk
P processors
n input items

Each time step a processor can:
read, write, operate (+,-,*,<<,...)

shared memory: CRCW (although CREW more realistic)

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Sum (n):
INPUT A = [a_1, a_2, ..., a_n]

Sequential? O(n)

PRAM: O(n/p + log n)

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for i=1 to n PARDO
   B(0,i) := A(i)

for h = 1 to log n DO
   for i=1 to n/2^h PARDO
      B(h,i) := B(h-1,2i-1) + B(h-1,2i)

return B(log n, 1)
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(log n) rounds:
A=B0 = 7 4 2 5 1 4 9 2
B1 = 11 7 5 11
B2 = 18 16
B3 = 34

O(n) work, O(log n) time

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PRAM = BSP

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Prefix Sum
INPUT A = \([a_1, a_2, \ldots, a_n]\)
OUTPUT B = \([a_1, a_1+a_2, a_1+a_2+a_3, \ldots]\)
\[b_i = \text{sum}_{j=1}^{i} a_i\]

Sequential?  O(n)


for i=1 to n PARDO
    \(B(0,i) := A(i)\)
for h = 1 to \(\log n\) DO
    for i=1 to \(n/2^h\) PARDO
        \(B(h,i) := B(h-1,2i-1) + B(h-1,2i)\)
for h = \(\log n\) to 0 DO
    for i=1 to \(n/2^h\), even PARDO
        \(C(h,i) := C(h+1,i/2)\)
    \(C(h,1) := B(h,1)\)
    for i=3 to \(n/2^h\), odd PARDO
        \(C(h,i) := C(h+1, (i-1)/2) + B(h,i)\)
Output C  (PAROUT)

Builds sum, then distributes back down.

log \(n\) rounds up, log \(n\) rounds down.

(log \(n\)) rounds:
\(A=\emptyset\) = 7 4 2 5 1 4 9 2
\(B1\) = 11 7 5 11
\(B2\) = 18 16
\(B3\) = 34
\(C3\) = 34
\(C2\) = 18 34
\(C1\) = 11 18 23 34
\(C0\) = 7 11 13 18 19 23 32 34

O(n) work, O(log n) time