

Prob Stats LISh

# Hypothesis Testing

April 11, 2023

# Hypothesis Testing part of Statistics

$X_1, X_2, \dots, X_n \stackrel{iid}{\sim} f(\theta)$

use  $X_1, \dots, X_n$  to study  $f(\theta)$

## Hypothesis

null hypo.  $H_0$ : "baseline", standard status quo  
specifies exactly  $\theta_0$

alternative hypo.  $H_1$ : new claim  
not fully specified  $\theta > \theta_0$  interesting if true

# Hypothesis Testing

Step 1 : Hypothesis Formulation

null  $H_0$  , alternative  $H_1$ .

Step 2 : Design Experiment

think about Random Variables  
 $x_1, x_2, \dots, x_n$

Step 3 : Run Experiment

collect data  $x_1, x_2, \dots, x_n$

consider  $H_0, H_1$  hypo

Example

Height

Step 1

Alternative Hypothesis that Utah population is taller than average.

$$H_1: \mu_{Utah} > 67$$

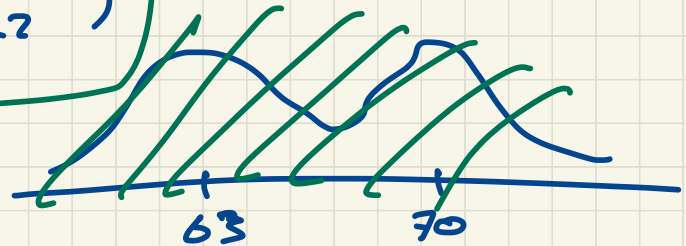
Null

People of Utah same height as everywhere else

get average height of U.S. 67 inches.

$$H_0: N(\underbrace{67}_{\mu}, \underbrace{25}_{\sigma^2})$$

std dev 5 inches.



## Step 2

## Design Experiment

$$H_0: N(\mu, \sigma^2)$$

random sample  $X_1, X_2, \dots, X_n$  RVs

test statistic  $T = T(X_1, \dots, X_n)$   
 $= \frac{1}{n} \sum_{i=1}^n X_i$

confidence level  $(1-\alpha)$  (e.g.  $\alpha = 0.05$ )

critical value at  $\alpha$

$t_\alpha$

$X_1, \dots, X_n \sim H_0$

$$X_i \sim N(\mu, \sigma^2)$$

$$\bar{X}_n \sim N(\mu, \sigma^2/n)$$

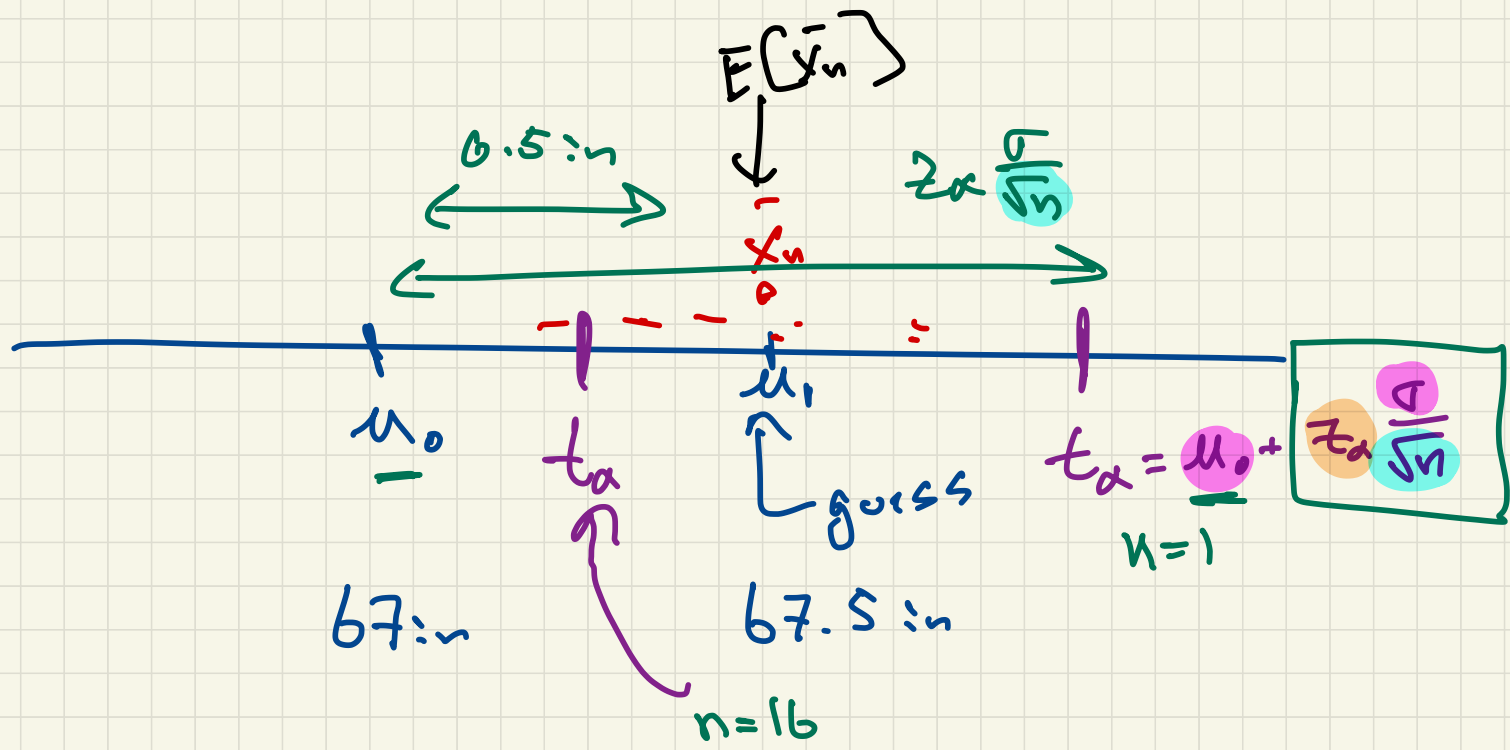
$$Z_n = \frac{\bar{X}_n - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$$

$$P_c(T \leq t_\alpha) = 1 - \alpha$$

$T \sim$

$$P_c(Z_n \leq z_\alpha) = 1 - \alpha$$

$$t_\alpha = \mu + z_\alpha \frac{\sigma}{\sqrt{n}}$$



$$P(\bar{y}_n > t_\alpha) = 1 - \alpha$$

### Step 3 Run Experiment

(lower case)

realization of R.V.  $x_1, x_2, \dots, x_n$

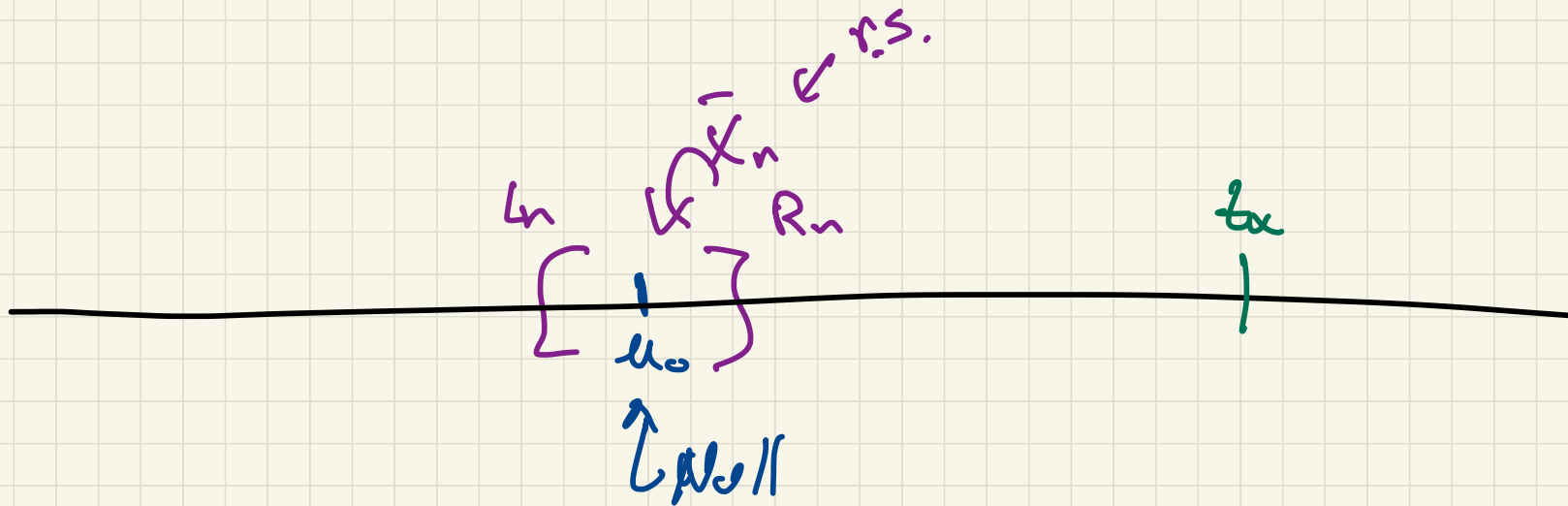
realization of test stat:  $t = T(x_1, x_2, \dots, x_n) = \bar{x}_n = \frac{1}{n} \sum_{i=1}^n x_i$

• if  $t > t_\alpha$  "reject the null hypothesis"

evidence in favor of  $(1-\alpha)\%$  confidence  
that the alternative hypo.  
more likely than null hypo.

• if  $t \leq t_\alpha$  "do not reject null hypothesis"  
does not confirm null hypothesis

$$P_c(L_n \leq \bar{x}_n \leq R_n) = 1 - \alpha$$





P-value

calculator

realization

of test  
statistic  $t$

$$P_r(T \leq t) = 1 - P$$

RV. test stat  $\sim N_0$

Solve for value  $P$