Introduction to Parallel Algorithm Analysis

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C. A. Petri [1962] introduced analysis model for concurrent systems.

- Flow chart
- Described data flow and dependencies.
- Very low level (we want something more high-level)
- Reachability EXP-SPACE-HARD, Decidable
Critical Regions Problem

Edsger Dijkstra [1965]

- Mutex: “Mutual exclusion” of variable
- Semaphores: Locks/Unlocks access to (multiple) data.
- Semaphore more general - keeps a count. Mutex binary.
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Important, but lower level details.
Amdahl’s and Gustafson’s Laws

**Amdahl’s Law:** Gene Amdahl [1967]
- Small portion (fraction $\alpha$) non-parallelizable
- Limits max speed-up
  \[ S = \frac{1}{\alpha}. \]

**Gustafson’s Law:** Gustafson+Barsis [1988]
- Small portion (fraction $\alpha$) non-parallelizable
- $P$ processors
- Limits max speed-up
  \[ S(P) = P - \alpha(P - 1). \]

\[
S = \frac{T_{seq}}{T_{par}}
\]

\[
S(P) = \frac{T_{seq}}{T_{par}(P)}
\]
Logical Clocks

Leslie Lamport [1978]
- Posed parallel problems as finite state machine
- Preserved (only) partial order: “happens before” mutex
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Highlights nuances and difficulties in clock synchronization.
DAG Model

Directed Acyclic Graph:
- Each node represents a chunk of computation that is to be done on a single processor
- Directed edges indicate that the from node must be completed before the to node
- The longest path in the DAG represents the total amount of parallel time of the algorithm
- The width of the DAG indicates the number of processors that can be used at once
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Steve Fortune and James Wyllie [1978]. “shared memory model”

- $P$ processors which operate on a shared data
- For each processor read, write, operation (e.g. $+$, $-$, $\times$) constant time.
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- **CREW**: Concurrent read, exclusive write
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Message Passing Model

Emphasizes Locality

- \texttt{send}(X, i) : sends X to \( P_i \)
- \texttt{receive}(Y, j) : receives Y from \( P_j \)
- Fixed topology, can only \texttt{send/receive} from neighbor
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Common Topologies:

- Array/Ring Topology
  - ●
- Mesh Topology
  - ●
- Hypercube Topology
  - ●
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\[ \text{Diagram of message passing model and topologies} \]
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  - (Ω(√p) rounds)
- Hypercube Topology
  - (Ω(log p) rounds)
Programming in MPI

Open MPI:

▸ (Open Source High Performance Computing).
▸ http://www.open-mpi.org/
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When to use MPI?
- Critical to exploit locality (i.e. scientific simulations)
- Complication in only talking to neighbor
Les Valiant [1989] BSP
Creates “barriers” in parallel algorithm.

1. Each processor computes on data
2. Processors send/receive data
3. Barrier: All processors wait for communication to end globally

 Allows for easy synchronization. Easier to analyze since handles many messy synchronization details if this is emulated.
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Many parameters:
  ▶ $P$: number of processors
  ▶ $M$: Memory/Cache Size
  ▶ $B$: Block Size/Cost
  ▶ $L$: Synchronization Costs

Argues: any portable and efficient parallel algorithm, must take into account all of these parameters.
Bridging Model for Multi-Core Computing

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Advantages:

- Analyzes all levels of architecture together
- Like Cache-Oblivious, but not oblivious

Matrix Multiplication, Fast Fourier Transform, Sorting
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At depth $d$ uses parameters: $\bigcup_i (p_i, g_i, L_i, m_i)$

- $p_i$: number of subcomponents (processors at leaf)
- $g_i$: communication bandwidth (e.g. I/O cost)
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MapReduce

Each Processor has full hard drive, data items $<\text{KEY, VALUE}>$. Parallelism Proceeds in Rounds:

- **Map**: assigns items to processor by KEY.
- **Reduce**: processes all items using VALUE. Usually combines many items with same KEY.

**Repeat** M+R a constant number of times, often only one round.

- Optional post-processing step.

**Pro**: Robust (duplication) and simple. Can harness Locality

**Con**: Somewhat restrictive model
General Purpose GPU

Massive parallelism on your desktop. Uses Graphics Processing Unit. Designed for efficient video rasterizing. Each \textit{processor} corresponds to pixel $p$

- depth buffer:
  $$D(p) = \min_i ||x - w_i||$$

- color buffer:
  $$C(p) = \sum_i \alpha_i \chi_i$$

- ...

Pro: Fine grain, massive parallelism. Cheap.
Con: Somewhat restrictive model. Small memory.
... and Beyond

Google Sawzall / Dremel

- Compute statistics on massive distributed data.
- Separates local computation from aggregation.
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Massive, Unorganized, Distributed Computing
- Bit-Torrent (distributed hash tables)
- SETI @ Home
- Twitter Storm / Facebook Casandra