

HW8: Probabilistic Modeling

1 Written Exercises

1. Suppose we have a binomial distribution with the “probability of heads” $\pi = 0.8$. Compute (show all the steps) the expected value and variance of this distribution.
2. Suppose we have a Gaussian with known mean $\mu = 1$ and known variance $\sigma^2 = 1$. What is the *density* of the distribution $\mathcal{Nor}(\mu, \sigma^2)$ at the following points: 0, 1, 2?
3. Consider the previous question, but where $\sigma^2 = 0.1$. What is the density at the given points? For $x = 1$, the density should be *greater than one*. How is this possible given that the Gaussian is normalized (i.e., sums to one).
4. The *Poisson* distribution is a distribution over *positive count values*. It has the form $p(k | \lambda) = \frac{1}{e^\lambda} \frac{\lambda^k}{k!}$, where k is the count and λ is the (single) parameter of the Poisson. Suppose we have a bunch of count data (for instance, the number of cars to pass an intersection on a given day, measured on N -many days) called k_1, k_2, \dots, k_N . Compute the maximum likelihood estimate for λ given this data. (Hint: write down the likelihood, then take the log. Do some algebra to simplify and then take the derivative with respect to λ .)