Hexahedral Mesh Generation for Volumetric Image Data

Jason Shepherd
University of Utah
March 27, 2006
Outline

• Hexahedral Constraints
  – Topology
  – Boundary
  – Quality
• Zhang et al. papers
• Smoothing/Quality
• Existing algorithm modifications?
• From Mitchell’s existence paper:
  – 1. Each internal 2-cell is contained in exactly two distinct 3-cells.
  – 2. Each face contains at least one lower dimensional face (excepting centroids).
  – 3. Each chord segment must contain two distinct centroids.
  – 4. Every internal cell contains at most one surface cell of one lower dimension.
  – 5. Each internal chord segment must be contained in exactly four distinct 2-cells.
  – 6. Each centroid is contained in six chord segments. Note, also, that each chord segment at a centroid is paired with another chord segment belonging to the same chord.
  – 7. Two 3-cells have, at most, one 2-cell in common.

• Interpreted:
  – 1. Only three sheets can intersect at any given centroid.
  – 2. Sheets cannot be tangent with another sheet.
  – 3. Sheets must span the space, or form a closed surface within the space.
  – 4. When traversing the centroids along a single chord, consecutive instances of a single centroid are not permitted.
Boundary Constraints

• **Surfaces**
  – **Lemma** - The boundary of any hexahedral mesh is a quadrilateral mesh.
  – **Theorem** - For each surface of a hexahedrally meshed solid, there exists a set of sheet patches which, taken together, are geometrically similar to the surface but offset a distance that is a function of the size of the mesh local to that boundary (i.e. the local chord length).
Sheet Insertion

- Modify mesh to capture geometry, through STC sheets
• **Curves** –
  
  – **Theorem** - For each curve on a hexahedrally meshed solid, there exists a set of sheet patch pairs such that the lines of intersection between each of the patch pairs, taken together, is a piecewise approximation of the curve, only offset a distance, which is a function of the mesh sizes local to the curve.
Grafting
Boundary Constraints

• Vertices –
  – Theorem - There exists at least one triple-sheet pairing that corresponds to each vertex on the boundary. This triple-sheet pair is equivalent to a centroid, and is offset a distance related to the mesh size local to the vertex.

A  B  C
Quality Considerations

• Skew –

• Sizing -
Quality Considerations

• Curvature -
Quality Constraints?

• High sheet curvature is necessary to produce negative jacobian elements, but is not sufficient...
  – Each hex is formed by the interaction of 3 sheets.
  – Poor quality (negative Jacobian) are found when the at least one of the three sheets has high curvature and the other one or both interact such that the other sheets are highly curved or skewed...
    • i.e. 2-3 of the sheets contain high curvature
    • Or 1 has high curvature and another is non-orthogonal
    • Etc.
Quality Constraints?
Review - Octree methods

Figure 3: Octree decomposition

Review - Octree Methods

Review – Octree Methods

- Octree
  - Advantages
    - Can mesh arbitrary shapes
    - Interior mesh is highly structured
  - Disadvantages
    - Orientation sensitive
    - No surface mesh control

(Schneiders, 96)
Solid Models

<table>
<thead>
<tr>
<th>Entity</th>
<th>Dimensionality</th>
<th>“Owned” Entities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertex</td>
<td>0D</td>
<td>None</td>
</tr>
<tr>
<td>Curve</td>
<td>1D</td>
<td>Two Vertices</td>
</tr>
<tr>
<td>Surface</td>
<td>2D</td>
<td>One or more Curves</td>
</tr>
<tr>
<td>Volume</td>
<td>3D</td>
<td>One or more Surfaces</td>
</tr>
</tbody>
</table>
Recent Work by Zhang et al.


Results
Demo

- Myknee
  - Smoothing and quality
- Mache
- Head
- Cut head
  - sharp features
Review

Structured & Semi-structured
- Mapping
- Submapping
- Primitives
- Sweeping

Unstructured
- Direct
  - Grid-Based
  - Plastering
  - Medial Axis
  - Whisker Weaving
- Indirect
  - H-Morph
  - THex
  - ...
Review

• Using the knowledge of all hexahedral constraints, can we modify other algorithms to work on more geometric classes?
• Can we automate all of these processes?
• Will this be helpful?