Moving Least Squares Surfaces 2

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Adamson and Alexa

- Many variants
- Basic intuition: blend many signed point-plane distance functions, weighted by Gaussian
Implicit MLS Surfaces

- Class of functions

\[ f(x) = n(x)^T (a(x) - x) \]

- Implicit surface: \( f(x) = 0 \)

- Main intuition: implicit surface is defined where the two terms are orthogonal to one another
\( a(x) = \text{Average of points} \)

- \( a(x) \) is simply the weighted average of the points, weighted by the distance to \( x \)
- \( a(x) - x \) is the vector that points to the average
- Direct consequence: if \( a(x) \) is stationary, it’s on the surface
\( n(x) = \text{Some direction} \)

- \( n(x) \) is defined to be some oriented direction
- There are many options
- The one most related to the MLS surfaces uses the eigenvectors of the covariance matrix
Covariance Matrix

- Measures the correlation between variances in different directions
- It’s symmetric, positive definite
  - Remember, from the SVD theorem, that a symmetric matrix is essentially a rotation followed by a scaling followed by the inverse rotation
  - Eigenvalues are all positive
Eigenanalysis of the Covariance Matrix

- illustration about SPD matrix eigenvalues
Back to $n(x)$

- $n(x)$ is defined as the smallest eigenvector of the covariance matrix.
- Oh-oh: eigenvectors are unsigned.
- $f(x) = n(x)^T (a(x) - x)$
- $f(x) \geq 0$
- Increase stability: define $n(x)$ as the average of normals given as input.
Ray tracing

- \( f(x) \) is an approximation of the distance to the surface
- Iteratively compute ray-polynomial intersections until it converges
  - Find support plane, using point of previous intersection
  - Intersect with local polynomial over support plane
  - Use intersection point to start next iteration
Ray tracing
Results
Defining Point-set Surfaces

- Focus: MLS surfaces without step 2
  - The projection is just the center of the reference frame
  - Big contribution: an MLS surface is kind of an implicit surface
  - Helps analysis and intuition
  - Only “kind of”, though
Extremal Surfaces

- Define two functions
- \( e(x) \), a scalar field
- \( n(x) \), a direction field
- If at a point \( x \), \( e(x) \) is a minimum along the line that goes through \( x \) in the direction of \( n(x) \), then \( x \) is a point in the surface
MLS for surfels

- Easy
- Instead of \( \mathbf{n}(x) \) being given by the covariance analysis, use predefined normals
- at each \( x \), take vector averages weighted by \( \theta(\cdot) \)
Implicit Surface Definition

- \( g(x) = \vec{n}(x) \cdot \nabla_y e(y, \vec{n}(x)) \mid_x \)
- Watch out, this also picks out the maxima! (this is the “kind of”)
- If \( \vec{n}(x) \) and \( e(x, a) \) are well defined where \( g(x) = 0 \), then the surface is a manifold
The Domain of a PSS

- Thesis: the set of points that can be projected on the MLS surface is surprisingly small
- Proposes some alternate, better-behaved definitions
- Buyers beware: we could not reproduce figure 1.
Alternate definition

- They argue it’s a **sampling** issue
- Define distance from hyperplane to points, instead of distance from points to hyperplane
  - Integrate it over the hyperplane
Results