

Extension of Star Coordinates into three dimensions

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ABSTRACT

Traditional Star Coordinates displays a multi-variate data set by mapping it to two Cartesian dimensions. This technique facilitates cluster discovery and multi-variate analysis, but binding to two dimensions hides features of the data. Three-dimensional Star Coordinates spreads out data elements to reveal features. This allows the user more intuitive freedom to explore and process the data sets.

Three-dimensional Star Coordinates is implemented by extending the data structures and transformation facilities of traditional Star Coordinates. We have given high priority to maintaining the simple, traditional interface. We simultaneously extend existing features, such as scaling of axes, and add new features, such as system rotation in three dimensions. These extensions and additions enhance data visualization and cluster discovery.

We use three examples to demonstrate the advantage of three-dimensional Star Coordinates over the traditional system. First, in an analysis of customer churn data, system rotation in three dimensions gives the user new insight into the data. Second, in cluster discovery of car data, the additional dimension allows the true shape of the data to be seen more easily. Third, in a multi-variate analysis of cities, the perception of depth increases the degree to which multi-variate analysis can occur.

Keywords: multi-variate data visualization, cluster discovery

1. INTRODUCTION

Huge databases of information exist for everything from cities to mushrooms. Conceptually, these databases are large tables with many columns for the variables and many rows for the data points. Presented as a table, this data is difficult to interpret and conceptualize, often denying a user an intuitive feeling for the data. Our goal is to put the table into a picture that can be displayed on a computer screen.

The traditional approach to this problem is to collapse the dimensions onto a flat (two-dimensional) surface. Several successful collapsing efforts have been reported, including Parallel Coordinates,¹ Vector Fusion,² Trellis Displays,³ and Star Coordinates.⁴ While most users accept two-dimensional graphs, such presentations hardly represent the full potential of pictures. Expecting the user to perceive a two-dimensional picture as three-dimensional is a standard practice, due to our familiarity with three dimensions. A reason for not initially collapsing the dimensions into a three-dimensional display is that volumetric displays are not widely available. Furthermore, many users fail to comprehend presentations with only two dimensions.

We have chosen a set of goals for a new system based on our experience with multi-variate displays. First, a multi-variate display system should be three-dimensional. Second, the system should be interactive. Third, it should capitalize upon the advantages of any successful two-dimensional system which it extends.

We gain several advantages by putting a system into three dimensions. Coming from higher dimensions, reduction to at most three dimensions increases the likelihood that a layperson will understand the representation. Many multi-variate display systems already exist such as XGobi,⁵ Parallel Volumes,⁶ and XmdvTool.⁷ Hyper-dimensional displays can require

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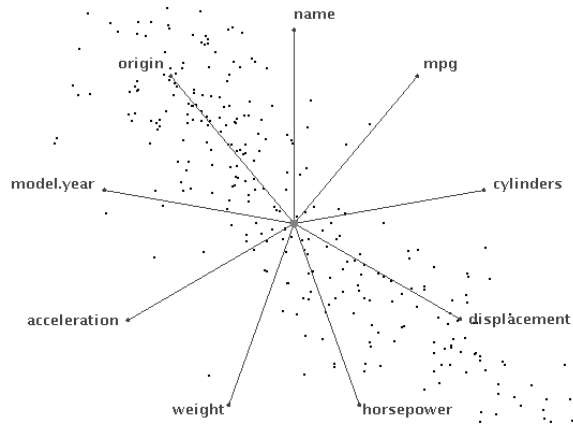


Figure 1. Kandogan’s implementation of his Star Coordinates system

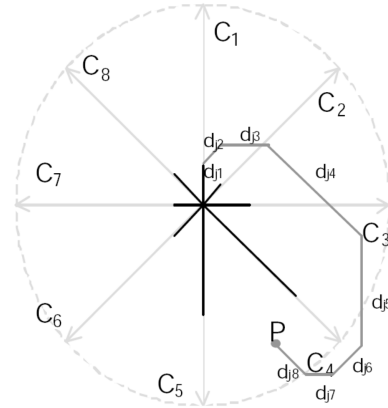


Figure 2. Computation for a single star location in Star Coordinates⁴ (Used with author’s permission)

training and experience before they can be used successfully. However, the strength of three-dimensional viewing is evident when compared with two dimensions: the information loss associated with the dimensional reduction is costly.

While visual cues can promote the perception of three dimensions on a two-dimensional screen, an effective way to actually make the system three-dimensional is through interactivity. This is what changes a view from being a cognitive task to being a perceptual task.⁸ Without interactivity, any two-dimensional picture presented on the monitor is still just two-dimensional. Interactivity allows the user to immerse himself or herself in the system, at which point it then feels three-dimensional.

Accommodating the first two ideas, three-dimensionality and interactivity, is not enough. The problem is that three-dimensional, interactive, multi-variate display systems often lack the intuitiveness of a simpler system and bog the user down with complex three-dimensional transformations. For this reason, our approach is to take an existing, intuitive, multi-variate display system in two-dimensions and build upon it. This allows the three-dimensional system and its interface to remain intuitive by consistently following the original system’s implementation.

Kandogan presents an intuitive multivariate viewing system in traditional two-dimensional Star Coordinates.⁴ Star Coordinates abandons the orthogonality of axes in order to fit the higher-dimensional system into two dimensions. Figure 1 shows a Star Coordinates display of multi-variate automobile data. Each star is plotted by calculating the vector sum of attribute values along the axes (Figure 2). In this way the star reflects the contribution of all its attributes. However, when more than two variables are involved, each star may be reached through an infinite number of different vector sum combinations. The Star Coordinates technique is elaborated in Section 3.

The contribution of this paper is the presentation of three-dimensional Star Coordinates as an enhanced multi-variate visualization system. This system essentially preserves the two-dimensional Star Coordinates⁴ interface while transforming the existing multi-variate system to display three dimensions. Hence, it achieves the three goals stated previously. We show that the system frees users from constrictive two-dimensional displays by mapping real data to familiar dimensions. The result is a powerful tool for cluster discovery and multi-variate analysis.

The paper proceeds as follows. Section 2 presents related work motivating the enhancement to three dimensions. Section 3 introduces our three-dimensional Star Coordinates system, explains its features, and establishes the theoretical foundation for our work. Section 4 examines our implementation. Section 5 compares two-dimensional and three-dimensional Star Coordinates in three applications and shows the advantage of the extra dimension in each application. Section 6 proposes ways to enhance the system and suggests other future work. Section 7 concludes.

2. RELATED WORK

Exploration of Star Coordinates has continued to be an area of interest since Kandogan introduced the technique. Teoh and Ma introduce StarClass, which uses user-driven “painting” to produce a decision tree and therefore improve clustering.⁹ PolarEyes, developed by Jayaraman and North, creates a heat map in a system that has the appearance of Star Coordinates

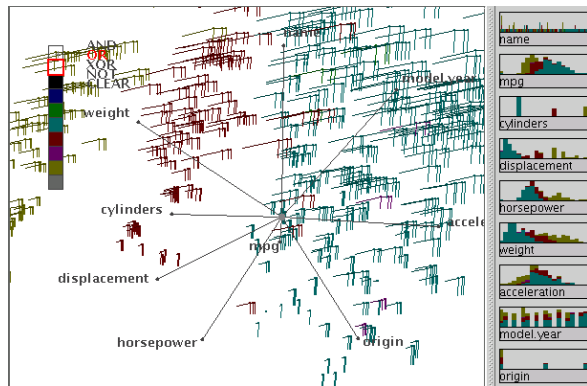


Figure 3. Two-dimensional Star Coordinates demonstrating many different interaction techniques it provides, including footprints, and glyphs

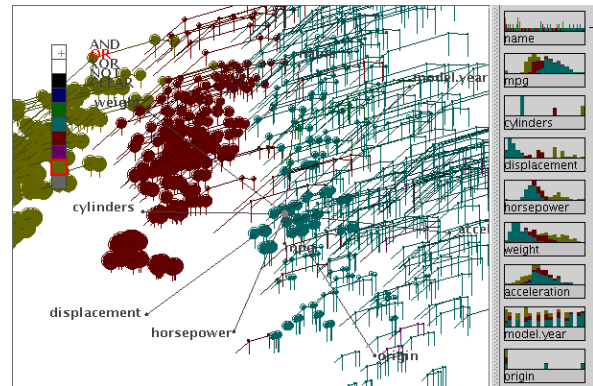


Figure 4. Three-dimensional Star Coordinates demonstrating additional interaction techniques it provides, including footprints, glyphs, and the size depth cue

but uses a different algorithm for vector summation.¹⁰ Chen and Ling’s VISTA adapts the idea of Star Coordinates specifically to handle skewed data sets with abnormal clusters.¹¹ This abundant and continued attention to Star Coordinates encourages further development of the technique.

Inselberg developed a technique for multi-variate viewing called Parallel Coordinates.¹ This work provided inspiration for Kandogan’s original work with Star Coordinates. Kandogan cites Parallel Coordinates as being very powerful, but points out that it remains difficult to understand for the layperson.⁴

Inselberg’s work also provided a foundation for two other systems: Parallel Planes¹² and Parallel Volumes.⁶ In Parallel Planes, Tan and Burton extend Parallel Coordinates by one dimension. Instead of plotting a point through the parallel axes, it is plotted through the parallel axis-pairs. Ping and Burton take this one step further in Parallel Volumes by plotting points through parallel and three-dimensional spaces. This idea of extending systems to higher dimensions provided the motivation for our work.

Tavanti and Lind explore the effect of extending systems to higher dimensions in their work.¹³ They perform two experiments to test the effect of adding a dimension to a tree-structure interface, similar to a file system explorer. While the paper makes no conclusions about the contribution of specific aspects of the three-dimensional interface, it does conclude that it is more effective in their application. These results support our experience in adding a dimension to traditional two-dimensional Star Coordinates.

Vector Fusion is a technique similar to Star Coordinates;² however, its primary uses and purposes differ greatly from Kandogan’s objectives. Johnson presents Vector Fusion as a way to view geometric shapes and structures that have more than three dimensions. The goal is to show the user a structure that normally is difficult to visualize. In contrast, Star Coordinates gives the user raw multi-variate data and lets the user interact with it. Its goal is to allow the user to discover any structures that may exist in the data. Star Coordinates and Vector Fusion are tools for different ends of the analysis spectrum.

A predecessor technique similar to Star Coordinates is Radviz.¹⁴ The two methods use different abstractions to view data points in almost the same manner. Star Coordinates uses the plotting vector sums and Radviz uses a spring equilibrium calculation. The two differ in terms of their interfaces. Radviz presents points statically by not exposing the springs to the user. Star Coordinates facilitates data analysis through dynamic interaction. Its component vectors of the sum may be resized, redirected or completely ignored.

3. STAR COORDINATES

Kandogan describes Star Coordinates as “simply an extension of typical 2d and 3d scatter-plots to higher dimensions with normalization”.⁴ It is a form of interactive multidimensional scaling. The system creates an axis for each component attribute and then maps the values of that attribute for the data to vectors along each axis. As explained in Kandogan’s

original work,⁴ Star Coordinates plots a star by summing the vectors of each component attribute for the piece of data the star represents.

Figure 2 illustrates the calculation of a single data star in Star Coordinates. Axes exist for each of the eight attributes, labeled C_1 through C_8 . The d -labeled vectors are computed for each attribute based on the orientation of the axes and the mapping of values. The smallest value in the set for an attribute is mapped to the origin and the largest is mapped to the end of the attribute's axis. In this way, no data values are "out of range." By computing the sum of the d vectors, Star Coordinates determines the location of star P .

3.1. The Third Dimension

Three-dimensional Star Coordinates follows the model of plotting stars based on axis locations and orientations, but the axes may extend into three Cartesian dimensions instead of just two. Adding a third dimension to traditional Star Coordinates allows for three-dimensional interaction, but maintains the two-dimensional display. Three-dimensional Star Coordinates extends traditional two-dimensional Star Coordinate in several ways:

- Stars distribute in a volume instead of a plane, giving users more space to exploit.
- Depth cues allow users to include more meaningful variables simultaneously in an analysis.
- Transformations are extended to three dimensions.
- System rotation is introduced as a powerful new transformation.

By distributing stars in a volume instead of a plane, three-dimensional Star Coordinates provides a user with a richer visualization space. While our examples demonstrate this later in the paper, the property is intuitive. As an example, consider a collection of tennis balls with diameter d , filling a box with width $w > 2d$, length $l > 2d$, and height $h > 2d$. Without disfiguring the tennis balls or stacking them, it is impossible to fit them all on top of a square of width w and length l . For example, with $l = w = h = 3d$, the square contains at most nine unstacked tennis balls while the box holds 27 tennis balls. Similar to how there is more room in the box than in the square, there is more room in three-dimensional Star Coordinates than in two-dimensional Star Coordinates.

A volume is a full dimension richer than a plane. Tavanti and Lind articulate the following four theoretical conclusions, relative to 2D:

- a*) 3D displays can be exploited to visualize large sets of hierarchical data;
- b*) The perspective nature of 3D representations makes it possible to show more objects in a single screen (objects shrink along the dimension of depth);
- c*) if more information is visible at the same time, users gain a global view of the data structure;
- d*) there is experimental evidence that 3D ecological displays enhance subjects' spatial performances.¹³

These four points support the intuition that moving Star Coordinates into three dimension gives users more space to exploit.

The depth cues that display this extra space add analytical power to the system. A star in three-dimensional Star Coordinates has a distance from the user in addition to its location on the screen. This additional reference allows for a visual comparison on multiple fronts simultaneously. For example, a star may be an outlier in the yz plane but average when compared to outliers in the x direction. Three-dimensional Star Coordinates provides the extra analytical power necessary for this comparison.

Three-dimensional Star Coordinates extends Kandogan's intuitive and easy-to-use transformations. Traditional Star Coordinates originally included only single- and multi-attribute rotation or scaling⁴ but Kandogan later expanded it to include marking, range selection, histograms, footprints and sticks¹⁵ (Figure 3). Three-dimensional Star Coordinates also uses rotation, scaling, marking, histograms, footprints, and sticks, together with new features described below (Figure 4). These are all useful transformations for cluster discovery and multi-variate analysis. Much of the power of Star Coordinates comes through user interaction.

In three-dimensional Star Coordinates we introduce the powerful transformation of system rotation in three dimensions. It allows overlapping clusters to be distinguished and axes to be isolated. Rotation is also the most powerful tool for system

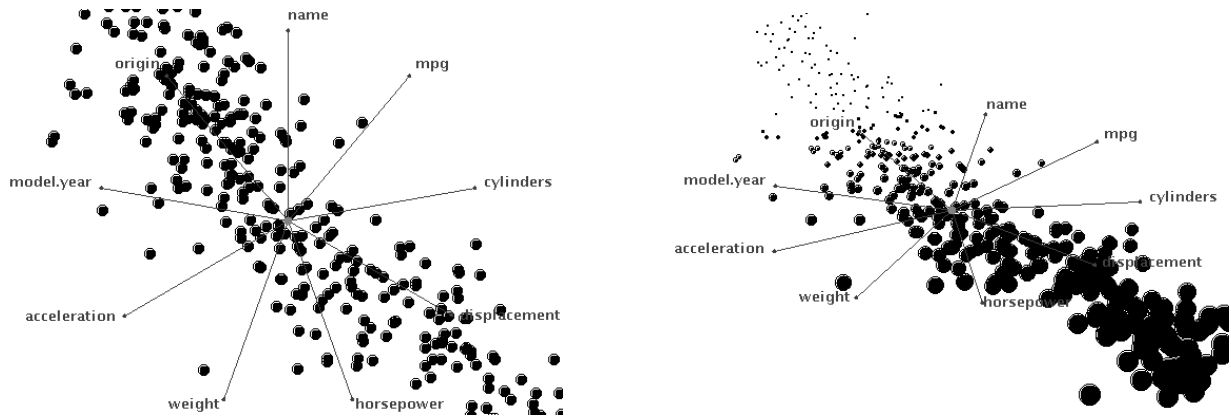


Figure 5. 3D Star Coordinates before and after rotating the system

interaction in two-dimensional Star Coordinates, but only z axis rotation is available in two-dimensional Star Coordinates. Three-dimensional Star Coordinates allows the user to rotate the entire system around the x or y axis (Figure 5). This coordinated movement of axes allows the user to move the stars in a familiar way, as if they are inside a ball in front of the user. This familiarity promotes complex exploration of the data set, which is illustrated in Section 5.

4. IMPLEMENTATION

To implement three-dimensional Star Coordinates, we started with Kandogan's two-dimensional implementation. Significant problems needed to be solved during the non-trivial extension into three dimensions.

4.1. System

We wanted to make the two-dimensional device imitate three dimensions to the greatest extent possible. Our first efforts changed the stars from solid squares to partially filled-in circles. The circles are filled in with a smaller, solid circle to imitate a specular highlight. This makes the stars more distinguishable and voluminous while maintaining computational efficiency. Although not significant in terms of computer graphics, this approach does add to the perception in a three-dimensional system.

In order to reduce the occlusion of distant stars and to facilitate pointing (discussed below), we used a parallel projection. Because of this, we also implemented star size as a depth cue to allow users to perceive the third dimension. The stars closer to the user appear larger than those which are further away (Figure 5). We disabled the user's ability to change the size of stars in the system (a feature available in two-dimensional Star Coordinates) in order to preserve the cue. We introduced artificial constraints on the minimum and maximum sizes of the stars. This prevented far stars from disappearing and near stars from inappropriately obscuring the view.

The third dimension introduces the problem of stars obscuring the user's view. Fortunately, the effect of occlusion on the system is not significant. First, all the stars are plotted initially using the same color: white. Because of this, occlusion has no effect before the user starts to highlight axes and stars. Which color is displayed becomes meaningful after the user begins highlighting stars. The user highlights in Star Coordinates to mark clusters of stars. By this time, he or she is manipulating the system and has a mental picture of the star layout.¹³ Also, the star size depth cue puts the data in its proper "perspective." Hence, we chose not to address occlusion specifically in our initial system.

4.2. Interface

While a mouse provided all the user input for two-dimensional Star Coordinates, two keyboard keys are used to provide a three-dimensional interface. The user performs system rotation using the arrow keys. Rotation or scaling in the third dimension can be performed using two keyboard keys. The mouse moves in two dimensions, which is why the third dimension is provided by the keyboard. By adding only minimal keystrokes, the user is able to perform operations intuitively using the hand that is not operating the mouse.

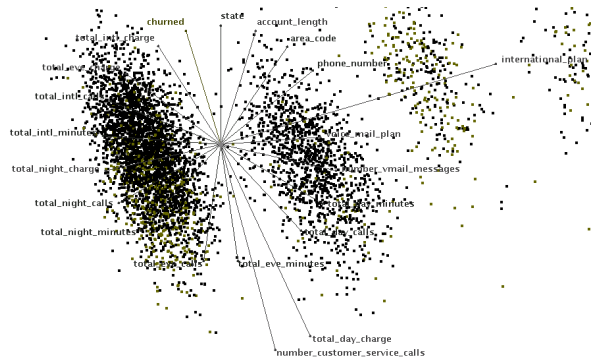


Figure 6. Two-dimensional Star Coordinates performing cluster analysis on churn data. Churned and non-churned customers are clustered together confusingly.

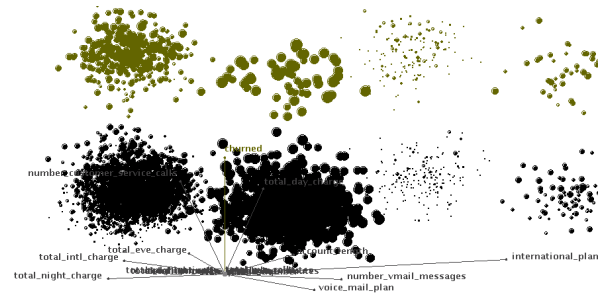


Figure 7. Three-dimensional Star Coordinates performing cluster analysis on churn data. Churned customers are separated from non-churned customers.

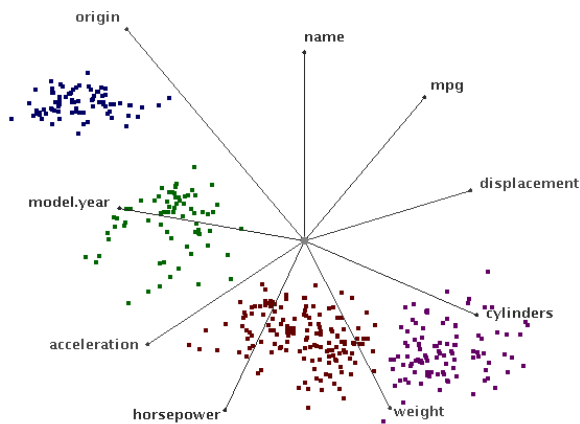


Figure 8. Two-dimensional Star Coordinates performing cluster analysis on car specification, revealing four clusters

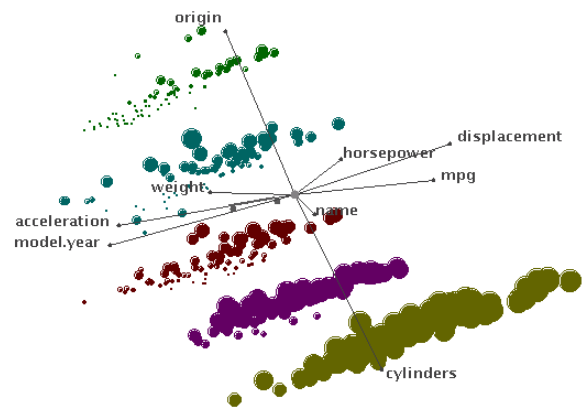


Figure 9. Three-dimensional Star Coordinates performing cluster analysis on car specification, revealing five clusters

Although the keyboard provides alternative input methods, the mouse’s location on the screen remained a two-dimensional pointer in a three-dimensional world. However, much like an individual can point across a room at an object and convey three-dimensional meaning, the mouse can point across space at a star or an axis and convey the mouse’s intent. In areas where occlusion disrupts the mouse’s line of sight, slight rotation of the system reveals the hidden objects. While we tried both parallel and perspective projections for displaying the data, we found parallel projection remained the most intuitive for user pointing.

Introducing a third dimension brought with it the possibility of disorienting the user. The potential for disorientation is one concern that motivated building upon a successful two-dimensional system. We facilitate acclimatization by using a system already known to be effective. Some disorientation still occurs in three-dimensional Star Coordinates and we suggest ways to reduce it further in future work. Our experience is that when dealing with large data sets with many axes, rotations can put the system in an orientation where it is easier to start over than to continue.

5. THREE APPLICATIONS

We evaluated three-dimensional Star Coordinates using three data sets used previously to evaluate two-dimensional Star Coordinates. The power of three-dimensional Star Coordinates comes from the new star size depth cue and system rotation transformation. The star size provides an additional attribute for comparison besides location. System rotation allows for easy multi-axis transformation in a way different from two-dimensional Star Coordinates. The following three examples show how increasing the dimensions of two-dimensional Star Coordinates has improved the technique.

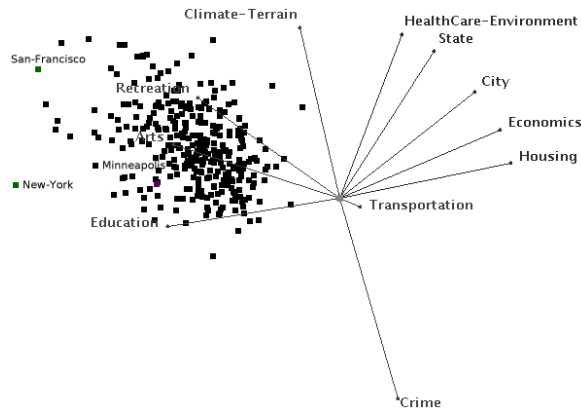


Figure 10. Two-dimensional Star Coordinates performing multi-variate analysis of cities, limited to fewer variables by space constraints

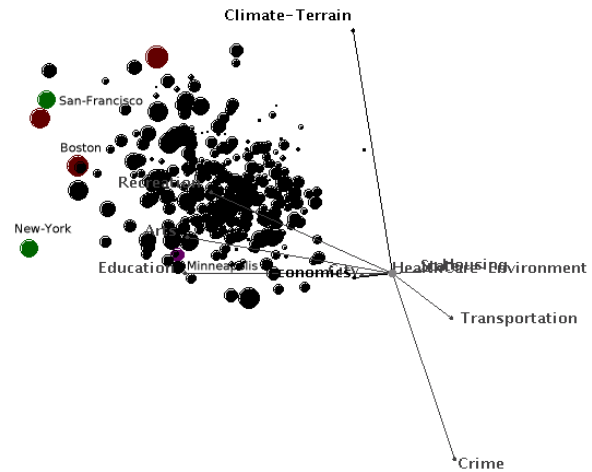


Figure 11. Three-dimensional Star Coordinates performing multi-variate analysis of cities, using all variables to compare cities

5.1. Churn

The first application we consider is customer churn in telecommunication companies. In this data set, the usage and plan information for 5000 customers is considered. The attribute of most importance is “churning,” which occurs when a customer cancels services. This data set is analyzed to discover why customers churn. This case has significantly more stars and axes than in the two subsequent examples.

Kandogan uses two-dimensional Star Coordinates to draw some conclusions about the data set.¹⁵ By turning off uninteresting and insignificant (in his estimation) axes, clusters emerge in the data. By performing an axis spectrum highlight on the “churned” axis, he highlights all customers who churn. In Figure 6 the churning customers are highlighted. Observing the movement of the stars during axis scaling allows an experienced user to judge the impact of attributes on churning. It is apparent from Figure 6 that the large data set can be overwhelming to view, especially for new users of the system.

While the three-dimensional system does not reduce the vastness of the data set, it does make it easier to manage. Figure 7 shows three-dimensional Star Coordinates’ presentation of this data set. In order to make a comparison, we scaled down the same axes which Kandogan turned off in two-dimensional Star Coordinates so as to have no effect in three-dimensional Star Coordinates. However, instead of turning off the churn axis as Kandogan does, we rotated it in the third dimension. After we did this, we rotated the system and scaled attributes in order to view their impact on churn. Because of the set isolation, it becomes apparent that customers who make more customer service calls are more likely to churn.

This makes sense, but is not obvious in two-dimensional Star Coordinates. Three-dimensional Star Coordinates is able to isolate this effect by system rotation. After the initial clusters around “international plan” and “voice mail plan” are found, the data can remain grouped but be rotated to highlight churning customers. In our example, these customers appear “above” the non-churning customers. It is then easy to see which attributes affect which group more, as all eight groups (churn and no churn for international plan, voice mail plan, both plans, and no extra plans) are isolated.

5.2. Cars

When introducing Star Coordinates, Kandogan used an example containing car specification for 406 cars manufactured world-wide. His analysis produced a figure similar to Figure 8. Two-dimensional Star Coordinates can be arranged in this manner after turning off several axes, scaling some axes, and rotating the axes to isolate active attributes. While the interface makes these individual actions fairly easy, several steps are required and there are options to explore as the data is analyzed. The analysis Kandogan presents reveals four clusters, each highlighted differently in Figure 8.

These four clusters represent four classes of cars. The three on the left represent European, Japanese and American cars. All three of these clusters represent low-weight, low-displacement, high acceleration cars. The final cluster represents “American-made, heavy, low-acceleration, high displacement cars”.⁴ This is a clear categorization of the data, but “low” and “high” are not discrete measurements, which makes classifying a borderline car difficult without the use of the visualization. Users would not want to take their computers along to enter in new data points at the car lot!

Figure 9 shows that same data set using three-dimensional Star Coordinates. We rotated the “origin” and “cylinders” axes to be orthogonal to the rest of the axes. We then rotated one away from the user (into the screen) and the other in the opposite direction (out of the screen). After performing these two transformations, we rotated the entire system so that all axes were viewable. These simple transformations reveal five distinct clusters. Each of these clusters is shaded differently in Figure 9.

The new clusters show that the weight and displacement are closely correlated to a different attribute: the number of cylinders. Fewer cylinders means less displacement and less weight. More cylinders means greater displacement and greater weight. By choosing the number of cylinders as the metric over the combination of multiple variables, a user has a discrete measurement to use for comparison. Comparing the number of cylinders between different cars becomes easy!

By choosing different transformations in two-dimensional Star Coordinates, we can discover these five clusters. The real advantage of three-dimensional Star Coordinates is that the third dimension gives the user more space to spread out the axes and thus makes the five cluster groupings easier to discover. This example also highlights the intuitive understanding that develops from system rotation. As all parts of the system move in three dimensions, the user visualizes the system and detects the clustering of the stars. This visualization is much more difficult in the traditional, flat, two-dimensional system.

5.3. Cities

The last example we consider is a multi-variate analysis of cities. In this example, we evaluated a set of 329 cities based on several criteria established by the user. Our goal is to discover a city that maximizes desirable attributes while minimizing undesirable ones. We disregard attributes that are less important. Of course, user preference determines which attributes are classified as desirable, undesirable or unimportant. Properties can be examined, regardless of personal preference in city attributes.

In Kandogan’s original work,⁴ he chooses climate, recreation, arts, and education as desirable attributes. He rotates these attributes to the upper left of the system. Kandogan rotates his undesirable attributes of transportation and crime to the lower right region of the system. He turns off the remaining attributes. The result is shown in Figure 10. The cities of San Francisco and New York, as pointed out in the original paper, score favorably using these attributes. They are highlighted in Figure 10.

Again, we analyzed the same data using three-dimensional Star Coordinates. The result is shown in Figure 11. In this example, we rotate Kandogan’s desirable and undesirable attributes to their corresponding locations, similar to the two-dimensional analysis. However, because of the extra degree of freedom in this system, we leave on the remaining attributes but move them to be near-orthogonal to the “interesting attributes.” In this way, the size of each star can reflect that star’s value for the orthogonal attributes. San Francisco and New York again are highlighted. However, after looking at their size, it appears that some cities which are located similarly are actually larger and hence more favorable (for example, Boston). These cities are also highlighted in Figure 11.

This illustrates the power of the star size depth cue. By adding the third dimension and representing it using a depth cue, it is possible to compare even more attributes meaningfully and simultaneously. For this city example, a user can specify a primary set of attributes (such as desirable and undesirable attributes), and then a secondary set (such as the remaining attributes) and then compare using both sets at the same time. This type of comparison is not possible in two-dimensional Star Coordinates.

6. FUTURE WORK

Several theoretical aspects of Star Coordinates can be studied. An interesting unstudied topic is an analysis of the geometry of this “collapsed” and non-orthogonal space. That work should include exploring the adjacency of axes, which is a property Tan and Burton introduce as a measurement of axis orthogonality.¹² This would help with understanding the stars’ movements in the space.

These ideas also can be exported usefully elsewhere. First, Star Coordinates could be combined with image recognition techniques to facilitate automatic cluster discovery in data. Other successful two-dimensional systems could be expanded to three dimensions. In doing so, new ways of directly reducing multi-variate data to three dimensions may become apparent.

7. CONCLUSION

We have presented three-dimensional Star Coordinates as a valuable extension of two-dimensional Star Coordinates. It retains all the utility of traditional Star Coordinates and has several advantages over the two-dimensional technique. First, system rotation allows for configurations of data to be maintained while considering different views. Second, the infinitely enlarged space of volumes relative to surfaces allows the structure of the data to be discovered more easily. Third, depth cues provide attribute references which may be used to perform more complex multi-variate analysis. These features significantly enhance cluster discovery and data analysis.

These features have been provided by extending Kandogan's implementation of Star Coordinates in significant ways. In traditional Star Coordinates, the underlying representation of the data is inherently two-dimensional, the display is inherently two-dimensional, and the input device (mouse) is inherently two-dimensional. We overcome these limitations through a combination of methods and techniques. Three-dimensional Star Coordinates maintains the intuitiveness of two-dimensional Star Coordinates while providing and capitalizing on the new, three-dimensional aspects of the system.

Our work achieved the three goals we imposed at the outset: first, that the technique be three-dimensional; second, that it be interactive; and third, that it be built upon a successful two-dimensional system, preserving the desirable features of the interface. By implementing these ideas, we move the data into the user's familiar space. The extra dimension gives the user freedom to interact naturally with the data. The result is a system that facilitates the release of information contained in the data. Three-dimensional Star Coordinates does that for all datasets considered.

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