FLOATING POINT OPERATIONS

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Overview

- Notes
  - Homework 6 will be posted tonight
    - Deadline: Mar. 5th

- This lecture
  - Floating point operations
  - Basics of logic design
Recall: Floating Point Addition

- Numbers maintain only 4 decimal digits and 2 exponent digits
  - $9.999 \times 10^1 + 1.610 \times 10^{-1}$

- Convert to the larger exponent
  - $9.999 \times 10^1 + 0.016 \times 10^1$

- Add
  - $10.015 \times 10^1$

- Normalize
  - $1.0015 \times 10^2$

- Check for overflow/underflow

- Round
  - $1.002 \times 10^2$

- Re-normalize
Recall: Floating Point Addition

- Numbers maintain only 4 decimal digits and 2 exponent digits
  - $9.999 \times 10^1 + 1.610 \times 10^{-1}$

- Convert to the larger exponent
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- Round
  - $1.002 \times 10^2$

- Re-normalize

If we had more fraction bits, these errors would be minimized
Floating Point Addition

- Numbers maintain only 4 binary digits and 2 exponent digits
  - $1.010 \times 2^1 + 1.100 \times 2^3$

- Convert to the larger exponent
  - $0.0101 \times 2^3 + 1.100 \times 2^3$

- Add
  - $1.1101 \times 2^3$

- Normalize
  - $1.1101 \times 2^3$

- Check for overflow/underflow
Floating Point Addition

- Numbers maintain only 4 binary digits and 2 exponent digits
  - $1.010 \times 2^1 + 1.100 \times 2^3$

- Convert to the larger exponent
  - $0.0101 \times 2^3 + 1.100 \times 2^3$

- Add
  - $1.1101 \times 2^3$

- Normalize
  - $1.1101 \times 2^3$

- Check for overflow/underflow

- IEEE 754 format (32-bit)

0 10000010 11010000000000000000000000000000
Floating Point Addition

Example: add the following two single-precision floating point numbers.

A: 0100000000110000000000000000000000
B: 01000001101001100000000000000000

Steps:
1. Convert to larger exponent
2. Add
3. Normalize
4. Round
Example: add the following two single-precision floating point numbers.

A: \( \begin{array}{c}
E_A = 128 \\
M_A = 1.11_{\text{two}}
\end{array} \)

B: \( \begin{array}{c}
E_B = 131 \\
M_B = 1.010011_{\text{two}}
\end{array} \)

A + B: \( \begin{array}{c}
E_A = E_B = 131 \\
M_A + M_B = 0.00111_{\text{two}} + 1.010011_{\text{two}} = 1.100001_{\text{two}}
\end{array} \)
Floating Point Multiplication

- Similar steps are required for multiplication
  - Compute exponent
    - Need to remove bias
  - Multiply significands
    - May end up unnormalized
  - Normalize
    - Shift the point
  - Round
    - Fit in the number of bits
  - Assign sign
    - Compute sign
Floating Point Multiplication

Example: multiply the following two single-precision floating point numbers.

A: \[
\begin{array}{cccccccccccccccc}
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}
\]

B: \[
\begin{array}{cccccccccccccccc}
0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0
\end{array}
\]

Steps:
1. Compute exponent
2. Multiply significands
3. Normalize
4. Round
5. Compute sign
Example: multiply the following two single-precision floating point numbers.

A: \[11000000001100000000000000000000000 \]

B: \[010000011010011000000000000000000 \]

\(E_A = 128\quad M_A = 1.11_{\text{two}}\)

\(E_B = 131\quad M_B = 1.010011_{\text{two}}\)

\(E_{AxB} = 128 + 131 - 127 = 132\quad M_{AxB} = 10.010011_{\text{two}}\)

\(E_{AxB} = 133\quad M_{AxB} = 1.010011_{\text{two}}\)

A x B: \[110000010100100010100000000000000000 \]
Floating Point Instructions

- MIPS employs separate registers for floating point
  - 32-bit registers: $f0, f1, ..., f31.
  - Each register represents a single-precision number
  - Register pairs are used for double-precision
    - Example: $f0 refers to {$f0, $f1}

<table>
<thead>
<tr>
<th>Example</th>
<th>Meaning</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>add.s</td>
<td>\texttt{$f2$,} \texttt{$f4$,} \texttt{$f6$} \texttt{} \texttt{} \texttt{}</td>
<td>\texttt{$f2 = f4 + f6$}</td>
</tr>
<tr>
<td>sub.s</td>
<td>\texttt{$f2$,} \texttt{$f4$,} \texttt{$f6$} \texttt{} \texttt{} \texttt{}</td>
<td>\texttt{$f2 = f4 - f6$}</td>
</tr>
<tr>
<td>mul.s</td>
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<tr>
<td>add.d</td>
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<td>\texttt{$f2 = f4 + f6$}</td>
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<tr>
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Floating Point Instructions

- Load/Store instructions by coprocessor 1 (c1)
  - Still use integer registers for address computation

- Comparison instructions
  - Set an internal bit (\texttt{cond}) to be inspected by branch instructions

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<td>lwcl</td>
<td>$f1,100($s2)</td>
<td>$f1 = \text{Memory}[$s2 + 100]</td>
</tr>
<tr>
<td>swcl</td>
<td>$f1,100($s2)</td>
<td>Memory[$s2 + 100] = $f1</td>
</tr>
<tr>
<td>bc1t</td>
<td>25</td>
<td>if (cond == 1) go to PC + 4 + 100</td>
</tr>
<tr>
<td>bc1f</td>
<td>25</td>
<td>if (cond == 0) go to PC + 4 + 100</td>
</tr>
<tr>
<td>c.lt.s</td>
<td>$f2,$f4</td>
<td>if ($f2 &lt; $f4) cond = 1; else cond = 0</td>
</tr>
<tr>
<td>c.lt.d</td>
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Code Example

- Convert a temperature in Fahrenheit to Celsius

  ```
  float f2c(float fahr) {
    return ((5.0/9.0)*(fahr-32.0));
  }
  ```

- Assume that constants are stored in global memory
Convert a temperature in Fahrenheit to Celsius

```c
float f2c(float fahr) {
    return ((5.0/9.0)*(fahr-32.0));
}
```

Assume that constants are stored in global memory

Memory:

```
$gp

const5

const9
```
Code Example

- Convert a temperature in Fahrenheit to Celsius

```c
float f2c(float fahr) {
    return ((5.0/9.0)*(fahr-32.0));
}
```

- Assume that constants are stored in global memory

```
f2c: mtcl $a0, $f12
    lwcl $f16, const5($gp)
    lwcl $f18, const9($gp)
    div.s $f16, $f16, $f18
    lwcl $f18, const32($gp)
    sub.s $f18, $f12, $f18
    mul.s $f0, $f16, $f18
    jr $ra
```