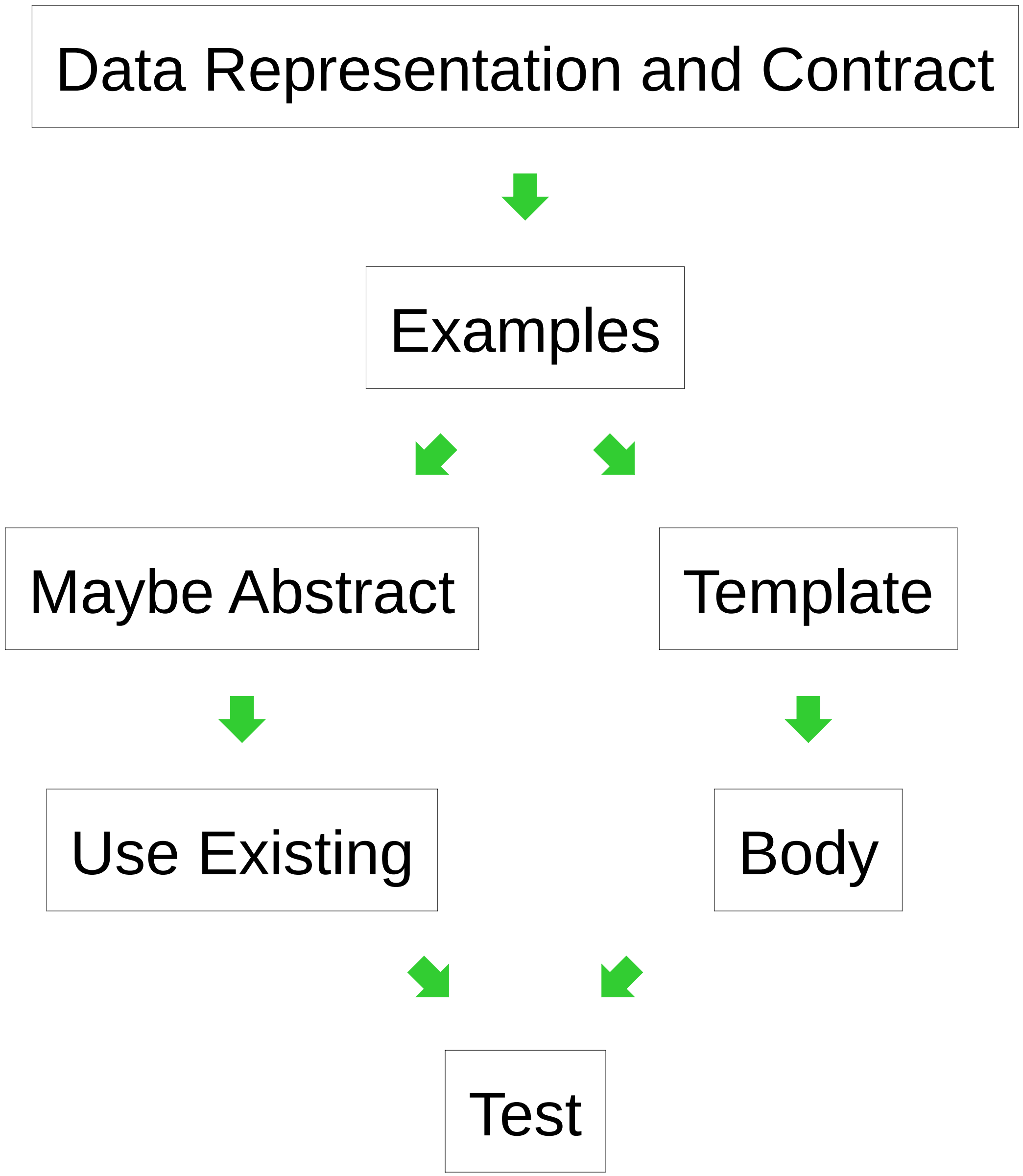
How to Design A Program (So Far)



Challenge Problem

- Implement the function **`odd-items`** which takes a list-of-X and produces a list-of-X containing every other item in the given list (including the first item)

Data Representation and Contract

Already done for us:

```
; odd-items : list-of-X -> list-of-X
```

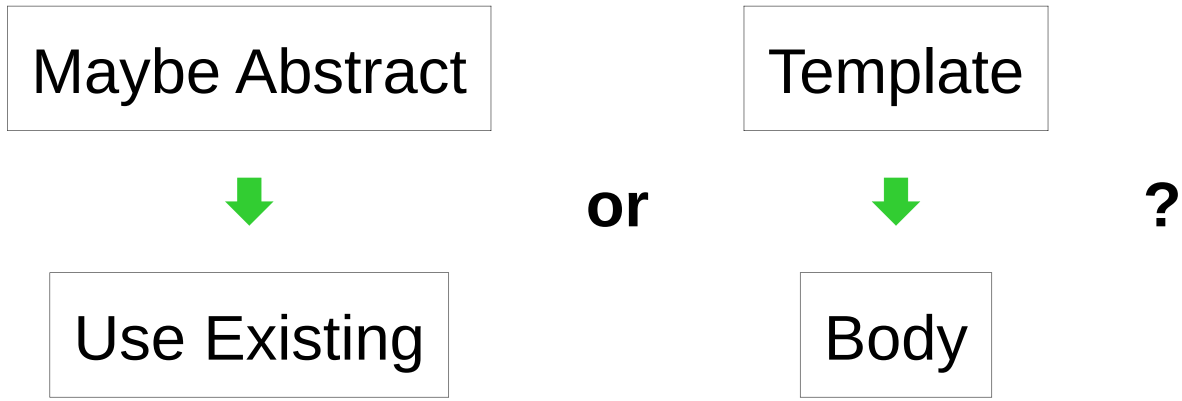
Examples

```
(odd-items empty) "should be" empty

(odd-items '(1 2 3 4 5))
"should be" '(1 3 5)

(odd-items '(apple banana cherry))
"should be" '(apple cherry)

(odd-items (list true false))
"should be" (list true)
```



We know that **`foldr`** captures the template for **`list-of-X`**, so choose the left branch – and abstraction is done already!

Maybe Abstract

↓

Use Existing

```
(define (odd-items l)
  (foldr (lambda (item odd-rest)
           ...)
         empty l))
```

Problem: the odd items of the rest of the list are useless for the odd items of the whole list

```
(odd-items '(1 2 3 4)) "should be" '(1 3)
```

but

```
(odd-items '(2 3 4)) "should be" '(2 4)
```

Template

↓ ?

Body

```
(define (odd-items l)
  (cond
    [(empty? l) empty]
    [(cons? l)
     ... (first l)
     ... (odd-items (rest l)) ...]))
```

Same problem – it's not just a reuse problem...

Structural Recursion

- For recursively defined data, our recipe so far always produces ***structurally recursive*** programs

- In a sense, it always works:

```
(define (odd-items l)
  (first
   (foldr (lambda (item odds+evens)
            (list (cons item
                        (second odds+evens))
                  (first odds+evens)))
          (list empty empty) l)))
```

But making structural recursion work sometimes requires more creativity than solving the problem a different way

Generative Recursion

Structural recursion is a powerful tool, but we need more tools

Our new tool is ***generative recursion***:

```
(define (func v)
  (cond
    [(trivially-solvable? v) ...]
    [else ...
          (func generated-v_1)
                      ⋮
          (func generated-v_n)
          ...]))
```

Structural recursion is a special case of generative recursion that is especially common

Back to Odd Items

When the list given to **odd-items** has less than two items, the problem is trivial to solve:

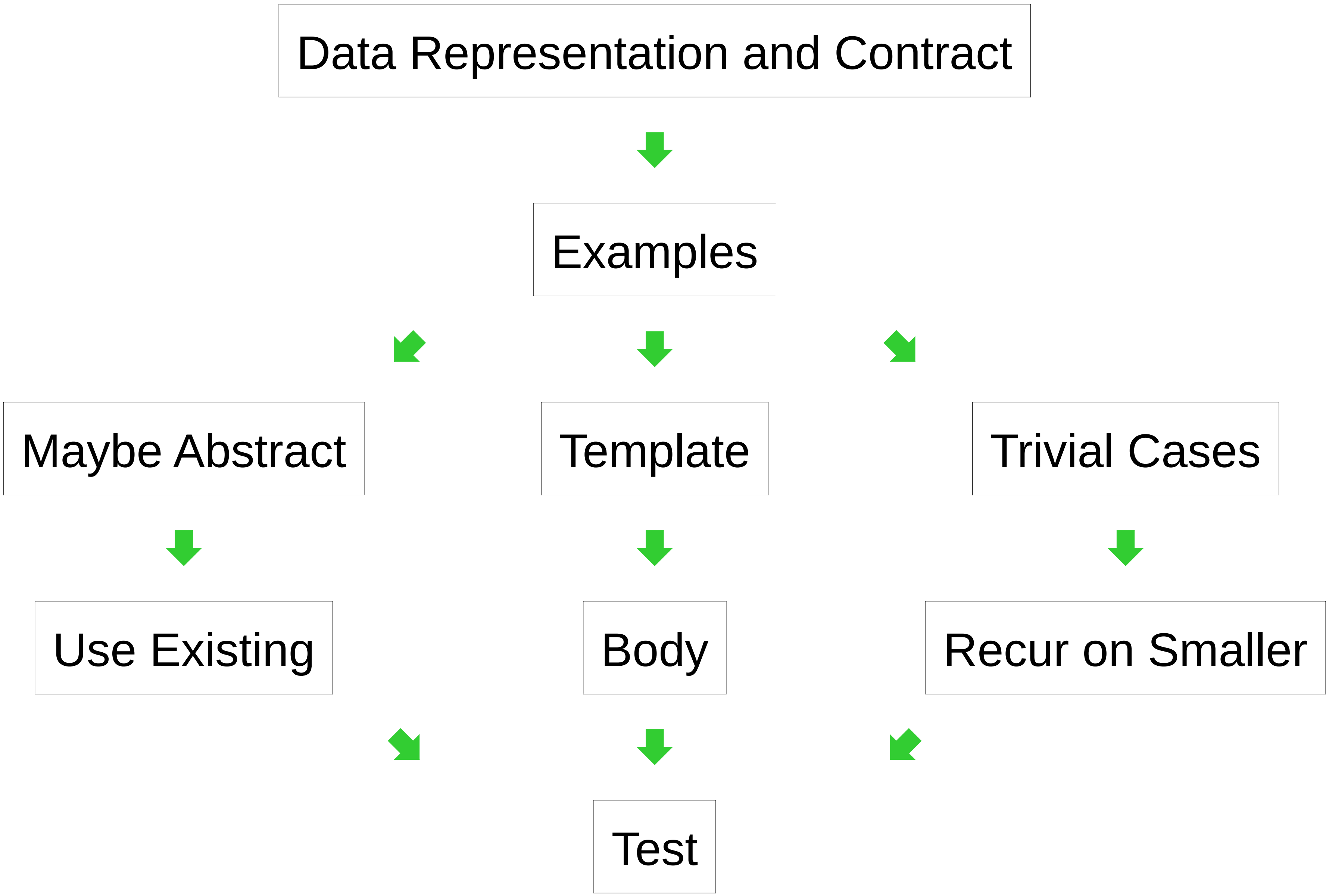
```
(define (odd-items l)
  (cond
    [(or (empty? l)
         (empty? (rest l)))
     l]
    [else ...]))
```

Back to Odd Items

Otherwise, it's helpful to have the **rest** of the ***rest***:

```
(define (odd-items l)
  (cond
    [(or (empty? l)
         (empty? (rest l)))
     l]
    [else (cons
           (first l)
           (odd-items (rest (rest l))))]))
```

How to Design A Program



Guessing a Number

```
; make-secret-checker : num -> (num -> sym)
(define (make-secret-checker n)
  (local [(define secret (random n))]
    (lambda (m)
      (cond
        [(= m secret) 'perfect]
        [(< m secret) 'too-small]
        [(> m secret) 'too-large]))))
```

- Implement the function **`discover-number`** which takes a number ***n*** and a function produced by **`(make-secret-checker n)`**, and returns the secret number in the function

Data Representation and Contract

Apparently done already:

```
; discover-number : num (num -> sym) -> num
```

Examples

```
(discover-number 1 (make-secret-checker 1))
"should be" 0

(discover-number 3 (make-secret-checker 3))
"should be" "0 or 1 or 2"
```

Maybe Abstract		Template		Trivial Cases	
⬇	or	⬇	or	⬇	?
Use Existing		Body		Recur on Smaller	

- Abstract/reuse: nothing obvious
- Template: nothing for **`num`**

... but is it really **`nat`**?

Yes, starting from 1

Template

↓

Body

```
; discover-number : nat (nat -> sym) -> nat
(define (discover-number n checker)
  (cond
    [(= n 1) ...]
    [else
     ...
     (discover-number (sub1 n) checker)
     ...]))
```

Template

↓

Body

```
; discover-number : nat (nat -> sym) -> nat
(define (discover-number n checker)
  (cond
    [(= n 1)   0]
    [else
     ...
     (discover-number (sub1 n) checker)
     ...]))
```

Template

↓

Body

```
; discover-number : nat (nat -> sym) -> nat
(define (discover-number n checker)
  (cond
    [(= n 1)   0]
    [else
     (cond
       [(symbol=? (checker n) 'perfect) n]
       [else
        (discover-number (sub1 n) checker)])]))
```

Template

↓

Body

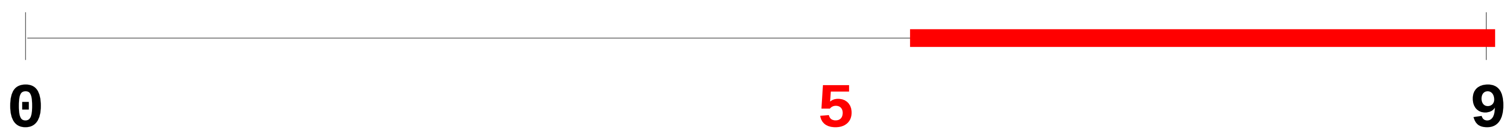
```
; discover-number : nat (nat -> sym) -> nat
(define (discover-number n checker)
  (cond
    [(= n 1)   0]
    [else
     (cond
       [(symbol=? (checker n) 'perfect) n]
       [else
        (discover-number (sub1 n) checker)])]))
```

This works, but is there a better way?

Guessing a Number

If you know a number is between 0 and 9:

and you only get **`'perfect`** or **`'imperfect`** answers to guesses, there's no better way to find the number



Guessing a Number

If you know a number is between 0 and 9:

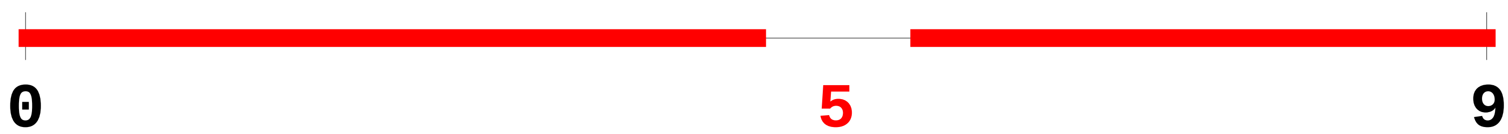
and you only get **`'perfect`** or **`'imperfect`** answers to guesses, there's no better way to find the number

`'perfect`

Guessing a Number

If you know a number is between 0 and 9:

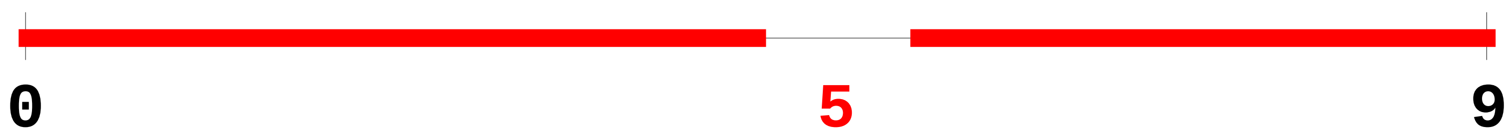
but you get **`'perfect`**, **`'too-small`**, or **`'too-large`** answers, it's better to guess in the middle



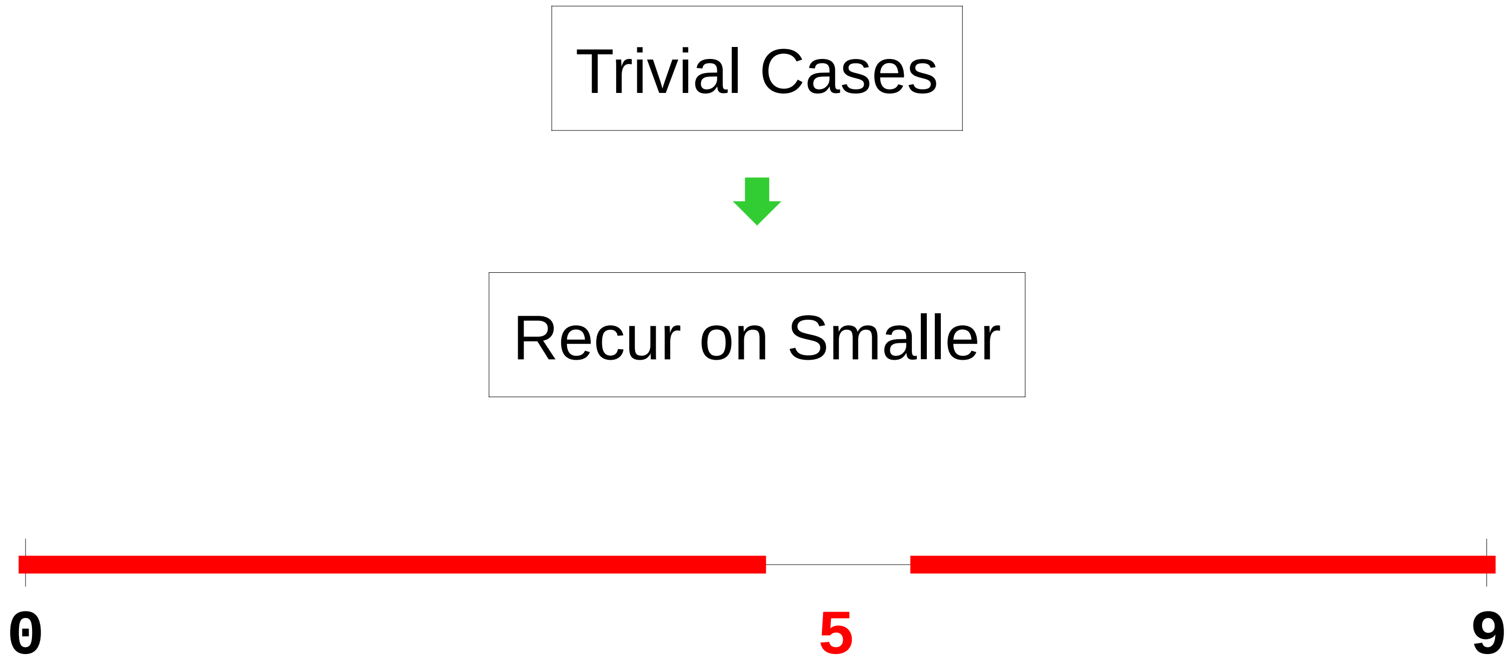
Guessing a Number

If you know a number is between 0 and 9:

but you get **'perfect**, **'too-small**, or **'too-large** answers, it's better to guess in the middle



'perfect



'perfect

- Trivially solvable if mid-point is **'perfect**
- Otherwise, mid-point results cuts the range in half **–** try again

Guessing A Number with Generative Recursion

```
(define (discover-number n checker)
  (discover-in-range 0 (sub1 n) checker))

; discover-in-range : nat nat (nat -> bool) -> num
; Finds the number between lo and hi (inclusive)
(define (discover-in-range lo hi checker)
  (cond
    [trivial? ...]
    [else
     ... (discover-in-range ...)
     ...]))
```

Guessing A Number with Generative Recursion

```
(define (discover-number n checker)
  (discover-in-range 0 (sub1 n) checker))

; discover-in-range : nat nat (nat -> bool) -> num
; Finds the number between lo and hi (inclusive)
(define (discover-in-range lo hi checker)
  (local [(define mid (quotient (+ lo hi) 2))]
    (cond
      [trivial? ...]
      [else
       ... (discover-in-range ...)
       ...])))
```

Guessing A Number with Generative Recursion

```
(define (discover-number n checker)
  (discover-in-range 0 (sub1 n) checker))

; discover-in-range : nat nat (nat -> bool) -> num
; Finds the number between lo and hi (inclusive)
(define (discover-in-range lo hi checker)
  (local [(define mid (quotient (+ lo hi) 2))]
    (cond
      [(symbol=? (checker mid) 'prefect) mid]
      [else
       ... (discover-in-range ...)
       ...])))
```

Guessing A Number with Generative Recursion

```
(define (discover-number n checker)
  (discover-in-range 0 (sub1 n) checker))

; discover-in-range : nat nat (nat -> bool) -> num
; Finds the number between lo and hi (inclusive)
(define (discover-in-range lo hi checker)
  (local [(define mid (quotient (+ lo hi) 2))]
    (cond
      [(symbol=? (checker mid) 'prefect) mid]
      [else
       ... (discover-in-range lo mid)
       ... (discover-in-range hi hi) ...]))))
```

Guessing A Number with Generative Recursion

```
(define (discover-number n checker)
  (discover-in-range 0 (sub1 n) checker))

; discover-in-range : nat nat (nat -> bool) -> num
; Finds the number between lo and hi (inclusive)
(define (discover-in-range lo hi checker)
  (local [(define mid (quotient (+ lo hi) 2))]
    (cond
      [(symbol=? (checker mid) 'prefect) mid]
      [else
       (cond
         [(symbol=? (checker mid) 'too-large)
          (discover-in-range lo mid)]
         [else
          (discover-in-range mid hi)])])))
```

Running the Guesser

```
(discover-number 10 check-7)
```

→

```
(discover-in-range 0 9 check-7)
```

using

```
(define (discover-number n checker)
  (discover-in-range 0 (sub1 n) checker))
```

Running the Guesser

```
(discover-in-range 0 9 check-7)
```

→

```
(cond
  [(symbol=? (check-7 4) 'perfect) 4]
  [else
   (cond
     [(symbol=? (check-7 4) 'too-large)
      (discover-in-range 0 4 check-7)]
     [else
      (discover-in-range 4 9 check-7)])])
```

using

```
(define (discover-in-range lo hi checker)
  (local [(define mid (quotient (+ lo hi) 2))]
    (cond
      [(symbol=? (checker mid) 'prefect) mid]
      [else
       (cond
         [(symbol=? (checker mid) 'too-large)
          (discover-in-range lo mid)]
         [else
          (discover-in-range mid hi)])])))
```

Running the Guesser

```
(cond
  [(symbol=? (check-7 4) 'perfect) 4]
  [else
   (cond
     [(symbol=? (check-7 4) 'too-large)
      (discover-in-range 0 4 check-7)]
     [else
      (discover-in-range 4 9 check-7)])])
```

→

```
(cond
  [(symbol=? (check-7 4) 'too-large)
   (discover-in-range 0 4 check-7)]
  [else
   (discover-in-range 4 9 check-7)])
```

Running the Guesser

```
(cond
  [(symbol=? (check-7 4) 'too-large)
   (discover-in-range 0 4 check-7)]
  [else
   (discover-in-range 4 9 check-7)])
```

→

```
(discover-in-range 4 9 check-7)
```

Running the Guesser

```
(discover-in-range 4 9 check-7)
```

→

```
(cond
  [(symbol=? (check-7 6) 'perfect) 6]
  [else
   (cond
     [(symbol=? (check-7 6) 'too-large)
      (discover-in-range 4 6 check-7)]
     [else
      (discover-in-range 6 9 check-7)])])
```

Running the Guesser

```
(cond
  [(symbol=? (check-7 6) 'perfect) 6]
  [else
   (cond
     [(symbol=? (check-7 6) 'too-large)
      (discover-in-range 4 6 check-7)]
     [else
      (discover-in-range 6 9 check-7)])])
```

→

```
(discover-in-range 6 9 check-7)
```

Running the Guesser

```
(discover-in-range 6 9 check-7)
```

→

```
(cond
  [(symbol=? (check-7 7) 'perfect) 7]
  [else
   (cond
     [(symbol=? (check-7 7) 'too-large)
      (discover-in-range 6 7 check-7)]
     [else
      (discover-in-range 7 9 check-7)])])
```

Running the Guesser

```
(cond
  [(symbol=? (check-7 7) 'perfect) 7]
  [else
   (cond
     [(symbol=? (check-7 7) 'too-large)
      (discover-in-range 6 7 check-7)]
     [else
      (discover-in-range 7 9 check-7)])])
```

→

```
7
```

Running the Guesser Again

```
(discover-number 3 check-2)
```

$\longrightarrow$

```
(discover-in-range 0 2 check-2)
```

Running the Guesser Again

```
(discover-in-range 0 2 check-2)
```

$\rightarrow$

```
(cond
  [(symbol=? (check-2 1) 'perfect) 1]
  [else
   (cond
     [(symbol=? (check-2 1) 'too-large)
      (discover-in-range 0 1 check-2)]
     [else
      (discover-in-range 1 2 check-2)])])
```

Running the Guesser Again

```
(cond
  [(symbol=? (check-2 1) 'perfect) 1]
  [else
   (cond
     [(symbol=? (check-2 1) 'too-large)
      (discover-in-range 0 1 check-2)]
     [else
      (discover-in-range 1 2 check-2)])])
```

→

```
(discover-in-range 1 2 check-2)
```

Running the Guesser Again

```
(discover-in-range 1 2 check-2)
```

→

```
(cond
  [(symbol=? (check-2 1) 'perfect) 1]
  [else
   (cond
     [(symbol=? (check-2 1) 'too-small)
      (discover-in-range 1 2 check-7)]
     [else
      (discover-in-range 1 2 check-2)])])
```

Running the Guesser Again

```
(cond
  [(symbol=? (check-2 1) 'perfect) 1]
  [else
   (cond
     [(symbol=? (check-2 1) 'too-small)
      (discover-in-range 1 2 check-7)]
     [else
      (discover-in-range 1 2 check-2)])])
```

$\longrightarrow$

```
(discover-in-range 1 2 check-2)
```

Running the Guesser Again

```
(discover-in-range 1 2 check-2)
```

→

```
(discover-in-range 1 2 check-2)
```

Running the Guesser Again

```
(discover-in-range 1 2 check-2)
```

$\rightarrow$

```
(discover-in-range 1 2 check-2)
```

Infinite loop!

Generative Recursion and Termination

- With structural recursion, a program always ***terminates***
  - Every value is finite
- With generative recursion, termination becomes more tricky
  - You have to argue that the problem size definitely gets smaller for every recursive call

Guessing a Number, Corrected

```
(define (discover-in-range lo hi checker)
  (local [(define mid (quotient (+ lo hi) 2))]
    (cond
      [(symbol=? (checker mid) 'prefect) mid]
      [else
       (cond
         [(symbol=? (checker mid) 'too-large)
          (discover-in-range lo (sub1 mid))]
         [else
          (discover-in-range (add1 mid) hi)])])))
```

Algorithms

Our `discover-in-range` function is an example of a general ***algorithm*** called ***binary search***

Many algorithms are less obvious than binary search

Mostly you'll use general algorithms, not invent them

- Algorithm textbooks are like "recipe" books
- Few people design new general algorithms

Generative recursion is far more common than general algorithms, and it's often merely structural recursion