Constraint Jacobians for Constant-Time Inverse Kinematics and Assembly Optimization

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Abstract
An algorithm for the constant-time solution of systems of geometric constraint equations is presented in this work. Constraint equations and their Jacobians may be used in conjunction with other numerical methods to solve for a variety of kinematics, dynamics, and assembly optimization problems. The use of constraint equations for these purposes is an under-utilized method in this area. The use of quaternions for coordinates in these constraint equations is shown to be a key choice in the optimization problem for avoiding local minima.

1 Introduction
The use of the constraint style of programming for the solution of kinematics, dynamics and automatic interactive re-assembly problems is a versatile and extensible framework in which to operate in. These equations express geometric relationships between bodies, such as the requirement that two joints stay together.

We will denote our coordinates for these geometric constraints with $\mathbf{q}$, our augmented Cartesian generalized coordinates. An alternative is to use joint-space (i.e. joint angle) coordinates, but such reduced coordinates limit our expressivity of the constraint equations and prohibit non-holonomic and other irregular constraints from being formulated [2].