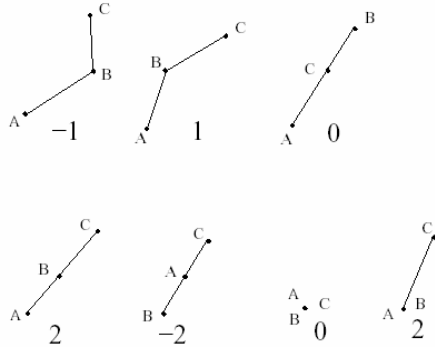
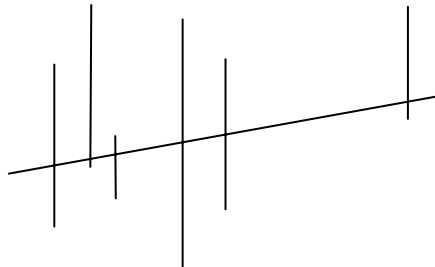


Computational Geometry Assignment 1  
Due 9/23 before class

1. Implement the primitive  $CCW(A,B,C)$ , which given 3 points  $A,B,C$ , computes the following information: If  $A \neq B$ , then the primitive returns 1 (resp. -1) if  $A,B,C$  make a right (resp. left) turn, 0 if  $C$  lies in the closure of  $AB$ , 2 if  $B$  lies in the relative interior of  $AC$ , -2 if  $A$  lies in the relative interior of  $BC$ . If  $A=B$  then the primitive returns 0 if  $A=B=C$  and 2 if  $A \neq C$ . Use  $CCW$  to implement a routine to check whether two (closed) segments intersect.

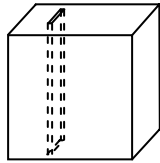


2. Design a linear algorithm that given a set of vertical segments sorted by their  $x$  coordinate, determines if a line can pass through all of them. How hard is it to compute an implicit description of all the stabbing lines?



3. A point  $\mathbf{p}$  is said to dominate another point  $\mathbf{q}$  if  $\mathbf{p}_x \geq \mathbf{q}_x$  and  $\mathbf{p}_y \geq \mathbf{q}_y$ . A point is called a maximum if no other point dominates it. Design and analyze an efficient algorithm for finding all the maxima of a given set.
4. Given  $n$  points in the plane, how can you construct a simple polygon having each point as a vertex and none other? Still more interesting is to prove that in the worst case  $\Omega(n \log n)$  operations are required. Why is that so?
5. Develop and analyze an algorithm for reporting whether or not two convex polygons of  $n$  and  $m$  vertices intersect. Try to achieve  $O(\log(n+m))$  time. (Chazelle & Dobkin 1987).

6. Let  $S$  be a set of  $n$  circles in the plane. Describe a plane sweep algorithm to compute all intersection points between the circles. (Because we deal with circles, not discs, two circles do not intersect if one lies entirely inside the other.) Your algorithm should run in  $O((n+k)\log n)$  time, where  $k$  is the number of intersection points.
  
7. Create an example of an object that cannot be decomposed into tetrahedra using only its vertices. Start with a cube of wood, and then cut “slits” out of it: thin holes going from one cube face to the opposite face with walls parallel to the other cube faces. You can use several of these, in the 3 possible orientations (X,Y,Z). Argue why the object you created cannot be decomposed. Hint: try to “almost” “wall off” with holes a small region of solid wood inside the cube. You should avoid having slits intersect, so that you don’t create any vertices inside the cube.



[Problems by Chazelle; O’Rourke; de Berg et al.]