Bresenham Circles

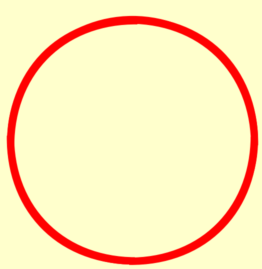
Lecture Set 2

CS5600 *Intro to Computer Graphics*
Rich Riesenfeld
Spring 2006

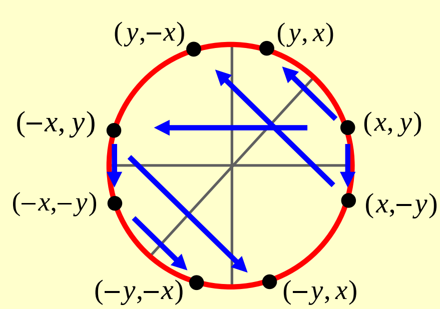
More Raster Line Issues

- Fat lines with multiple pixel width
- Symmetric lines
- End point geometry – how should it look?
- Generating curves, e.g., circles, etc.
- Jaggies, staircase effect, aliasing...

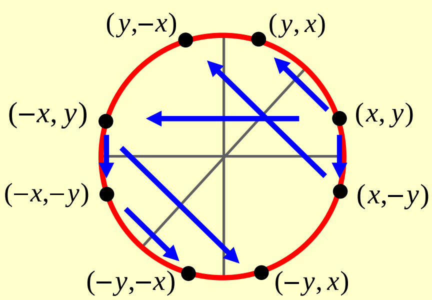
Generating Circles



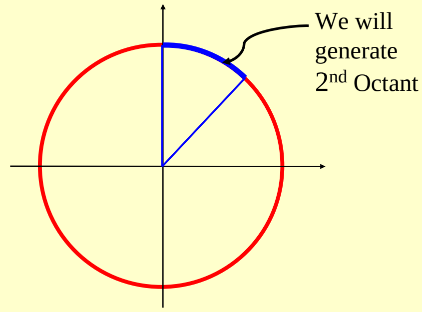
Exploit 8-Point Symmetry



Once More: 8-Pt Symmetry



Only 1 Octant Needed



Generating pt (x,y) gives

the following 8 pts by symmetry:

$\{(x,y), (-x,y), (-x,-y), (x,-y),$
$(y,x), (-y,x), (-y,-x), (y,-x)\}$

2^{nd} Octant Is a Good Arc

- It is a function in this domain
  - single-valued
  - no vertical tangents: $|slope| \leq 1$
- Lends itself to Bresenham
  - only need consider E or SE

Implicit Eq's for Circle

- Let $F(x,y) = x^2 + y^2 - r^2$
- For (x,y) on the circle, $F(x,y) = 0$
- So, $F(x,y) > 0 \Rightarrow (x,y)$ *Outside*
- And, $F(x,y) > 0 \Rightarrow (x,y)$ *Inside*

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Choose E or SE

- Function is $x^2 + y^2 - r^2 = 0$
- So, $F(M) \geq 0 \Rightarrow SE$
- And, $F(M) < 0 \Rightarrow E$

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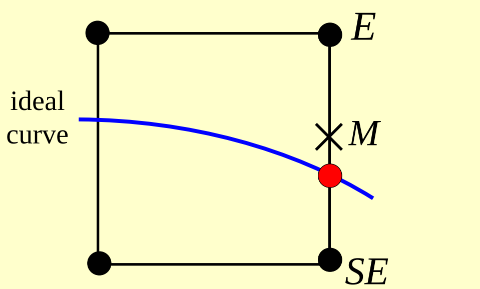
$F(M) \geq 0 \Rightarrow SE$



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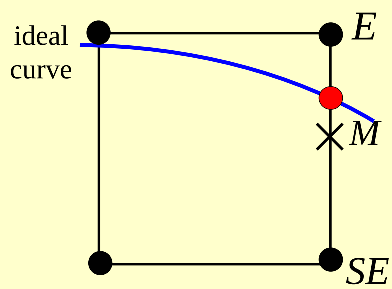
$F(M) < 0 \Rightarrow E$



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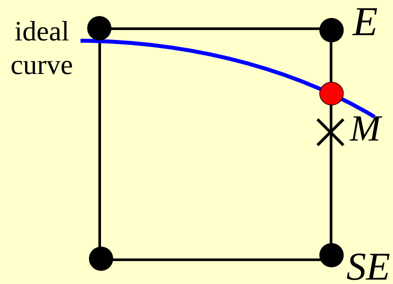
Decision Variable d

Again, we let,

$$d = F(M)$$

Look at Case 1: E



$d_{old} < 0 \Rightarrow E$

$$d_{old} = F(x_p + 1, y_p - \tfrac{1}{2})$$
$$= (x_p + 1)^2 + (y_p - \tfrac{1}{2})^2 - r^2$$

$d_{old} < 0 \Rightarrow E$

$$d_{new} = F(x_p + 2, y_p - \tfrac{1}{2})$$
$$= (x_p + 2)^2 + (y_p - \tfrac{1}{2})^2 - r^2$$

$d_{old} < 0 \Rightarrow E$

$$d_{new} = d_{old} + (2x_p + 3)$$

Since,

$$(x_p + 2)^2 - (x_p + 1)^2 = (4x_p + 4) - (2x_p + 1)$$
$$= 2x_p + 3$$

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$d_{old} < 0 \Rightarrow E$

$$d_{new} = d_{old} + \Delta_E \;,$$

where,

$$\boxed{\Delta_E = 2x_p + 3}$$

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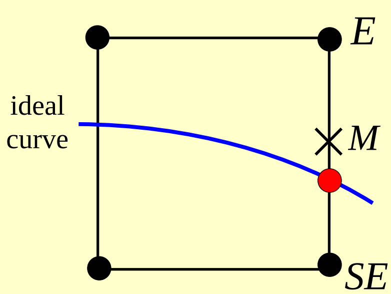
Look at Case 2: *SE*



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$d_{old} \geq 0 \Rightarrow SE$

$$d_{new} = F(x_p + 2,\; y_p - 3/2)$$
$$= (x_p + 2)^2 + (y_p - 3/2)^2 - r^2$$
$$d_{new} = d_{old} + (2x_p - 2y_p + 5)$$

Because,…, straightforward manipulation

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$d_{old} \geq 0 \Rightarrow SE$

$$d_{new} - d_{old} =$$

$$\underbrace{(2x_p + 3)} + \underbrace{(-3y_p + \tfrac{9}{4})} - \underbrace{(-y_p + \tfrac{1}{4})}$$

From Δ_E calculation

From *new* y-coordinate

From *old* y-coordinate

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$d_{old} \geq 0 \Rightarrow SE$

I.e.,

$$d_{new} = d_{old} + (2x_p - 2y_p + 5)$$

$$= d_{old} + \Delta_{SE}$$

$$\Delta_{SE} = 2x_p - 2y_p + 5$$

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Note: Δ´s Not Constant

Δ_E *and* Δ_{SE}

depend on values of x_p and y_p

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Summary

- Δ´s are no longer constant over entire line
- Algorithm structure is *exactly* the same
- Major difference from the line algorithm
  - Δ is re-evaluated at each step
  - Requires *real* arithmetic

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Initial Condition

- Let *r* be an integer. Start at $(0, r)$
- Next midpoint *M* lies at $(1, r - \frac{1}{2})$
- So, $F(1, r - \frac{1}{2}) = 1 + (r^2 - r - \frac{1}{4}) - r^2$

$$= \frac{5}{4} - r$$

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Ellipses

- Evaluation is analogous
- Structure is same
- Have to work out the Δ´s

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Getting to Integers

- Note the previous algorithm involves *real* arithmetic
- Can we modify the algorithm to use integer arithmetic?

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Integer Circle Algorithm

- Define a shift decision variable

$$h = d - \frac{1}{4}$$

- In the code, plug in $d = h + \frac{1}{4}$

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Integer Circle Algorithm

- Now, the initialization is $h = 1 - r$
- So the initial value becomes

$$F(1, r - \frac{1}{2}) - \frac{1}{4} = (\frac{5}{4} - r) - \frac{1}{4}$$
$$= 1 - r$$

Integer Circle Algorithm

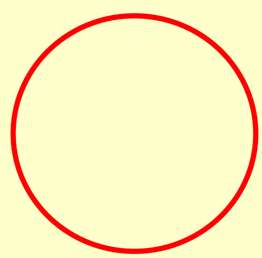
- Then, $d < 0$ becomes $h < -\frac{1}{4}$
- Since h an integer

$$h < -\frac{1}{4} \Leftrightarrow h < 0$$

Integer Circle Algorithm

- But, h begins as an integer
- And, h gets incremented by integer
- Hence, we have an integer circle algorithm
- Note: Sufficient to test for $h < 0$

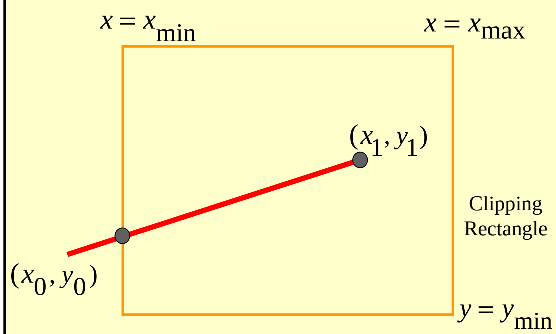
End of Bresenham Circles



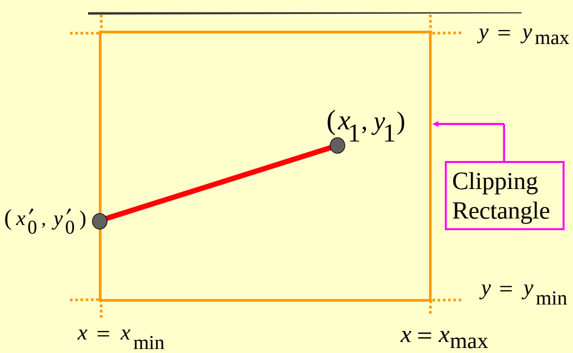
Another Digital Line Issue

- Clipping Bresenham lines
- The integer slope is not the true slope
- Have to be careful
- More issues to follow

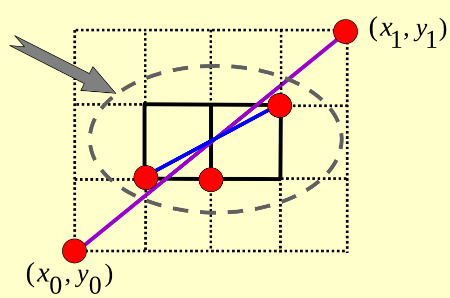
Line Clipping Problem



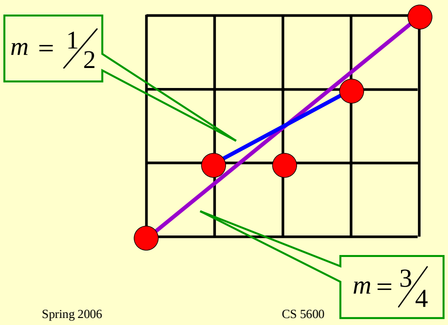
Clipped Line



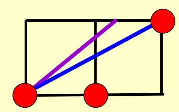
Drawing Clipped Lines



Clipped Line Has Different Slope !

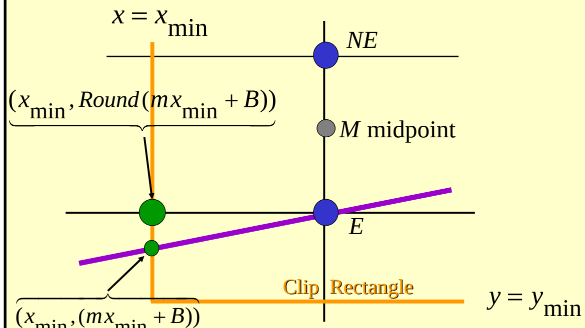


Pick Right Slope to Reproduce Original Line Segment

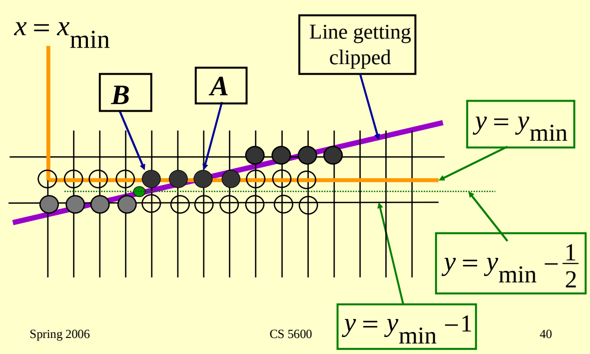


Zoom of previous situation

Clipping Against $x = x_{min}$



Clipping Against $y = y_{min}$



Clipping Against $y = y_{min}$

- Situation is complicated
- Multiple pixels involved at $(y = y_{min})$
- Want all of those pixels as "*in*"
- Analytic ∩, rounding x gives A
- We want point B

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Clipping Against $y = y_{min}$

- Use $Line$ ∩ $y = y_{min} - ½$
- Round *up* to nearest integer x
- This yields point B, the desired result

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Observations

- Lines are complicated
- Many aspects to consider
- We omitted many
- What about intensity of

  $y = x$ vs $y = 0$?

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The End

Bresenham Circles

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