Image Processing with Nonparametric Neighborhood Statistics

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See also: http://www.cs.utah.edu/~suyash/pubs/uinta.html

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Papers

- Suyash P. Awate and Ross T. Whitaker, "Higher-Order Image Statistics for Unsupervised, Information-Theoretic, Adaptive Image Filtering UINTA", IEEE Computer Vision and Pattern Recognition (CVPR) 2005, v 2, pp 44-51
- Suyash P. Awate and Ross T. Whitaker, "Nonparametric Neighborhood Statistics for MRI Denoising", Information Processing in Medical Imaging (IPMI) 2005, pp 677-688
- Tolga Tasdizen, Suyash P. Awate, Ross T. Whitaker, Norman Foster, "MRI Tissue Classification with Neighborhood Statistics: A Nonparametric, Entropy-Minimizing Approach", Medical Image Computing and Computer Assisted Intervention (MICCAI) 2005, v 2, pp 517-525
- Suyash P. Awate and Ross T. Whitaker, "Unsupervised, Information-Theoretic, Adaptive Image Filtering with Applications to Image Restoration UINTA", IEEE Trans. Pattern Analysis and Machine Intelligence (PAMI) 2006, 28(3):364-376
- Suyash P. Awate, Tolga Tasdizen, Ross T. Whitaker, "Unsupervised Texture Segmentation with Nonparametric Neighborhood Statistics", European Conference on Computer Vision (ECCV) 2006
- Suyash P. Awate, Tolga Tasdizen, Norman Foster, Ross T. Whitaker, "Adaptive, Nonparametric Markov Modeling for Unsupervised, MRI Brain-Tissue Classification", Medical Image Analysis (MEDIA) 2006, 10(5):726-739
- Suyash P. Awate and Ross T. Whitaker, "Feature-Preserving MRI Denoising using a Nonparametric, Empirical-Bayes Approach", IEEE Trans. Medical Imaging (TMI) 2007 (To Appear)

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Talk Overview

- Motivation
- Image denoising
- Density estimation
- UINTA filtering strategy overview
- Entropy minimization
- Implementation issues: statistics, image processing
- Other applications
- Final thoughts

Images

Denoising Vs Reconstruction

- Any geometric/statistical penalty can be applied in two ways:
  1. Gradient descent as filter (choose # iterations)
  2. With data (fidelity) term to steady state
     - Variational
     - Noise/measurement models, optimality, etc.

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Variational Methods
E.g Anisotropic Diffusion

- Perona&Malik (1990)

$$\frac{\partial f}{\partial t} = \nabla \cdot c(|\nabla f|)\nabla f$$

- Penalty:
  - Quadratic on grad-mag with outliers (discontinuities)
    - Nordstrom 1990; Black et. al 1998
  - Favors piecewise const. Images

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Other Flattening Approaches

- Total variation
  - Rudin et. al (1992)
- Mumford-Shah (1989) related
  - Explicit model of edges
  - Cartoon model
- Level sets to model edges
  - Chan & Vese (2000)
  - Tsai, Yezzi, Willsky (2000)
- Model textures + boundaries
  - Meyer (2000)
  - Vese & Osher (2002)

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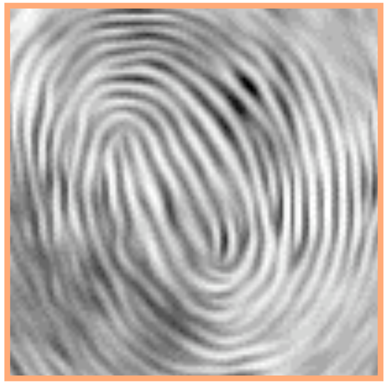
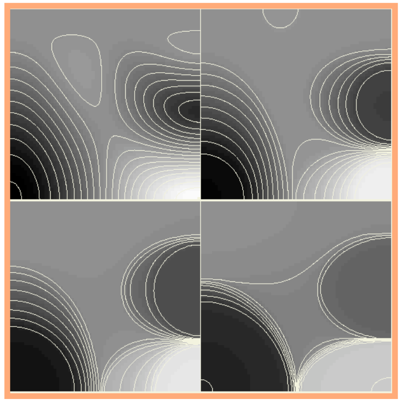
PDE Methods
Other Examples

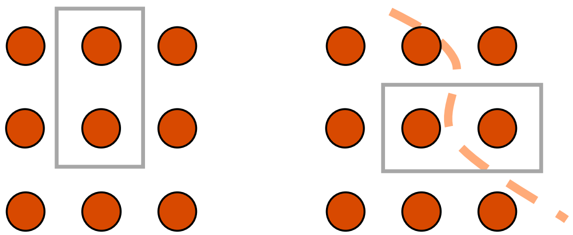
- Weickert (1998)
  - Coherence enhancing
- Tasdizen et. al (2001)
  - Piecewise-flat normals
- Wilmore flows
  - Minimize curvature





Markov Random Fields
E.g. Geman and Geman (1984)

- Gibbs energies on cliques
  - Quantify image preferences
    - Discrete geometric configurations
  - Given a priori
  - Hidden variables/processes to capture features



Issues

- Prioritize geometric configurations a priori
  - Works well of the model fits, otherwise...
- Free parameters
  - Thresholds -> determine when to apply different models (e.g. "preserve edge or smooth")
- Generality
  - Cartoon-like simplifications are disastrous in many applications
- Increasing the geometric complexity
  - Is there a better way?

Examples

Lena

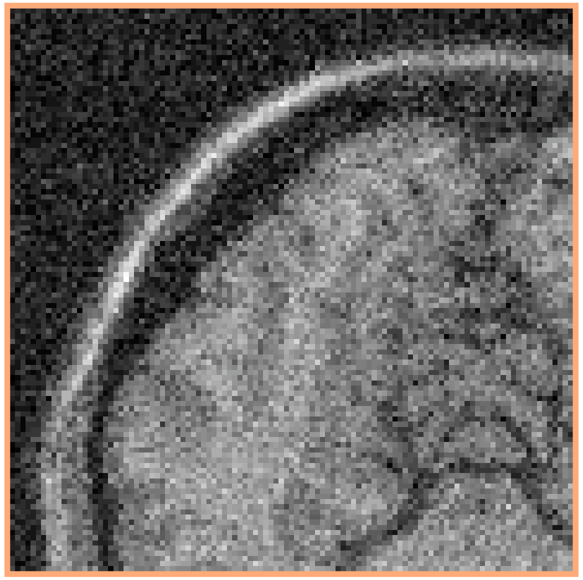
Anisotropic Diffusion

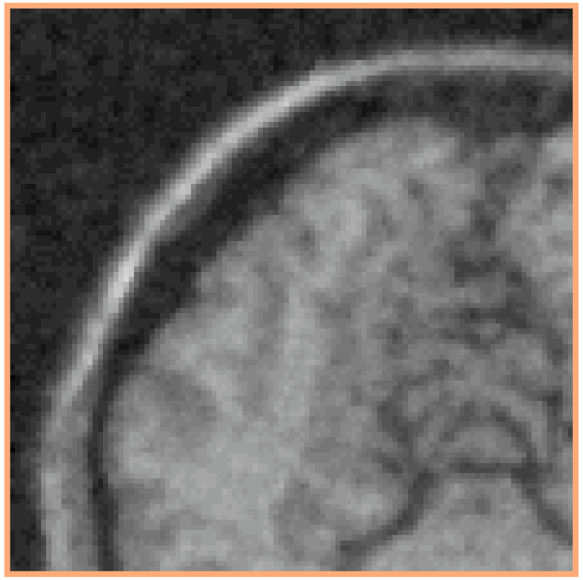
Coherence Enhancing

Curvature Flow

Examples

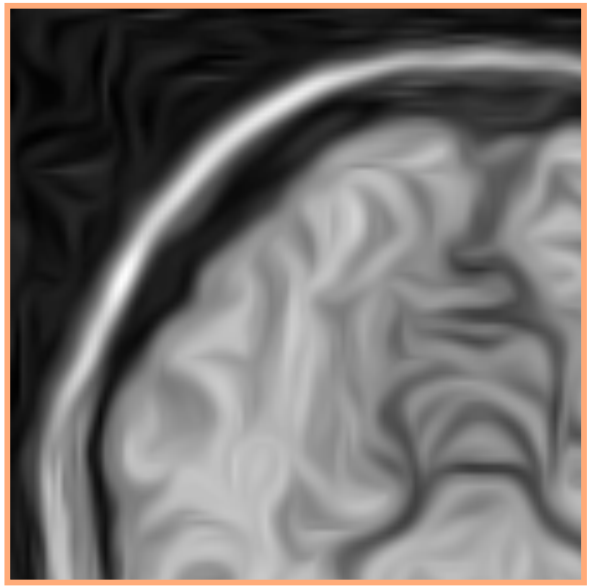
MRI (Simulated noise)

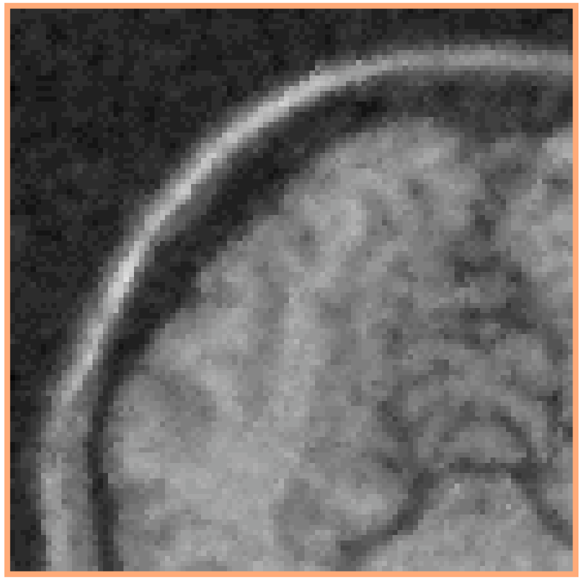




Bilateral Filtering

Coherence Enhancing

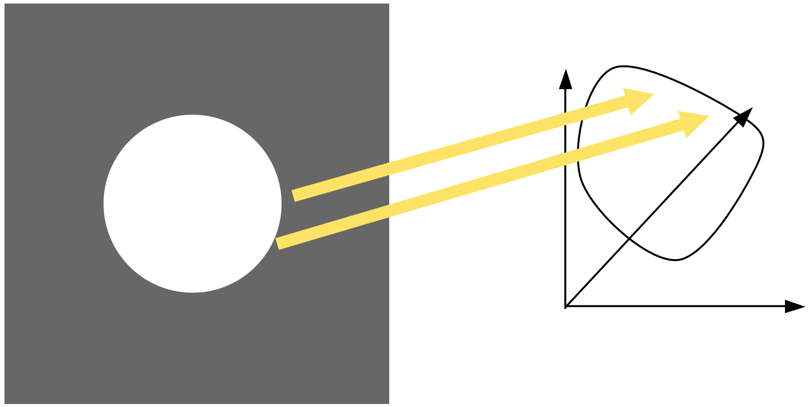


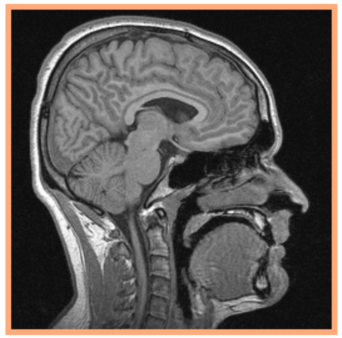


Anisotropic Diffusion

Observations About Images

- Statistics of *natural* images are not *so* random
  - Huang & Mumford (1999)
- But not so simple
  - Manifolds in high-dimensional spaces
  - de Silva & Carlsson (2003)





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Proposed Strategy

- Infer the appropriate Markovian relationships from the data
  - Images neighborhoods (nhds) as random processes
  - Move away from geometric formulations
- Increase redundancy (functional dependency) of image nhds
  - Information content
  - Entropy
- Optimal posteriori estimates (noise model)

Related Work

- DUDE algortihm–Weissman et. al (2003)
  - Discrete channels + noise model
  - MLE estimation
- Texture synthesis
  - Efros & Leung (1999)
  - Wei & Levoy (2002)
- NL-means, Baudes et al. (CVPR 2005)
  - Independent, simultaneously presented
  - More later…
- Sparsity in image neighborhoods
  - Roth and Black 2005
  - Elad and Aharon 2006

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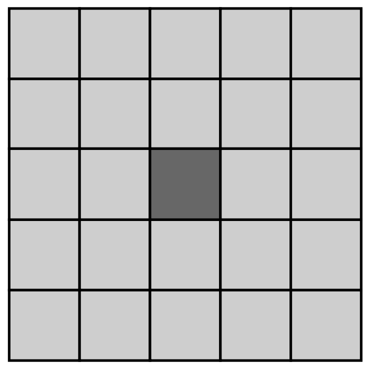
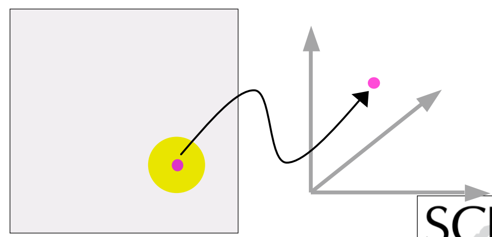
Image Model

- Pixels and neighborhoods Z = (X, Y)
  - P(Z), P(X|Y)
- Scenario
  - Corrupted image -> noise model
  - Prior knowledge P(X|Y)
  - Theorems:
    - Can produce most likely image x′ using P(X|Y = y′)
    - Iterate to produce optimal estimate





Modeling P(Z)

- Set of image neighborhoods
  - Large, complex, high-dimensions
- Approach
  - Represent complexity through examples
  - Nonparametric density estimation

Nonparametric, Multivariate Density Estimation

- Nonparametric estimation
  - No prior knowledge of densities
  - Can model *real* densities
- Statistics in higher dimensions
  - Curse of dimensionality (volume of n-sphere -> 0)
  + However, empirically more optimistic
  + Z has identical marginal distributions
  + Lower dimensional manifolds in feature space

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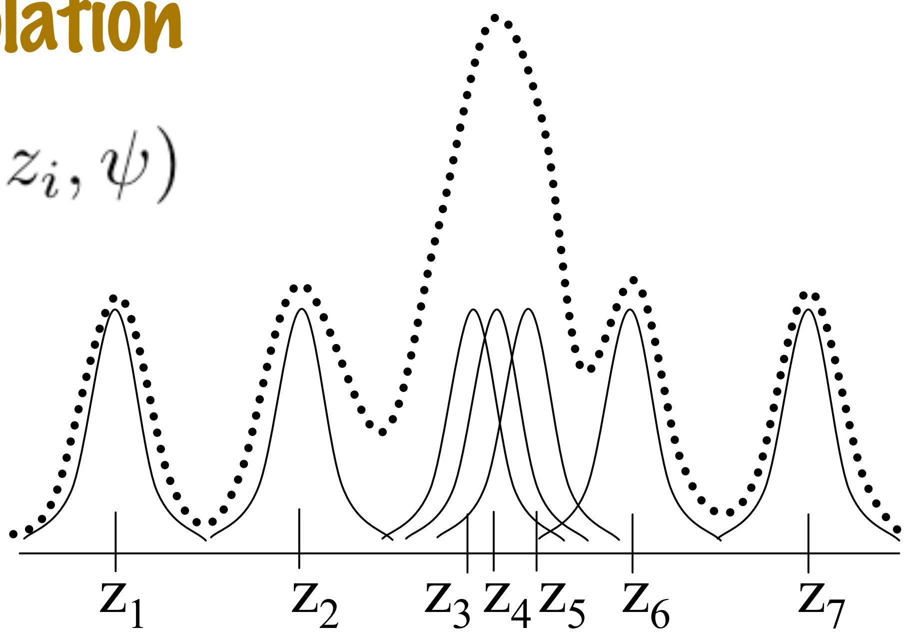
Parzen Windows (Parzen 1962)

- Scattered-data interpolation

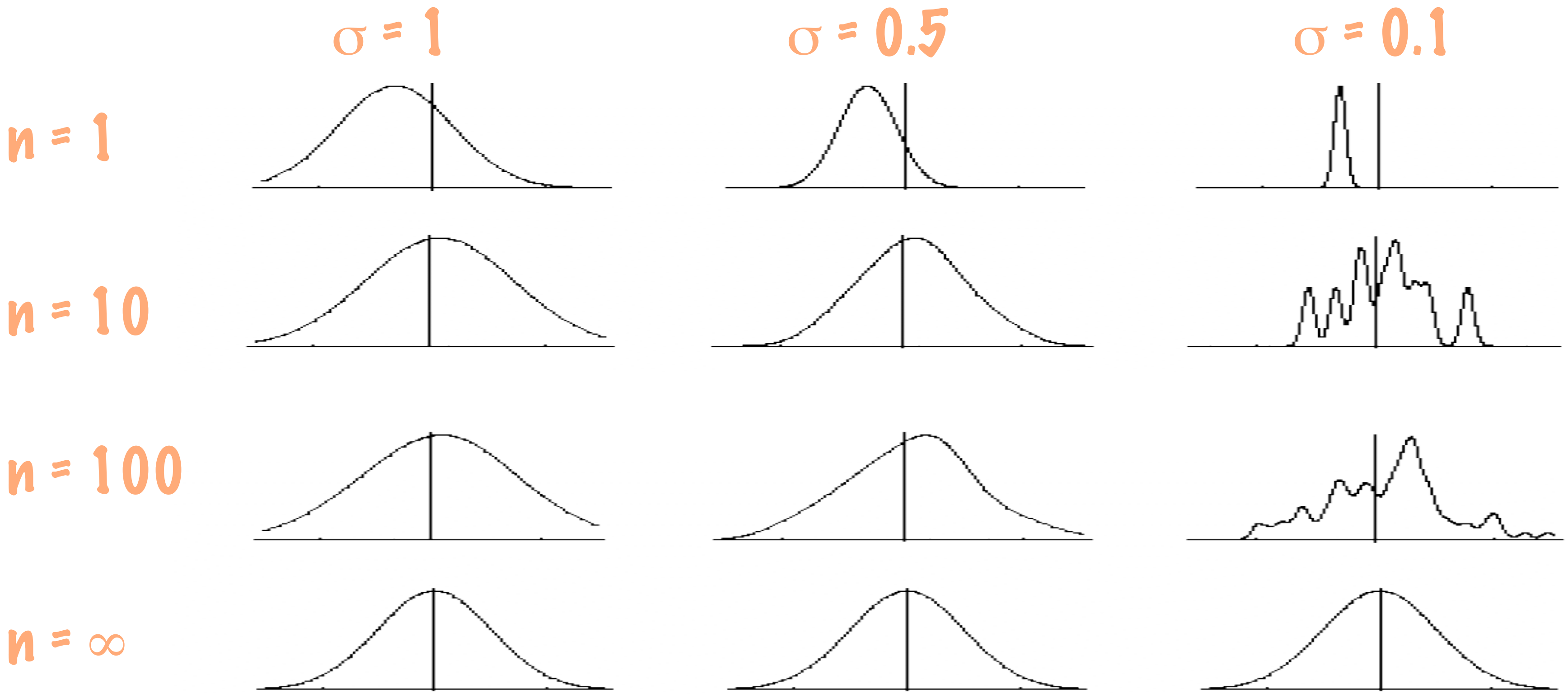
$$p(z) \approx \frac{1}{|A|} \sum_{z_i \in A} G(z - z_i, \psi)$$



- Window function
  - $G \equiv$ Gaussian
  - Covariance matrix: $\psi = \sigma^2 I$

Parzen Windows (Parzen 1962)

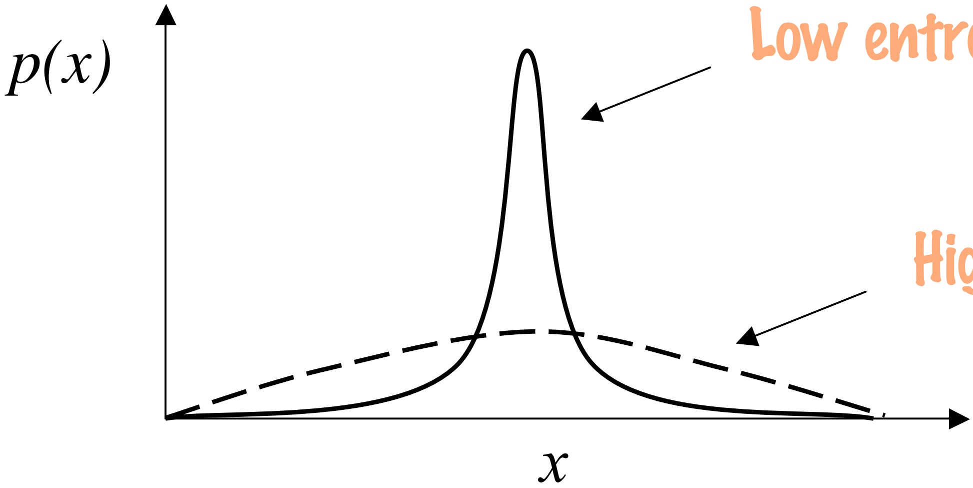
- Effects of finite sampling (Duda & Hart)



Entropy (Shannon 1948)

- Entropy of a random variable *X* (instance x)
  - Measure of *uncertainty* – information content of a sample

$$h(X) = -\int p(x) \log p(x) dx = -E_p \left[\log p(X) \right]$$



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UINTA Strategy
Awate & Whitaker CVPR 2005, PAMI 2006

- Iterative algorithm
- Progressively minimizes the entropy of image nhds Z = (X, Y)
  - Pixel entropies (X) conditioned on nhd values (Y)
  - Gradient descent (time steps -> mean shift)
- Nonparametric density estimation
  - Stochastic gradient descent

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Entropy Minimization

- Entropy as sample mean

$$
\begin{aligned}
h(Z) &= -E_p[\log p(Z)] \\
&\approx \frac{1}{|B|} \sum_{i \in B} \log p\left(z_i\right) \\
&\approx \frac{1}{|B|} \sum_{i \in B} \log \left(\frac{1}{|A|} \sum_{j \in A} G(z_i - z_j, \psi) \right)
\end{aligned}
$$

  - Set *B*: all pixels in image
  - Set *A*: a small *random* selection of pixels
  - z_i shorthand for $z(s_i)$
- Stochastic approximation

Entropy Minimization

- Stochastic approximation
  - Reduce $O(|B|^2)$ to $O(|A||B|)$
  - Efficient optimization
- Stochastic-gradient descent

$$\begin{aligned}\Delta x &= -\lambda \frac{\partial h(X|Y=y)}{\partial x} \\ &\approx \frac{\lambda \psi^{-1}}{|B|}\left[\sum_{j \in A} \frac{G(z_j - z, \Psi)}{\sum_{k \in A} G(z_k - z, \Psi)} x_j - x\right]\end{aligned}$$

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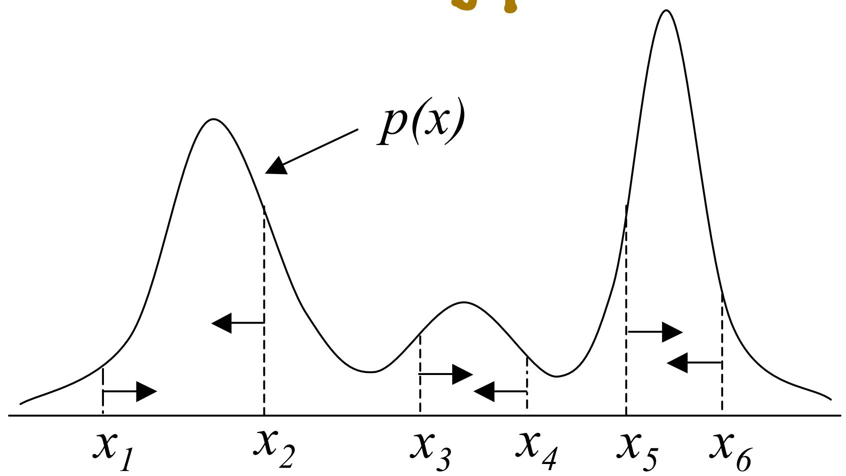
Mean-Shift Procedure (Fukunaga et al. 1975)

- Entropy minization <-> mean shift

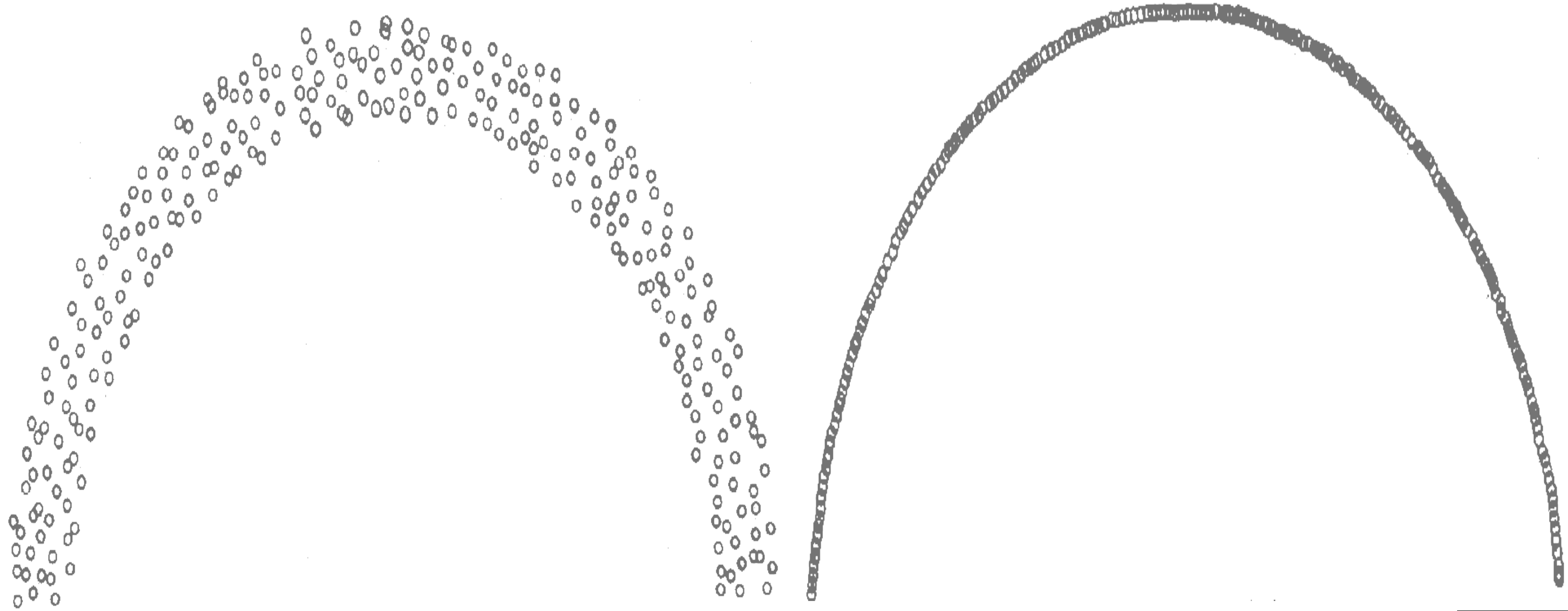
$$\lambda = \Psi|B| \quad x \longleftarrow \sum_j w_j x_j$$

- Mean-shift – a mode seeking procedure



Mean-Shift Procedure (Fukunaga et al. 1975)

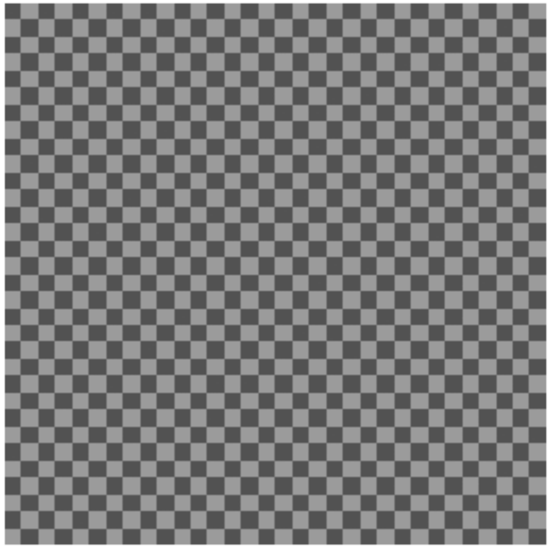
- Data filtering to reduce noise
  - Hand tuned parameters



Implementation Issues

- Scale selection for Parzen windowing
  - Automatic – min entropy with cross validation
- Rotational invariance
- Boundary neighborhoods
- Random sample selection – nonstationary image statistics
- Stopping criteria

Results



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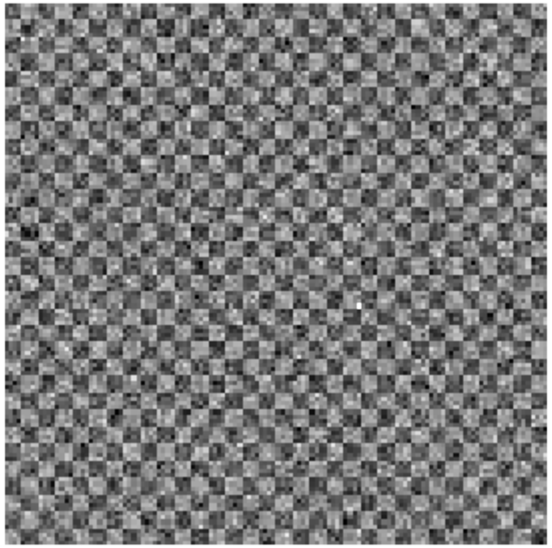
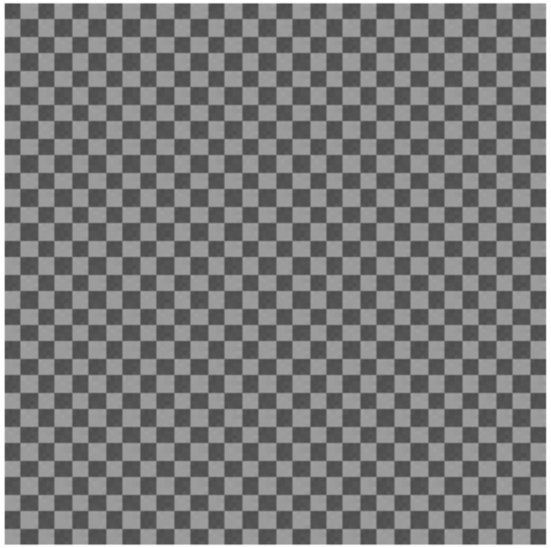


Noisy



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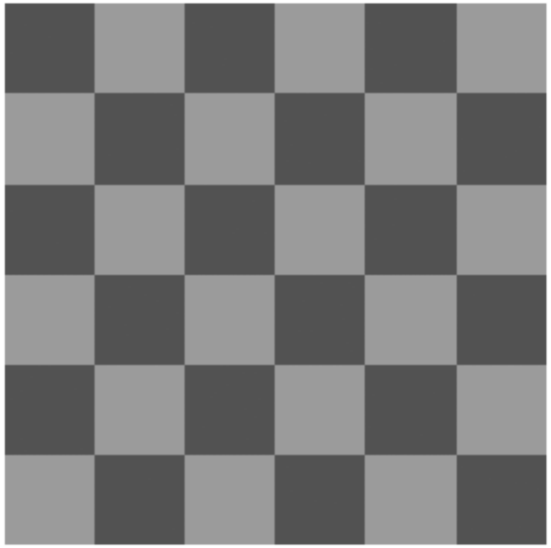
Checkerboard With Noise



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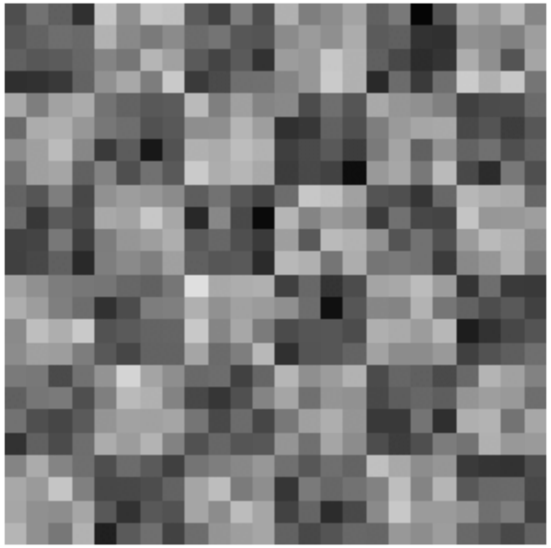

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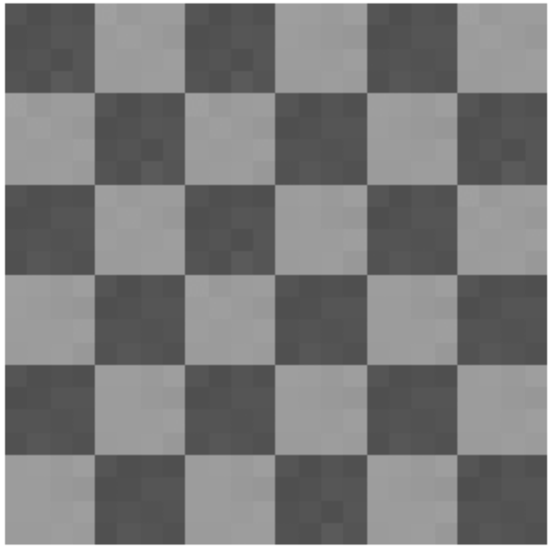

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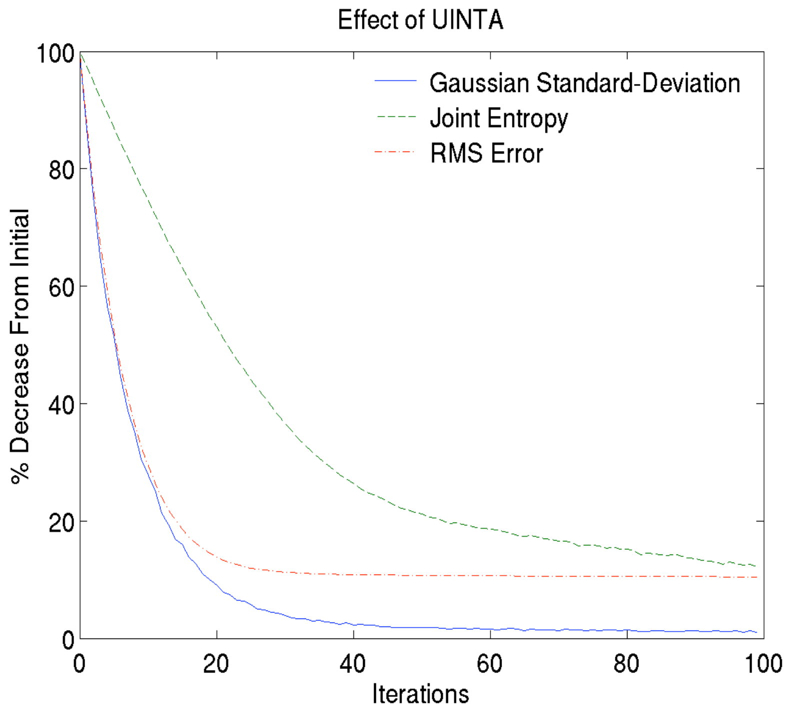
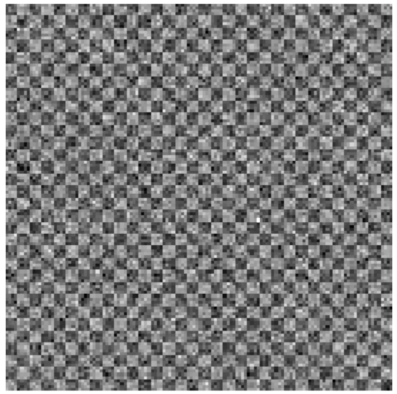
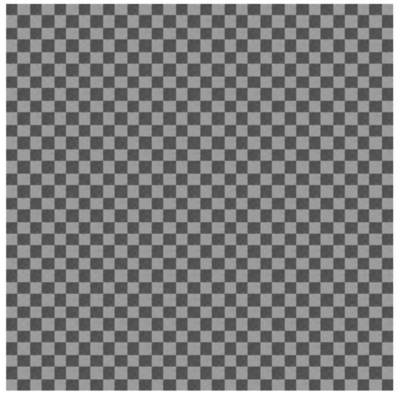
Quality of Denoising

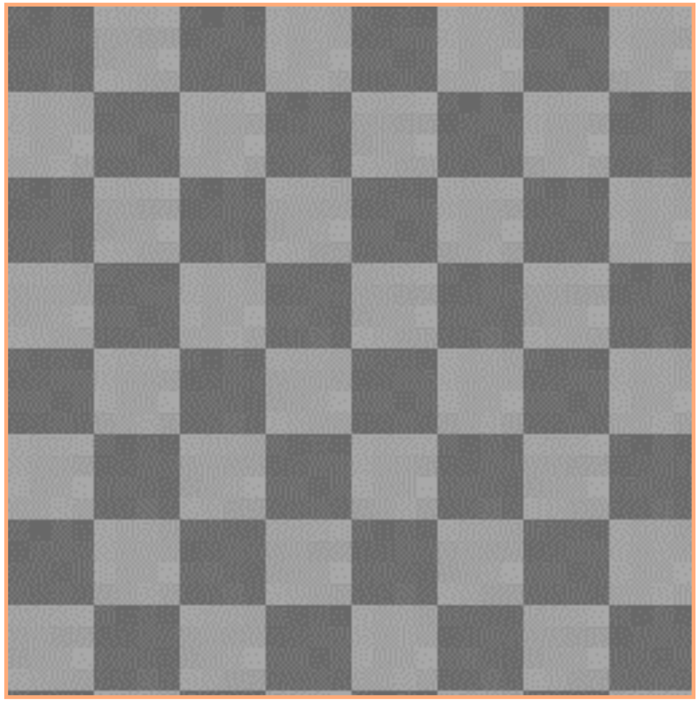
- σ, joint entropy, and RMS- error vs. number of iterations

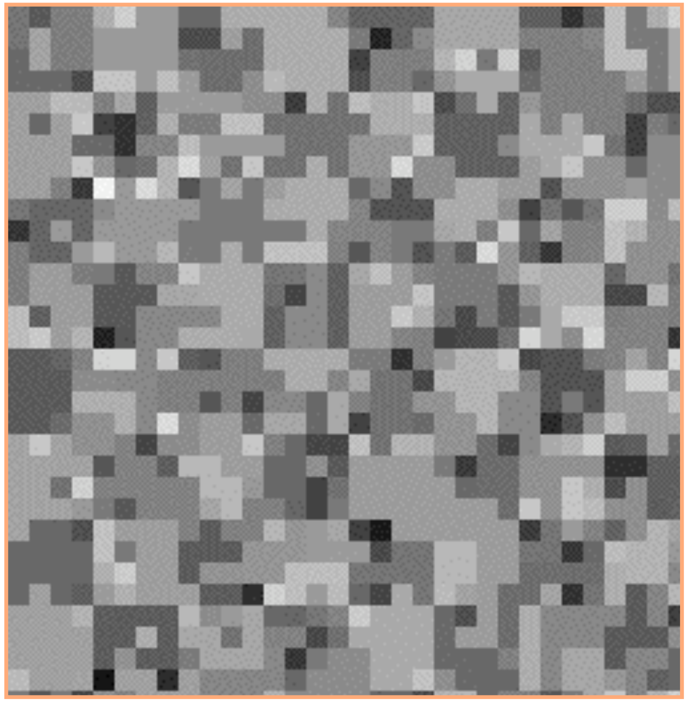




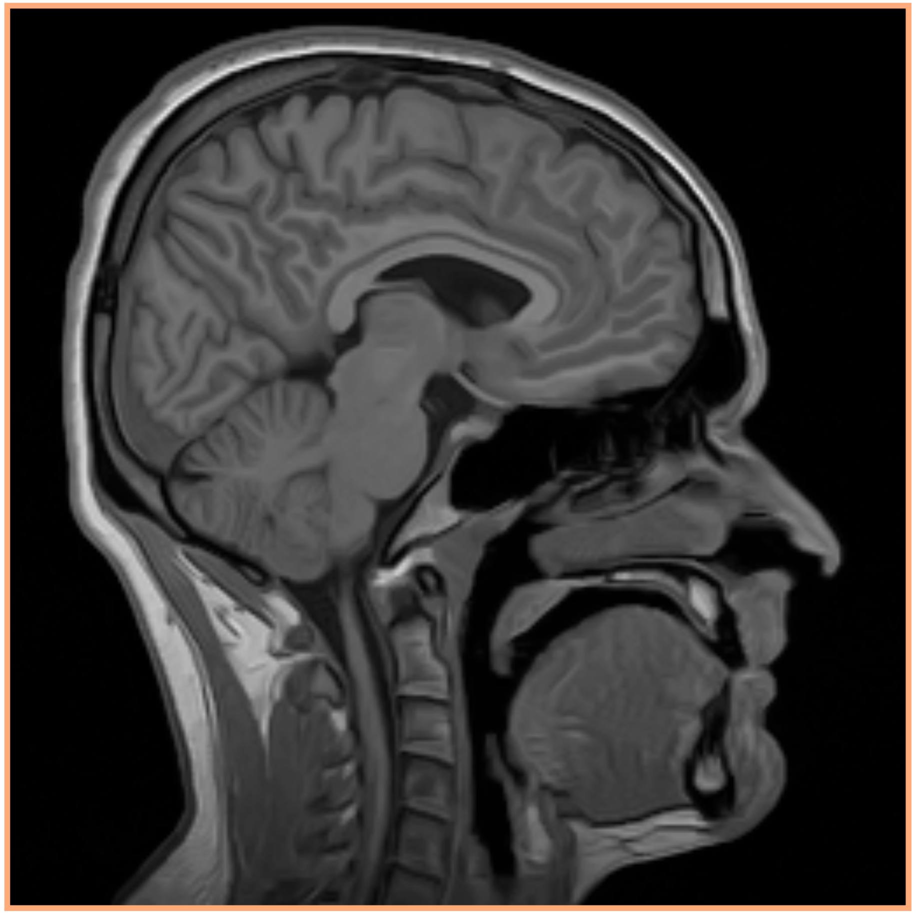


Vs Perona Malik





MRI Head



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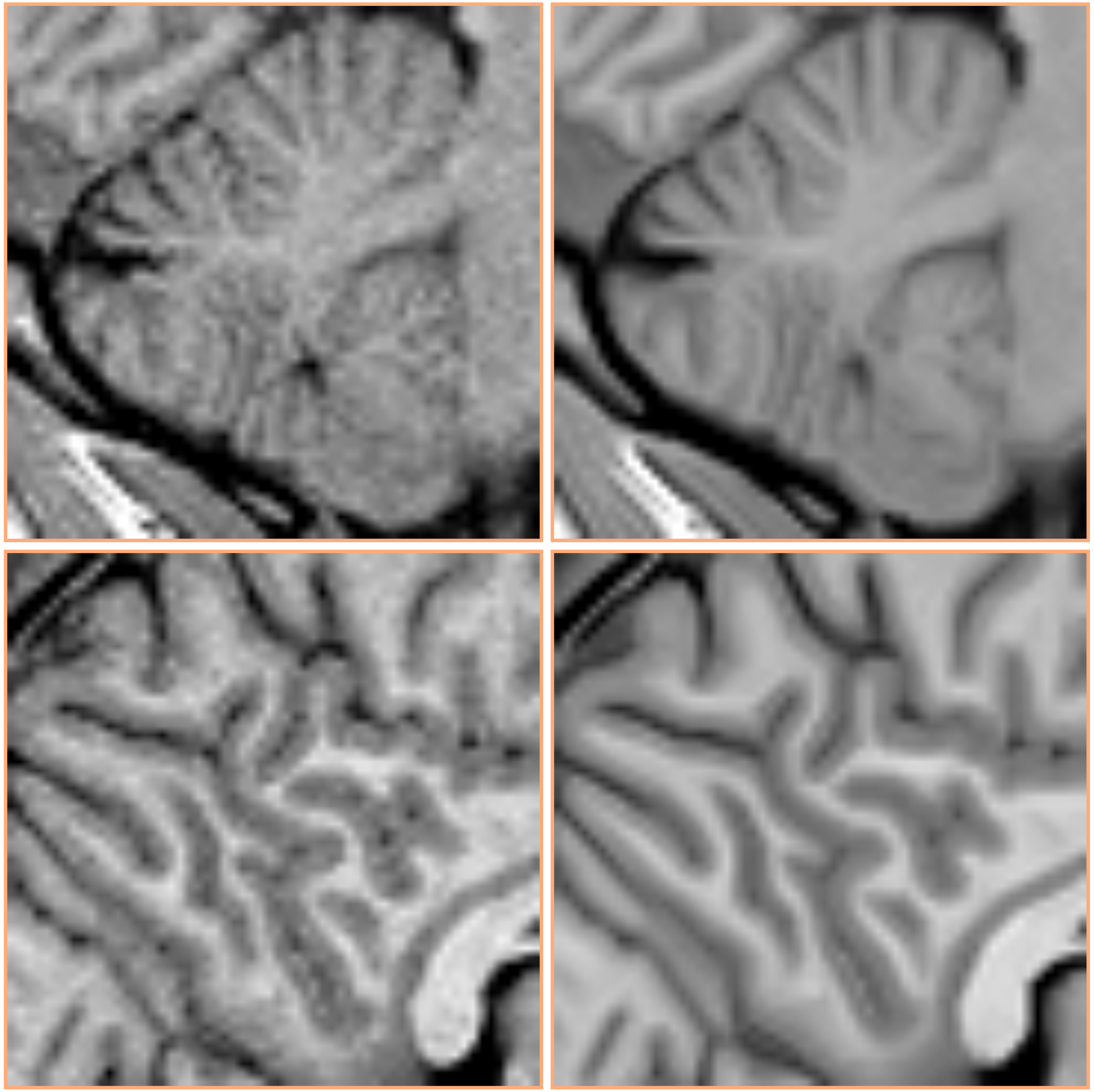
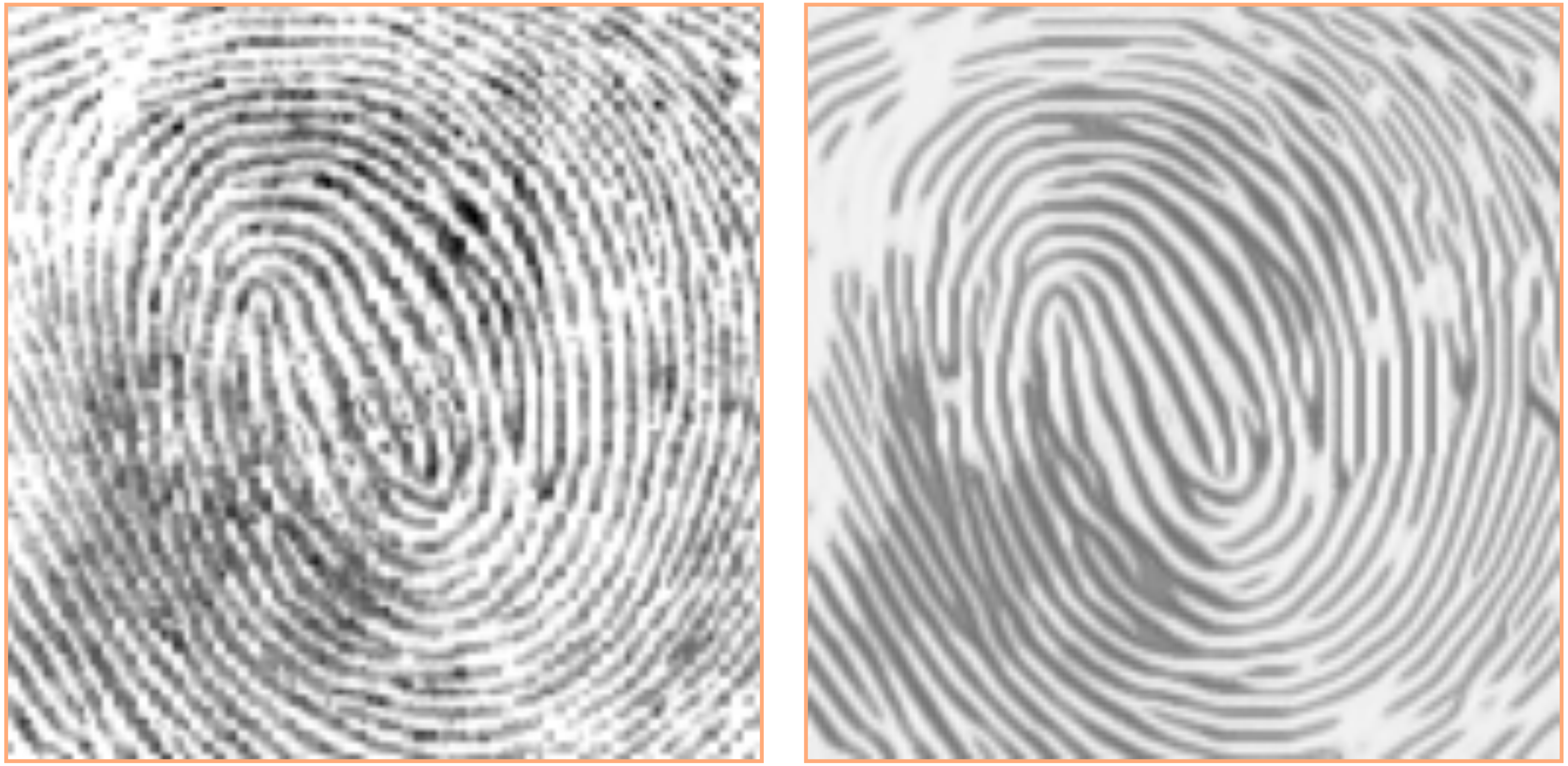
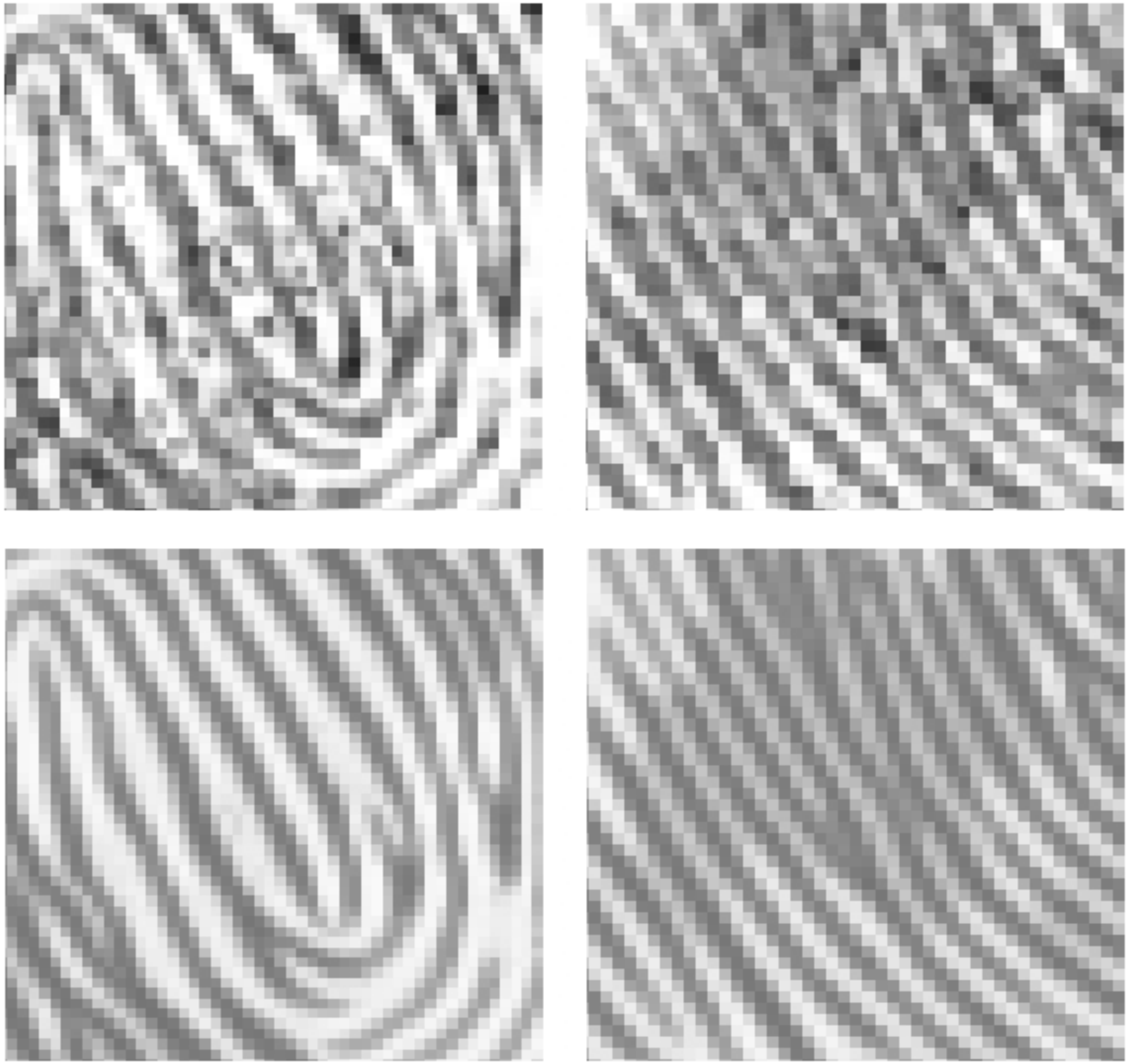
MRI Head



Fingerprint

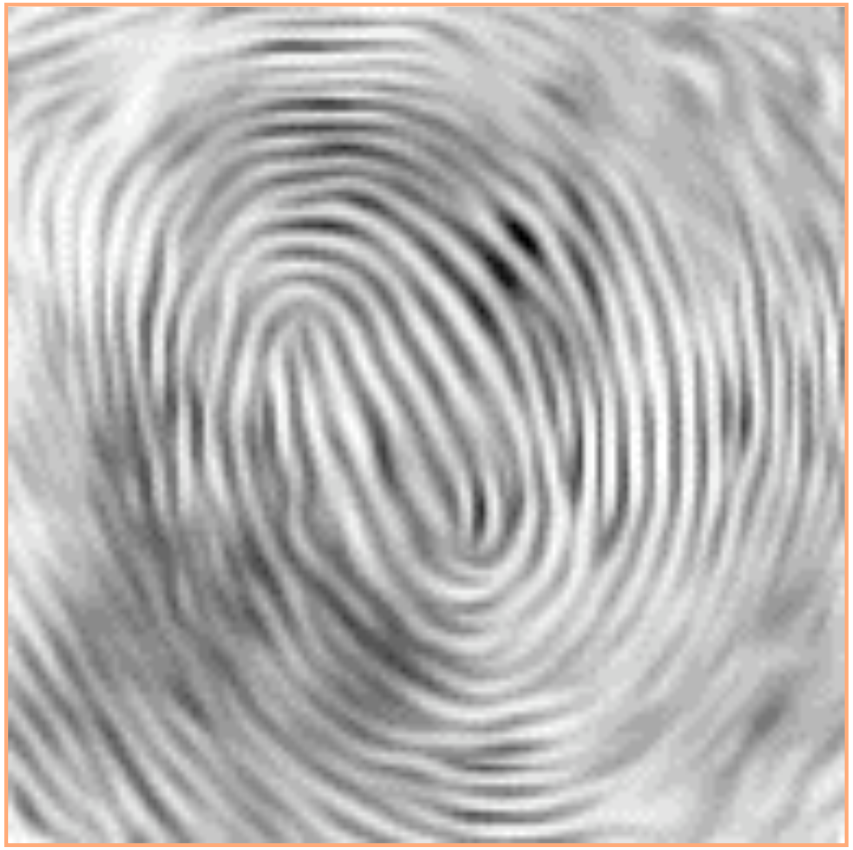


Fingerprint



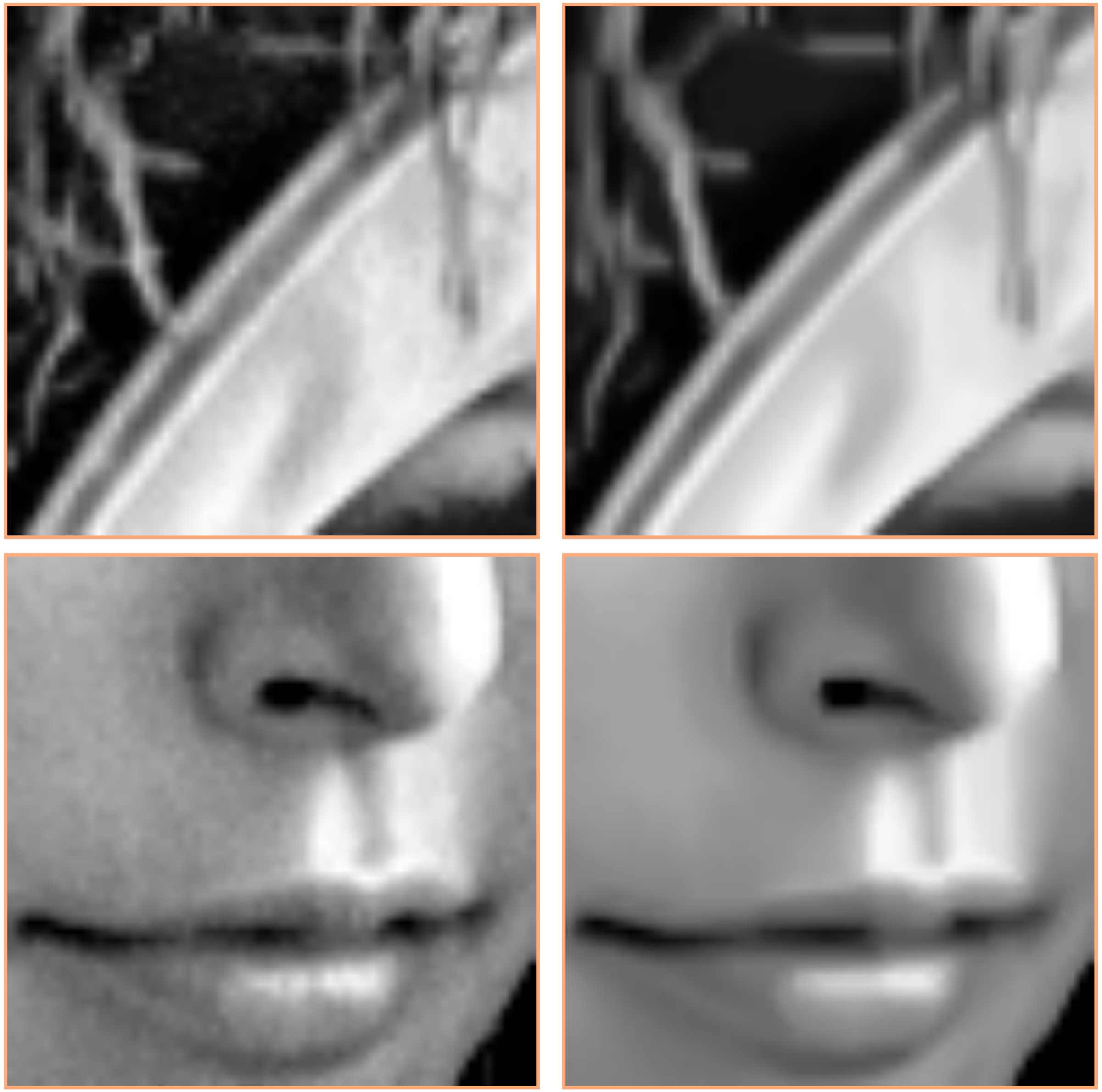
Vs Perona Malik

Vs Coherence Enhancing

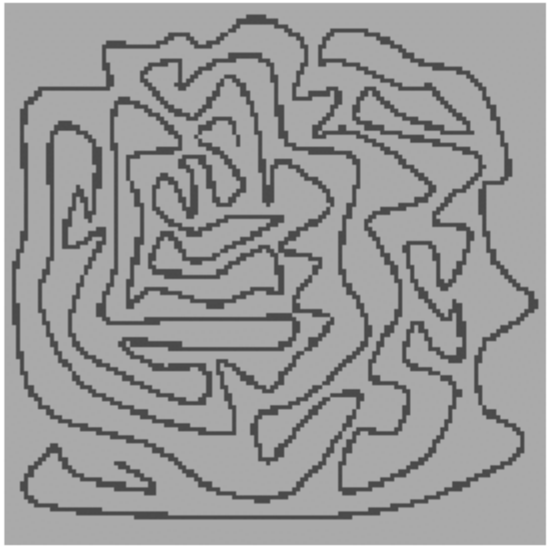


Lena

Lena



Results



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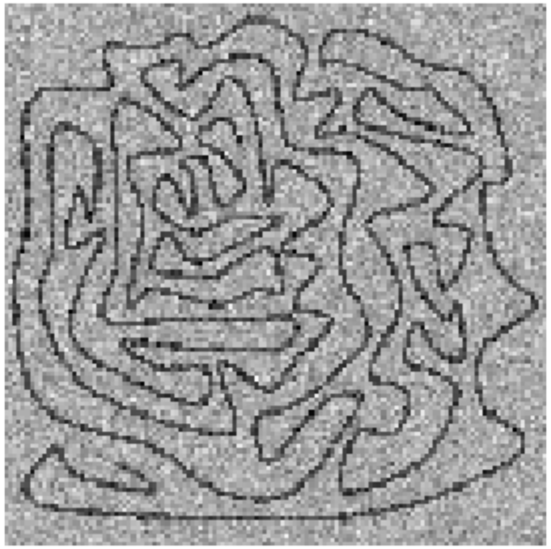
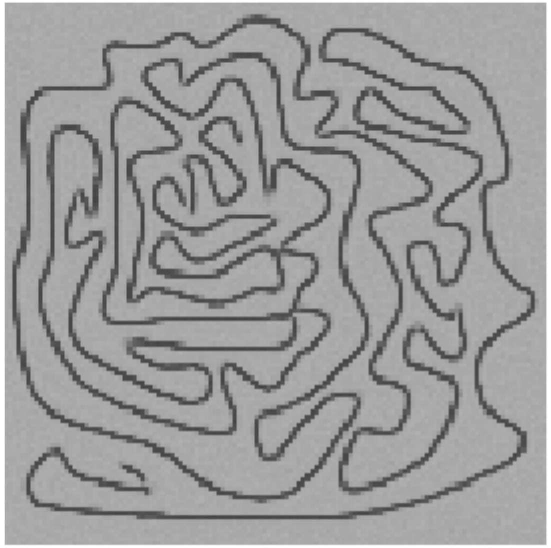


Noisy



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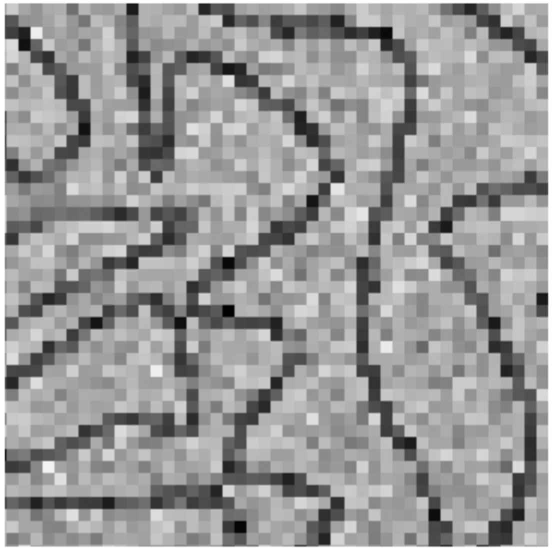
Results

Original



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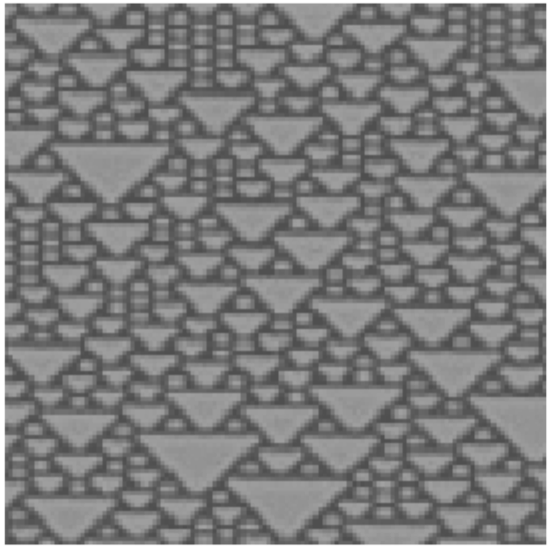
Results



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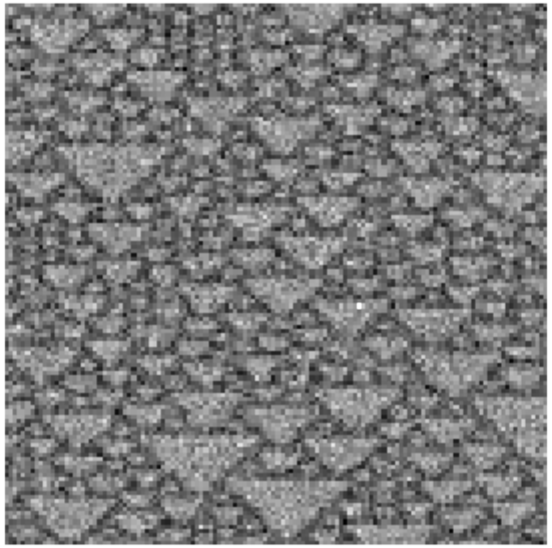

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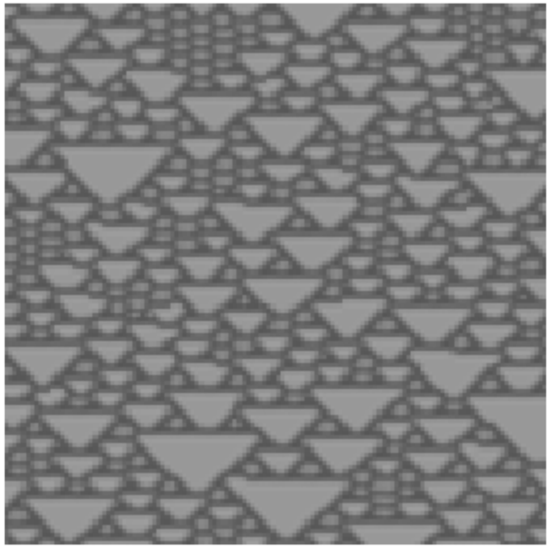

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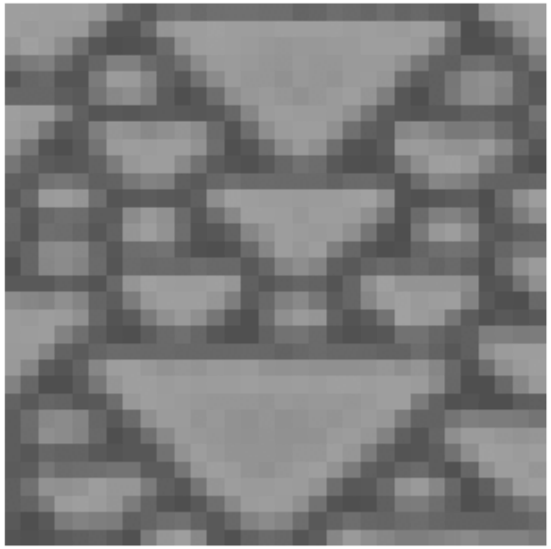
Fractal



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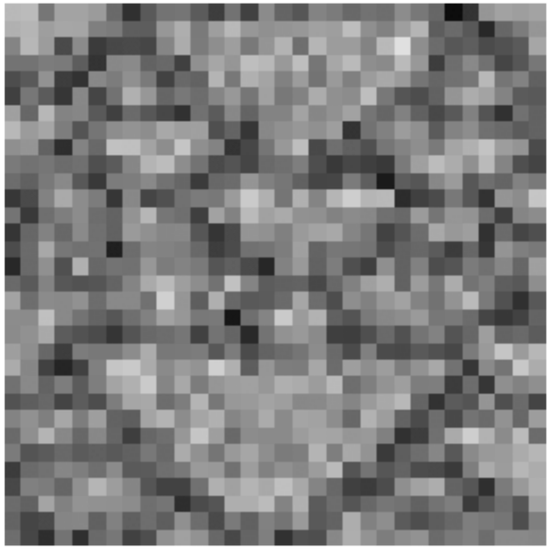

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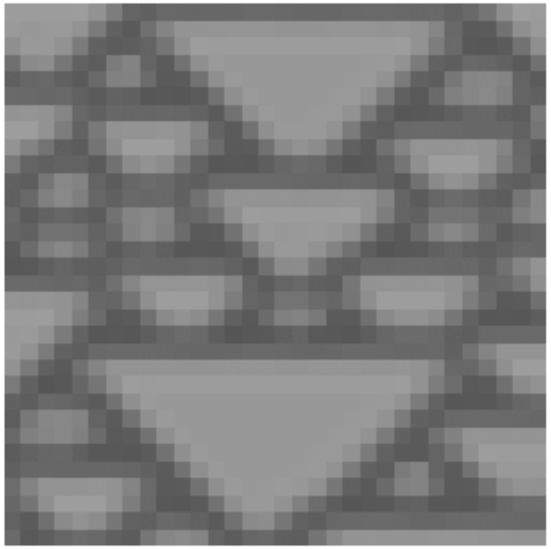

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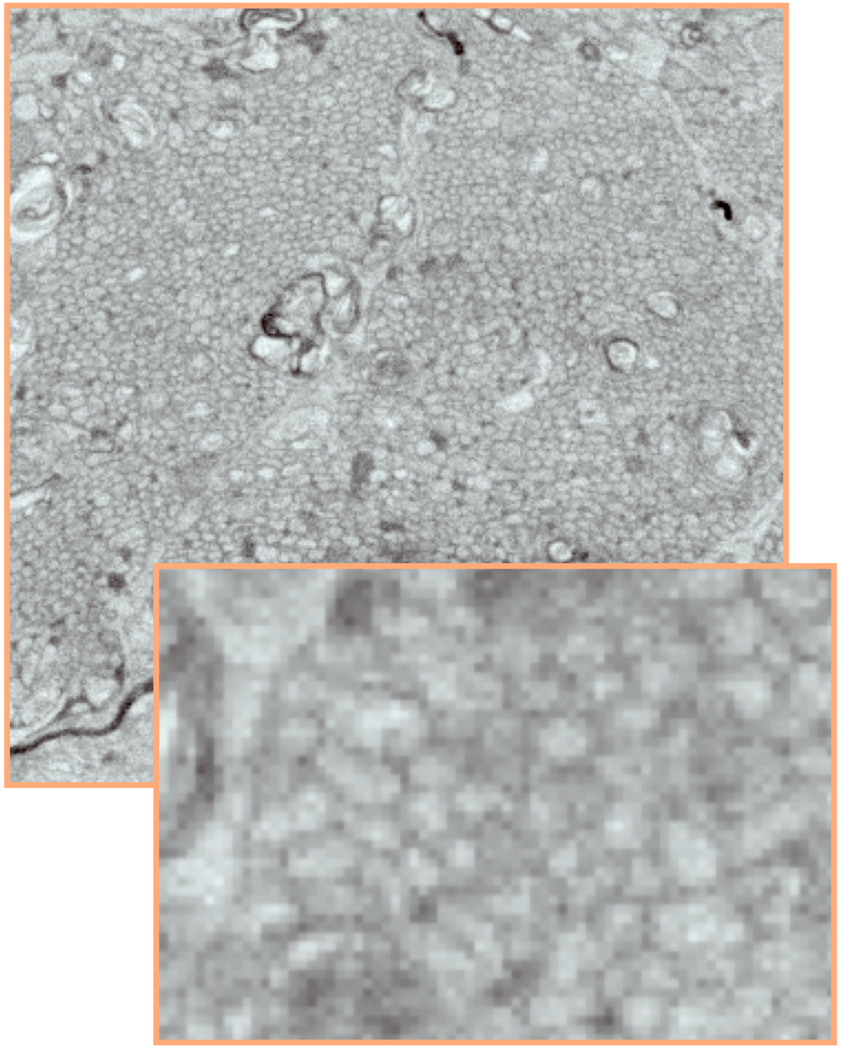
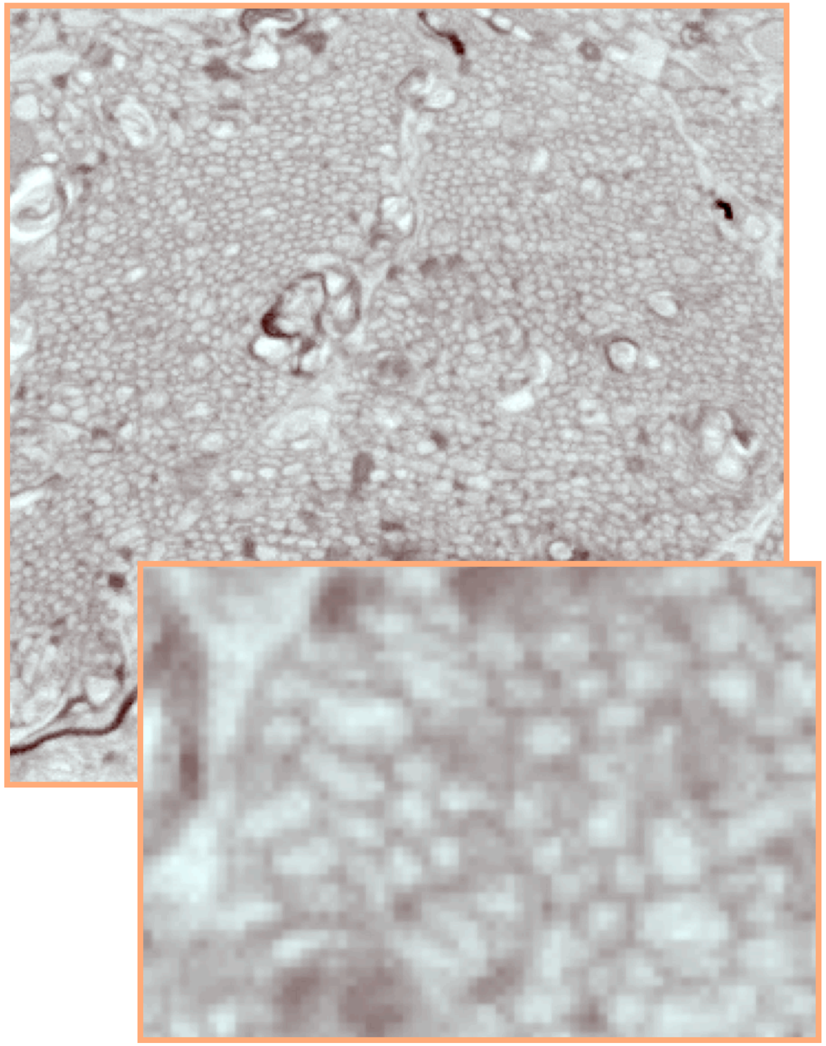
Microscopy

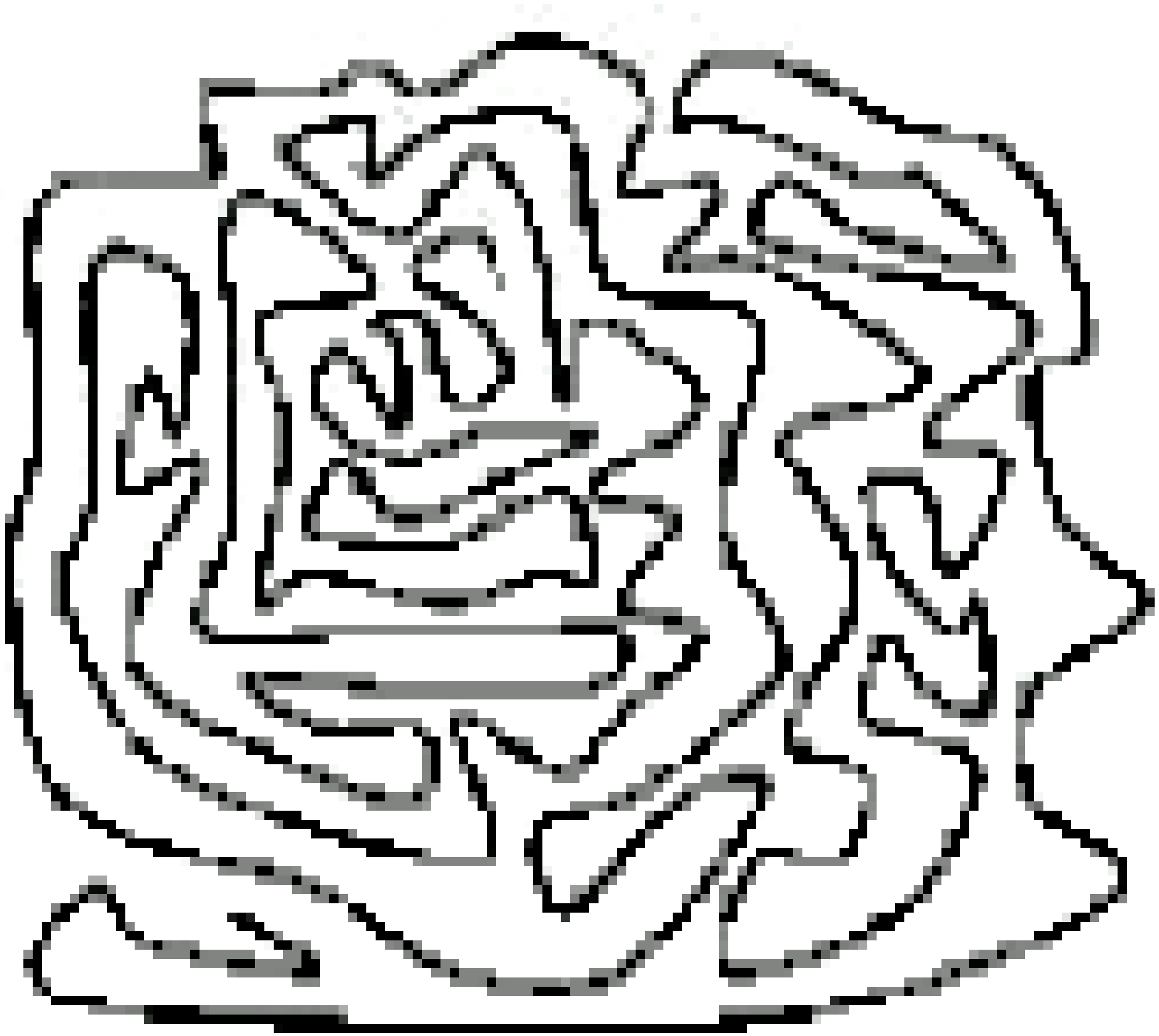




Quantitative Results

- Generalizes well
  - Relatively insensitive to a few parameters (e.g. nhd size)
- Compares favorably with s.o.t.a. wavelet denoisers
  - Close but worse for standard images (photographs)
  - Better for less typical images (defy wavelet shrinkage assumptions)
- Spectral data -> gets even better

Entropy Scale Space?

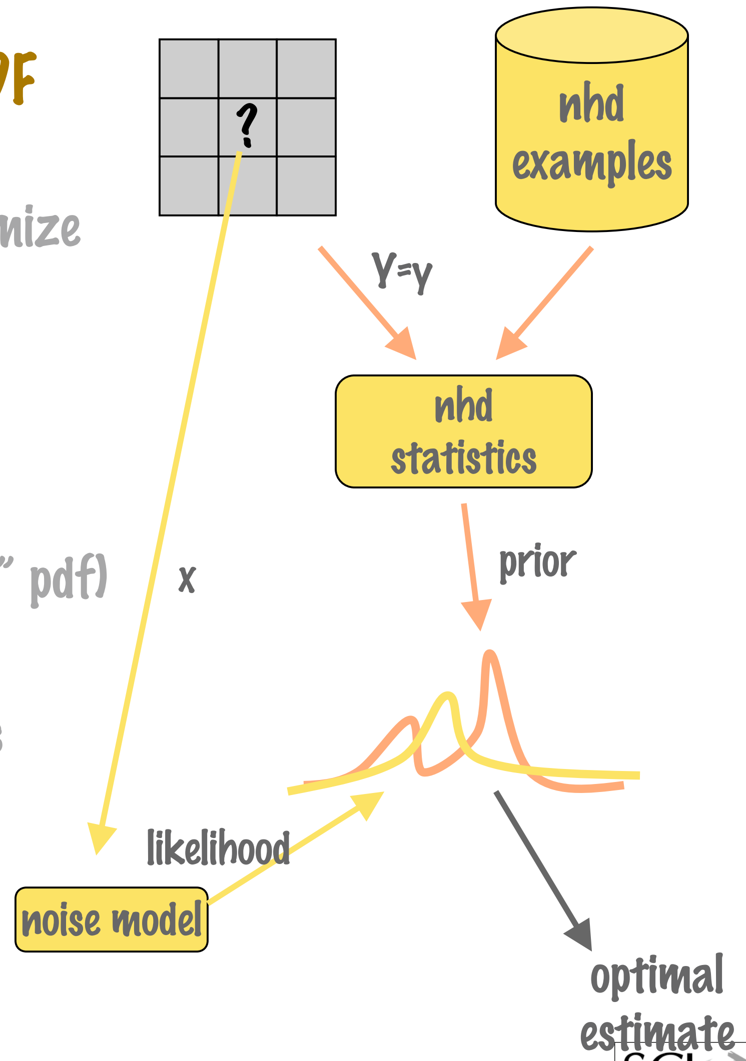


Other Applications

- Optimal reconstruction
  - Noise model
  - Awate&Whitaker, IPMI, 2005
- MRI head segmentation
  - Iterative tissue classification
  - Tasdizen et al., MICCAI, 2005
- Texture segmentation
  - Awate, Tasdizen, Whitaker, ECCV 2005

Optimal Reconstruction

- What if we had a noise model and a PDF conditioned on image nhds?
  - -> "Optimal" estimate for each pixel (minimize expected error)
- Image statistics (each nhd forms a "lookup")
  - Database of "perfect" image nhds
  - Bootstrap from the noisy images ("denoise" pdf)
- Noisy neighborhoods
  - Iterate on sequence of improving estimates



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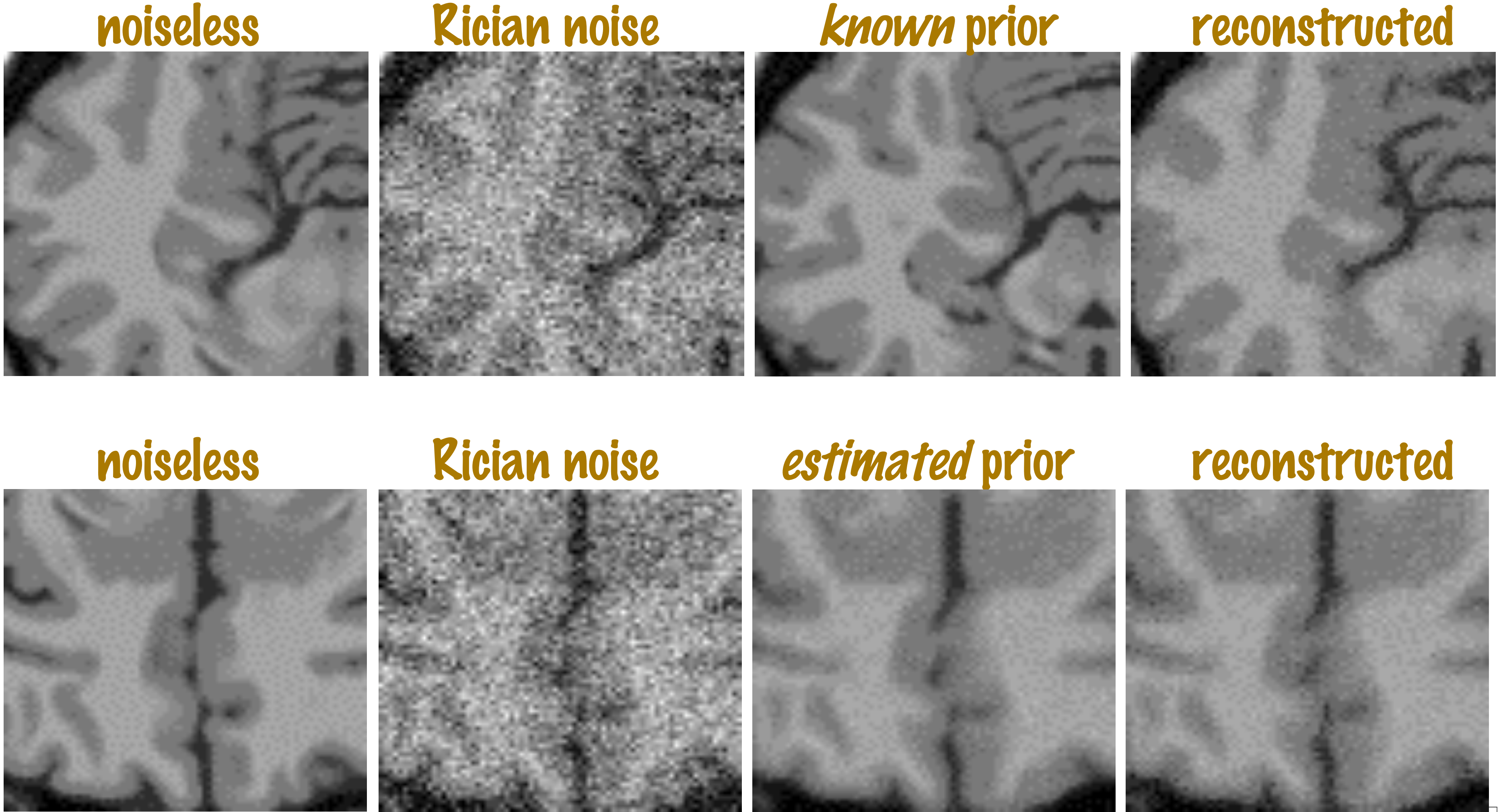
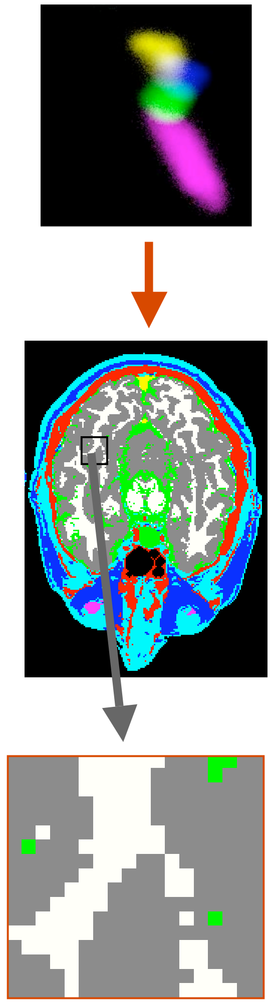
Optimal Estimation (MRI)



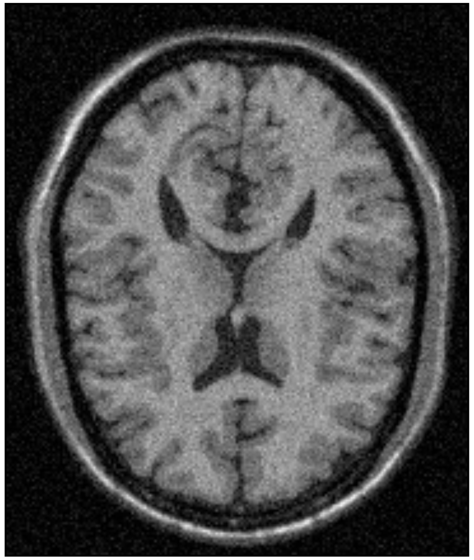
MRI Tissue Classification

- Classify pixels based on spectral (multi-modal) MRI measurements
- Overlapping (noisy) clusters
  - ambiguity and misclassified pixels
- Use spatial data to influence decision
  - Large-scale (absolute) relationships <-> statistical atlases
  - Local (nhd) relationships <-> Markov random fields (smooth configurations)
- Idea: *learn* nhd relationships from data
  - Classify (iterative) to reduce in-class nhd entropy

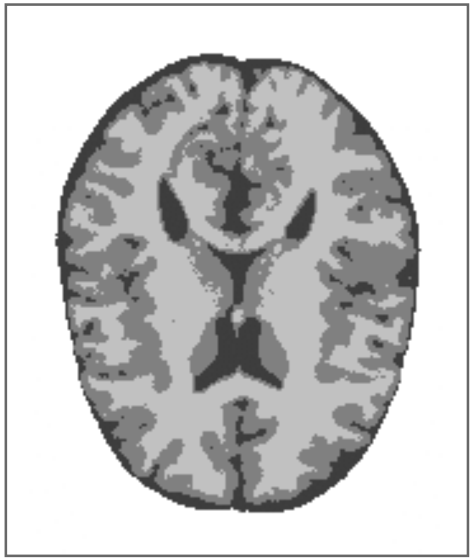


MRI Tissue Classification

- Algorithm: 1) initialize with atlas, 2) iteratively relabel to reduce tissue-wise nhd entropy

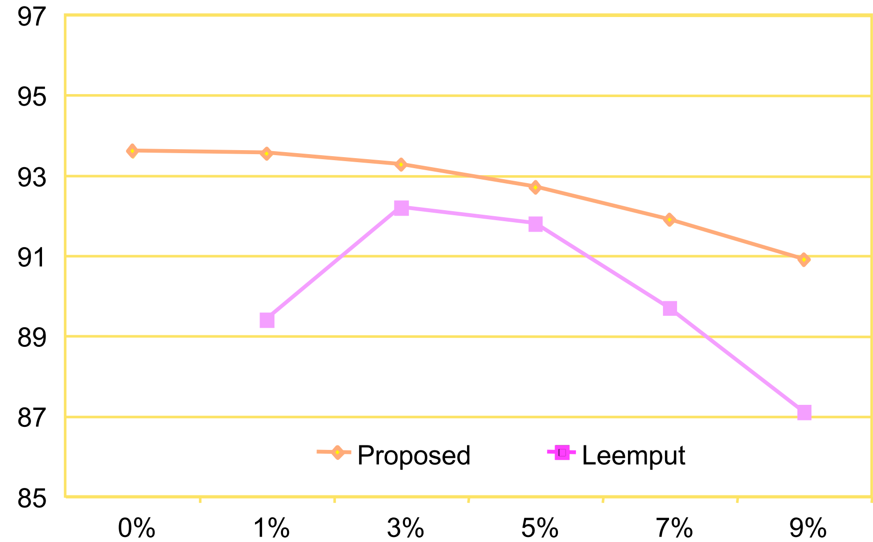


MRI Input



GM, WM, CSF Seg.

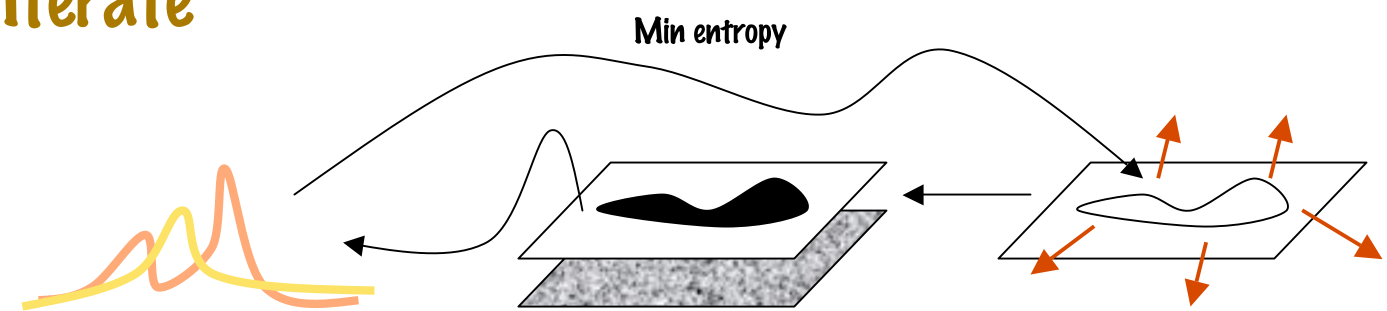
GM Classification Performance vs Noise Level



Comparison: SOTA-EM w/MRFs & Atlas (Leemput et al.)

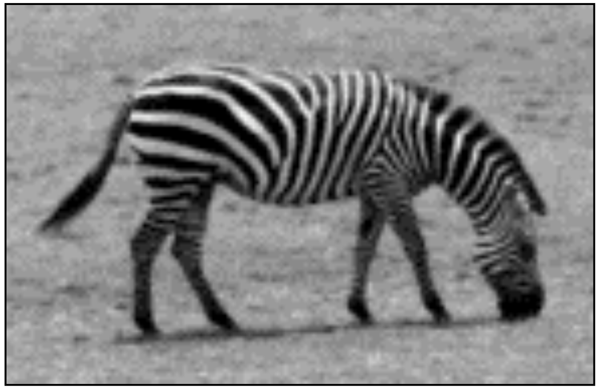
Texture Segmentation

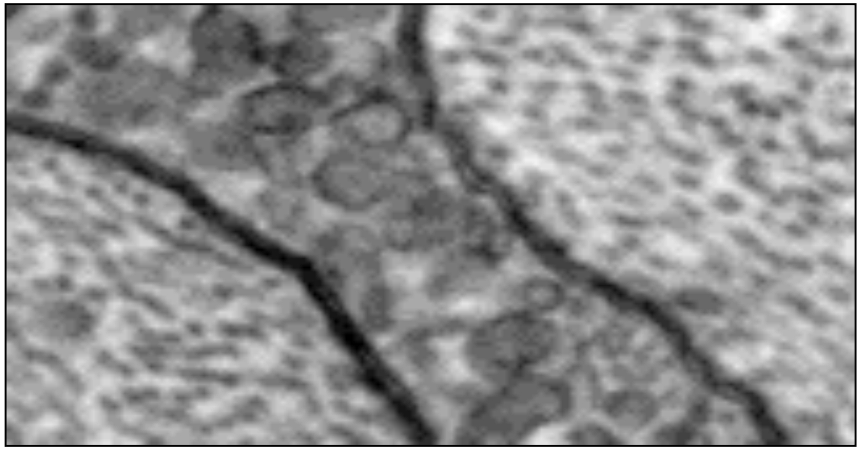
- Reassign class labels to reduce in-class entropy
  - Deformable model to keep spatial coherence
- Recompute pdfs from new class labels
  - Random samples + nonparametric nhd statistics
- Iterate



Texture Segmentation
Awate et al., 2005

- Initialization -> checkerboard
- Deformable model -> level sets (Tsai and Seglmi, 2004)





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Thoughts