Taylor series

$$f(x+dx,\, y+dy,\, t+dt) = f(x,y,t) + \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial t}dt + \dots$$

$$f(x+dx,\, y+dy,\, t+dt) = f(x,y,t)$$

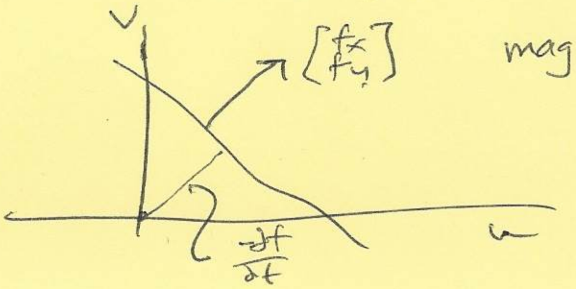
$$\Rightarrow \quad -\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x}\underbrace{\frac{dx}{dt}}_{=u} + \frac{\partial f}{\partial y}\underbrace{\frac{dy}{dt}}_{=v}$$

$$-\frac{\partial f}{\partial t} = \nabla f \cdot \bar{u}$$

$$f_x = \frac{\partial f}{\partial x} \qquad f_y = \frac{\partial f}{\partial y}$$

$$\Rightarrow \quad f_x u + f_y v + \frac{\partial f}{\partial t} = 0$$ equ of line

with normal $[f_x ; f_y]$ + dist $-\frac{\partial f}{\partial t}$ from origin



mag of velocity is $\dfrac{-f_t}{[f_x^2 + f_y^2]^{1/2}}$

minimize

$$E^2(x,y) = (f_x u + f_y v + f_t)^2 + \lambda(u_x^2 + u_y^2 + v_x^2 + v_y^2)$$

differentiate wrt u + v

write $\nabla^2 u$ as $u - u_{av}$ $\nabla^2 v$ as $v - v_{av}$

$$(\lambda^2 + f_x^2)u + f_x f_y v = \lambda^2 u_{av} - f_x f_t$$

$$f_x f_y u + (\lambda^2 + f_y^2)v = \lambda^2 v_{av} - f_y f_t$$

$$u = u_{av} - f_x \frac{P}{D} \qquad v = v_{av} - f_y \frac{P}{D}$$

where $P = f_x u_{av} + f_y v_{av} + f_t$

$$D = \lambda^2 + f_x^2 + f_y^2$$

$k = 0$; error = Inf initialize all u^k + $v^k = 0$

<u>while</u> error > threshold

$$u^k = u_{av}^{k-1} - f_x \frac{P}{D}$$

$$v^k = v_{av}^{k-1} - f_y \frac{P}{D}$$

$t = 0$ initialize $u(x,y,0)$, $v(x,y,0)$

<u>for</u> $t = 1$ <u>to</u> max frames

$$u(x,y,t) = u_{av}(x,y,t-1) - f_x \frac{P}{D}$$

$$v(x,y,t) = v_{av}(x,y,t-1) - f_y \frac{P}{D}$$