CS7960 L19 : MapReduce I triangle count

MapReduce
M = Massive Data

Mapper(M) -> \{(key,value)\}
Shuffle(\{(key,value)\}) -> group by "key"
Reducer (\{"key,value_i\}) -> ("key, f(value_i))
Can repeat, constant \# of rounds

Given graph $\mathrm{G}=(\mathrm{V}, \mathrm{E})$
Assume $|V|=n$ and $|E|=m=n \wedge\{1+c\}$
typical large graphs have c in [0.08, 0.5]
$N(v)=$ neighbors of $v$
cluster coefficient cc(V)
= fraction $N(v)$, neighbors themselves
How dense a subgraph is
** need to find all triangles for each v in $\mathrm{V}^{* *}$
(sequential)
for each $v$ in $V$
for each ( $u, w$ ) in $N(v)$
if (u,w) in E -> Triangle[v]++
$\mathrm{T}=\operatorname{sum}_{-}\{\mathrm{v}$ in V$\}|N(\mathrm{v})| \wedge 2$
$O\left(n^{\wedge} 2\right)$ if some $v N(v)=O(n)$
(parallel)

Map 1: $G=(V, E)$-> $(v, u),(u, v)$ for $(v, u)$ in $E$
Reduce 1: (v, $N(v)$ ) -> ((u,w),v) s.t. u,w in $N(v)$
Map 2: -> $((u, w), v)$ (output of R1)

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-> ((u,w),$) for (u,w) in E
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Reduce 2: ((u,w),\{v1,v2,v3,...vt,\$?\}
iff \$, then -> (vi,1/3)
Map 3: identity
Red 3: aggregate
:( running time still max_\{v in V$\}|\mathrm{N}(\mathrm{v})| \wedge 2$

LiveJournal
80\% reducers done in 5 min
99\% reducers done in 35 min
some 60 minutes

Idea 1: count each triangle once, with lowest degree
(sequential)
for each $v$ in $V$
for each ( $u, w$ ) in $N(v)$
if $\operatorname{deg}(u)>\operatorname{deg}(v) \& \& \operatorname{deg}(w)>\operatorname{deg}(v)$ if (u,w) in E -> \{Tri[v]++,Tri[u]++,Tri[w]++\}

In Reduce 1, add if condition.
In Reduce 2, -> (vi,1)
-> $(u, t),(w, t)$
Works better!
two types of nodes:
$L=\{v \mid N(v)<=\operatorname{sqrt}\{m\}\}$
$H=\{v \mid N(v)>\operatorname{sqrt}\{m\}\}$
|LI <= n -> produce $0(\mathrm{~m})$ paths
$|\mathrm{H}|<=2$ sqrt $\{\mathrm{m}\}$-> produce $0(\mathrm{~m})$ paths
if $m=0\left(n^{\wedge 2)}\right.$ (very dense)
n ~ sqrt\{m\}
-> $0(m \wedge\{3 / 2\})$ work (optimal!)

Idea 2 : Graph Split
partition $V$ into p equal-size sets \{V1,V2,...,Vp\}
For triples (Vi,Vj,Vk) -> subgraph G_\{ijk\} = G[Vi + Vj + Vk]
computer triangles on G_\{ijk\}
triangles counted $\left\{1, p-2\right.$, or $\left.p^{\wedge} 2\right\}$ times
figure out and adjust
subgraph has $0\left(m / p^{\wedge} 2\right)$ edges in expectation
work: $p^{\wedge} 3 * O((m / p \wedge 2) \wedge\{3 / 2\})=0(m \wedge\{3 / 2\})$
p about 20 worked best on LiveJournal graph

