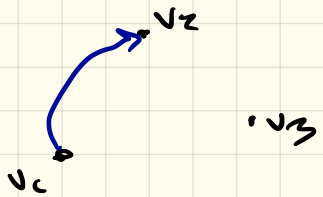
L25 - Communities + Influencers in Graphs.

Input $G = (V, E)$

social network

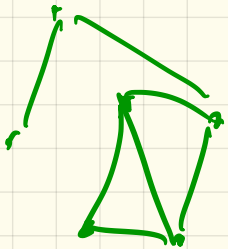


V = people (entities)

E = connections

Large Graphs

$|V|$ = (billion (facebook) further



generally (for public) not fully available.

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20 15 years ago

- Math World

Erdös-Renyi Model

$G(n,p)$ = n nodes each edge (i,j) exist w.p. p.

Not real world models

- Sociology

  - Karate Club ~30 nodes
  - High School $|V| = 200$

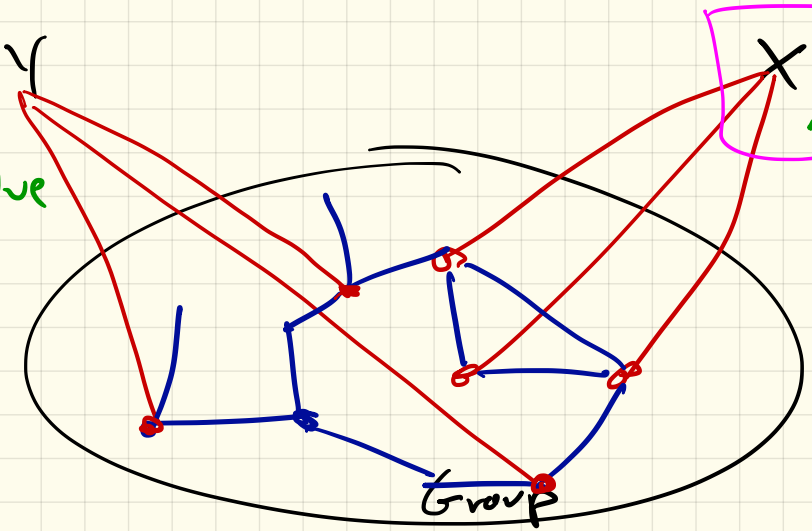
+ usually draw

+ small

clique

Why Do People Join Groups?

3 friends in group



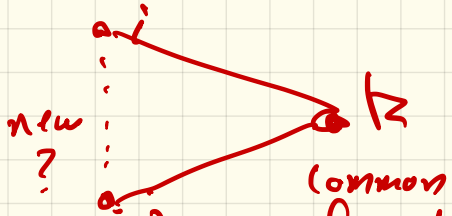
- broader perspective
- independent support
- gets to join close group.
- clique is more persuasive

Preferential Attachment

- new edges, more likely to form triangles than random.

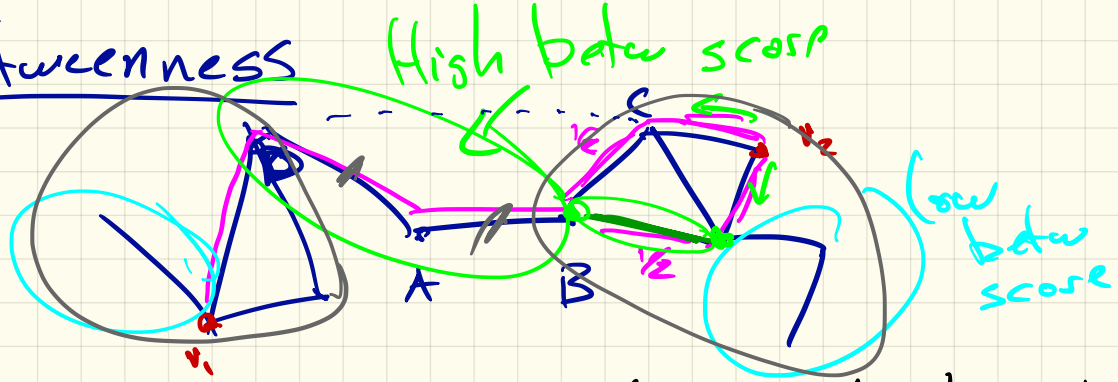
not Erdös-Renyi

If node i has
few triangles
↳ more likely
depressed



if too few. o_
too many triangles.
is Bot?

Betweenness



Betw(A,B) = fraction of shortest paths that use edge (A,B)

all pairs of $v_i, v_c \in V$

How important is edge (A,B)?

Can define Betw(v_i)

Remove high btw edges.

→ Remaining connected comp. communities

Communities are not partitions

→ tightly connected subset of graph.

→ can overlap

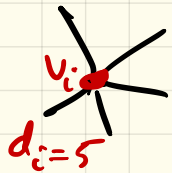
→ set does not need to partition V.

Modularity defined on subset $C \subset V$

$$Q(C) = \left(\text{fraction edges in } C\right) - \left(\text{Exp frac edges in group}\right)$$

$$\frac{1}{\cdots} \sum_{j \in C} \sum_{i \in C} A_{ij} \qquad \frac{1}{\cdots} \sum_{i \in C} \sum_{j \in C} E_{ij}$$

- Adjacency matrix $A_{ij} = \begin{cases} 1 & \text{if edge} \\ 0 & \text{o.w.} \end{cases}$
- Expected matrix $E_{ij} = \dfrac{d_i \cdot d_j}{2|E|}$



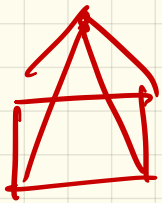
d_i = degree v_i $\equiv$ # edges which include v_i

$$Q(C) = \frac{1}{4|E|} \left[\sum_{j \in C} \sum_{i \in C} \left(A_{ij} - E_{ij} \right) \right]$$

regularizer

Clique in Graph $G=(V,E)$

motifs subset $S \subset V$



when all $v_i, v_j \in S$ have edge $(v_i, v_j) \in E$

Alg. to find all motifs like Apriori Alg.

① Find all edges in G

② Search  to check it triangle

③ Search  to check 4-clique