

Homework 2: Total Probability, Independence, and Bayes' Rule

Instructions: Your answers are due at the beginning of class on the due date. You can either turn in a paper copy, or a pdf version through canvas. We highly recommend using latex (<http://www.cs.utah.edu/~jeffp/teaching/latex/>) for producing the assignment answers. If the answers are too hard to read (e.g. **do not create pdf using your phone's camera!**) you will lose points (entire questions may be given 0).

Please make sure your name appears at the top of the page.

You may discuss the concepts with your classmates, but write up the answers entirely on your own. **Be sure to show all the work involved in deriving your answers! If you just give a final answer without explanation, you may not receive credit for that question.**

1. Independence: **Show the formula you use to check these!**

- (a) You roll a six-sided die. Are the following two events independent?

$A =$ “the number is less than 5”,

$B =$ “the number is even”.

- (b) You flip two coins. Are the following two events independent?

$A =$ “you flipped repeats (HH or TT)”,

$B =$ “you flipped at least one tails”.

2. A doctor sees a patient who may have a particular heart disease. According to the patient's family history and other risk factors, the doctor decides there is a 25% chance that the patient has the disease. The doctor then takes a blood test that turns out positive for the disease. However, there is a 5% chance that the test is positive when a patient does *not* have the disease. When a patient does have the disease, there is an 90% chance that the test will be positive. What is the probability that the patient has the disease given the positive test?
Hint: Think about what the two events are, and use Bayes' Rule.
3. You have a bucket of 8 red balls and 12 green balls. I remove one ball from the bucket. I make a note of the color of the ball, but do not tell you. Next, I replace the ball along with 4 new balls with the same color as the ball that I selected. After all of this, you select a ball from the bucket.
 - (a) Draw a probability tree diagram, where the first level is the color I select, and the second level is the color that you select. **Be sure to give all branches of the tree and their probabilities, as well as the joint probabilities at the end of the branches!**
 - (b) Using your diagram, what is the probability that the ball you selected is red?
 - (c) Consider the two events $A =$ “I pick green” and $B =$ “you pick red”. Are these events independent? (Don't just answer yes or no, show your work!)
 - (d) If you pick a red ball, what is the probability that I picked a red ball?
4. Exercise 3.18 from the book.